Banks' Net Interest Margin and Changes in the Term Structure*

Christoph Memmel[†] and Lotta Heckmann-Draisbach[‡]

June 24, 2022

Abstract

We model and analyze the impact of an interest rate shock on a bank's net interest margin, where this shock consists not only of a level shift, but also of a change in the steepness of the term structure. The outcome of our parsimonious, yet comprehensive model for a bank's interest business is broadly in line with the results of a quantitative survey among German small and medium-sized banks. In addition, we find that using only two parameters, the level and the steepness, is sufficient to describe the term structure of interest rates well .

Keywords: Net Interest Margin, Term Structure

JEL classification: G 21

---

*We thank Arved Fenner and the participants of the 38th International Conference of the French Finance Association (Saint-Malo, 2022). The views expressed in this paper are those of the author and do not necessarily reflect the opinions of the Deutsche Bundesbank.

†E-mail: christoph.memmel@bundesbank.de; phone: +49 (0) 69 9566 4421; Deutsche Bundesbank, Wilhelm-Epstein-Straße 14, 60431 Frankfurt.

‡E-mail: lotta.heckmann-draisbach@bundesbank.de; phone: +49 (0) 69 9566 6502; Deutsche Bundesbank, Wilhelm-Epstein-Straße 14, 60431 Frankfurt.

1 Introduction

The impact of changing interest rates on banks' interest business is of central importance for customers and investors on the one hand, and for policy makers and supervisors on the other hand. Especially during the long low-interest rate environment, it was and still is key to understand how a significant change in the term structure would affect bank interest rates and banks' net interest margin. When analysing interest rates, the focus often lies on the interest level, not on other characteristics of the term structure. As an approximation, this is empirically justified as changes in the interest level account for about 90% of the variances of the changes in the interest rates. Nevertheless, changes in the steepness of the term structure make up about 10 percent of the variance of interest rate changes. Therefore, to gain a fairly complete picture of the impact of term structure movements, it is advisable to take account of not only changes in the level, but of changes in the steepness of the term structure as well.

We set out to model the relationship between a bank's net interest income and the term structure of interest rates, and validate our model with data on the German banking sector. We think that this sample is particulary relevant, as net interest income is by far the largest source of income for German banks, not only for small and medium-sized banks, for instance for saving banks and credit cooperatives, but also for large banks.[1] Therefore, net interest income and the corresponding term structure are important to assess banks' profitability. However, our model is quite general and could also be applied to other jurisdictions.

Considering changes in the steepness of the term structure seems especially important in the context of banks' interest business, because a substantial part of their net interest income comes from making use of the usually positive steepness of the term structure, in other words, banks tend to grant long-term loans and finance these operations using short-term deposits, thereby benefiting from the usually higher interest rates for longer maturities. It is known that German banks are much engaged in this term transformation; Memmel (2011) and Busch and Memmel (2016) find that the contribution it makes to German banks' net interest income strongly depends on the time period under consideration and estimate that this contribution can account for up to around one-third of German banks' net interest income.

In this paper, we look at the impact of changes in the level and in the steepness of the term structure on banks' interest margins. We do this with the help of passive investment strategies to mimic a bank's interest business, where these investment strategies consist in investing in risk-free par-yield bonds of a certain maturity. These passive investment strategies are capable of incorporating changes in the level and in the steepness of the term structure. We check the empirical fit of the modeling with the results of a quantitative survey among small and medium-sized banks in Germany, known as the low-interest rate environment survey (LIRES). This survey data is especially suitable for this purpose, as it not only includes stress scenarios consisting of changes in the interest level, but also a scenario involving a change in steepness, more precisely a flattening of the term structure. Moreover, other effects that could have an impact on banks' net interest margin can be eliminated from the survey data.

In our analysis, we find that banks' interest business can be approximated by a portfolio of these passive investment strategies in bonds. We derive this conclusion from three observations: (i) A portfolio of these trading strategies provides a good theoretical fit to the concept of a continuing banking business model. (ii) A portfolio of these trading strategies explains more of the dynamics in banks' net interest margins than other sensible strategies. (iii) The results of the quantitative LIRES survey can be reasonably well explained by a portfolio of these trading strategy. In addition, we find that the term structure of interest

[1] In 2020, the share of net interest income with respect to German banks' operating profits was 67.3%; for savings banks and credit cooperatives, this share was 70.5% and 72.3%, for the large banks still 54.3%. See Deutsche Bundesbank (2021)

rates (in Germany) can be reasonably well described by just two parameters, namely its level and its steepness. This holds true for a period of nearly fifty years, including the low-interest rate environment.

This paper has two main contributions: first, the parsimonious modeling of banks' interest business while still reproducing empirical features of banks' interest business and, second, the analysis of the German term structure.

As to the first contribution, Memmel (2008) models the banks' interest business as a portfolio of many different bond portfolios. In this paper, we model the banks' interest business with one portfolio on the asset side and one portfolio on the liability side. The restriction to two portfolios makes the model more parsimonious, but still capable of reproducing some empirical features of banks' interest business, such as the incomplete pass-through to bank rates, term transformation and market power. Moreover, the restriction to two portfolios yields expressions for the change in the net interest margin that can be directly compared to supervisory reporting data.

It is empirically widely found that changes in interest level are positively correlated with banks' net interest margins (see, for instance, Albertazzi and Gambacorta (2009), Oesterreichische Nationalbank (2013) and Claessens et al. (2018)), i.e. an increase in the interest level is associated with higher net interest margins. This is especially true in the low-interest rate environment. The effects of the steepness of the term structure on banks are rarely investigated. One such study is carried out by English (2002) who analyzes the effect of the steepness of the term structure on banks' net interest margins, and he gets mixed results. Our parsimonious model is able to reproduce these empirical features.

We do not deal with changes in other determinants of banks' net interest margins, such as credit risk, credit standards or the exposure to interest rate risk. In the literature (see, for instance, Ho and Saunders (1981), McShane and Sharpe (1985), and Entrop et al. (2015)), their impact on the net interest margin is well documented, however we concentrate on changes in the term structure and assume the other determinants to be time-constant, knowing that earnings from term transformation and earnings from other determinants may be interrelated (see Chaudron et al. (2022)).

The empirical data we use in our study, i.e. the different waves of the low-interest rate environment survey (LIRES), have already been used in several studies to learn about banks' interest business, see e.g. Busch et al. (2017), Heckmann-Draisbach and Moertel (2020), Dräger et al. (2021) and Busch et al. (2021).

As to the second contribution, i.e. modeling the term structure, we find that level shifts explain about 91% of the yearly variance in interest rates. This is in line with studies from the U.S. (see, for instance, Bliss (1997)) and earlier studies for Germany (see, for instance, Memmel (2014)). Furthermore, about 8 per cent of the variance in interest rates is explained by changes in the steepness. This shows that while the term structure level has a major impact, it is important for more accurate modeling to include the steepness as well.

The paper is structured as follows. In Section 2, we explain the model setup for modeling the term structure and banks' net interest margin. In Section 3, the empirical data used in the study is described and, in Section 4, we give the results. Section 5 concludes.

2 Empirical Modeling

2.1 Term structure

We collect risk-free interest rates (zero-bond returns) of different maturities $m = m_1, ..., M$ in the vector R_t, where the index t gives the point in time and m_1 to M the maturities of the interest rates:

$$R_t = \begin{pmatrix} r_t(m_1) \\ \vdots \\ r_t(M) \end{pmatrix} \tag{1}$$

This spans, for every point in time, a term structure of interest rates, i.e. a collection of interest rates that mature after different periods. The vector R_t has two dimensions: a time dimension and a cross-sectional dimension of the different maturities. To make the cross-sectional dimension more manageable, Nelson and Siegel (1987) model the term structure (for each point in time) as a function, depending only on a small number of parameters (here: four, namely $\alpha_{0,t}$, $\alpha_{1,t}$, $\alpha_{2,t}$ and $\lambda_t > 0$):

$$r_t^{NeSi}(m) = \alpha_{0,t} + \alpha_{1,t}\frac{1 - exp(-\lambda_t \cdot m)}{\lambda_t \cdot m} + \alpha_{2,t}\left(\frac{1 - exp(-\lambda_t \cdot m)}{\lambda_t \cdot m} - exp(-\lambda_t \cdot m)\right) \tag{2}$$

Svensson (1994) added a further term (similar to the last one, but with a different parameter λ_t from that in Equation (2)), yielding a model with six parameters.

Often it is analytically easier to deal with linear relationships, for instance, one can then estimate the coefficients with ordinary least squares OLS (see Equation (9)). Diebold and Li (2006) turned Equation (2) into a linear relationship with three parameters by setting the parameter λ_t as time-constant, namely to $\bar{\lambda} = 12 \cdot 0.0609$ (see Equation (8)). Generally, a linear model for the term structure has the form:

$$r_t^{lin.Model}(m) = \alpha_{0,t} + \alpha_{1,t} \cdot f_1(m) + \ldots + \alpha_{n,t} \cdot f_n(m) \tag{3}$$

The interest rates do not seem to be stationary (see, for instance, Busch and Memmel (2017)), so we mainly deal with changes in interest rates (where the $\triangle$-operator represents monthly, quarterly or yearly changes). As the functions $f_i(m)$ do not depend on time-varying parameters (like λ_t in Equation (2)), we obtain:

$$\triangle r_t^{lin.Model}(m) = \beta_{0,t} + \beta_{1,t} \cdot f_1(m) + \ldots + \beta_{n,t} \cdot f_n(m) \tag{4}$$

with $\beta_{i,t} = \triangle\alpha_{i,t}$.

In this paper, we mainly deal with a model for the term structure that includes the level and the steepness. We choose the following simple model to describe the interest level and the steepness of the term structure:

$$r_t^{HeMe}(m) = \alpha_{0,t} + \alpha_{1,t} \cdot m \tag{5}$$

In this model, the parameter $\alpha_{0,t}$ corresponds to the short-term interest level in time t and the parameter $\alpha_{1,t}$ gives the steepness per unit of measurement, for instance months, quarters or years.[2]

By way of comparison, we consider the following three linear models, which are special cases of the model in Equation (3), namely a cross-sectionally constant interest level (leading to a parallel shift), the model from above in Equation (5) and the model of Diebold and Li (2006) and compare their explanatory power:

$$\triangle r_t^{Parallel}(m) = \beta_{0,t} \tag{6}$$

.

$$\triangle r_t^{HeMe}(m) = \beta_{0,t} + \beta_{1,t} \cdot m \tag{7}$$

[2]For the ease of interpretation, we refer to "years" in the tables.

$$\triangle r_t^{DiLi}(m) = \beta_{0,t} + \beta_{1,t}\frac{1-exp(-\bar{\lambda}m)}{\bar{\lambda}m} + \beta_2,t\left(\frac{1-exp(-\bar{\lambda}m)}{\bar{\lambda}m} - exp(-\bar{\lambda}m)\right) \quad (8)$$

We regress the estimated changes $\triangle\hat{r}_t^{Model}(m)$ of the three models on the true changes $\triangle r_t(m)$, having $N = dim(R_t) \cdot T_{Period}$ observations. To determine the parameter vector β_t, we run (for each point in time t) the following OLS estimation for $\beta_t = (\beta_{0,t})$ (Equation (6)), $\beta_t = (\beta_{0,t}, \beta_{1,t})'$ (Equation (7)) and $\beta_t = (\beta_{0,t}, \beta_{1,t}, \beta_{2,t})'$ (Equation (8)):

$$\hat{\beta}_t = (X'X)^{-1}X'\triangle R_t. \quad (9)$$

Depending on the model, the matrix X (of dimension $dim(R_t) \times c$) consists of up to three columns c, where the first column is always composed of ones. In the case of Equation (7), the second column includes the maturities $m_1, ..., M$. In the case of Equation (8), the second and third columns include time-constant function values of the maturities m_1 to M.

2.2 Banks' interest business

We model the interest business of a bank as in Dräger et al. (2021): On the asset side, there are default-free loans (share: ϕ_A) that are granted in a revolving manner, i.e. whenever a loan matures, it is replaced by a new one. These loans have a maturity M_A and a coupon c_t equal to the then prevailing par-yield bond rate. The other assets are cash (share: $1-\phi_A$). On the liability side, there are default-free loans (share: ϕ_L) with maturity M_L that the bank issues in a revolving manner; the rest of the liabilities consist of non-remunerated current accounts (share: $1-\theta_L$). Please note that the share ϕ_A can also be interpreted differently: Instead of the share of assets that have a pass-through of 100%, it can also be interpreted as the average pass-through on the asset side. The same reasoning applies to the liability side.

The interest business of such a bank can be modeled by a portfolio of passive trading strategies $S(m)$ that consist in investing in par-yield bonds of maturity m in a revolving manner (see Memmel (2014) for the properties of these strategies). The portfolio of these trading strategies allows us to determine the consequences of an interest rate shock for Bank i, thereby taking account of empirical features of banks' interest business:

- The net pass-through is modeled with the variables $\phi_{A,i}$ and $\phi_{L,i}$. If the average pass-through on the asset side $\phi_{A,i}$ is larger than the one on the liability side $\phi_{L,i}$, Bank i benefits in the long run from an increase in the interest level (i.e. an increase in the net interest margin (NIM)). Empirically, this is often found (see, for instance, Albertazzi and Gambacorta (2009) and Claessens et al. (2018)).

- In the short run, it is possible that the net interest margin (NIM) becomes smaller as a consequence of a positive interest level shock, especially for banks that carry out a lot of term transformation. In case $\frac{\phi_{A,i}}{M_{A,i}} < \frac{\phi_{L,i}}{M_{L,i}}$, we have such a situation. This is found by Alessandri and Nelson (2015) for banks in the UK and by Busch and Memmel (2017) for banks in Germany.

- Market power is modeled by the pass-through on the liability side, by the variable $\phi_{L,i}$. A low value for the variable $\phi_{L,i}$ is likely due to low remuneration of a bank's deposits. Busch and Memmel (2021), for instance, find that regional German banks with only few competitors in their home county pay less for their deposits than banks with many competitors.

An example may be helpful in understanding this point: Suppose Bank A grants loans with 4 years of maturity in a revolving manner. Further suppose that this business stands for 95% of the balance sheet (5% cash) and the liability side is composed of revolvingly issued bonds (70%, maturity 2.5 years) and of non-remunerated current accounts (30%).[3] Under these assumptions, this bank has a long-run pass-through of 25% (=95%-70%), i.e. when the interest level goes up by 100 bp, its net interest margin will ultimately increase by 25 bp. In the short run, however, we will observe a drop in its net interest margin by 4.25 bp (=95bp/4 - 70 bp/2.5) if the interest level goes up by 100 bp.[4]

One central assumption is that we can model a bank's interest business by portfolios of the passive trading strategy $S(m)$. We check the empirical validity of this assumption by running the following regression:

$$NIM_{t,i} = \alpha_i + \gamma_t + \beta \cdot \frac{F_{t,i}}{A_{t,i}} + \varepsilon_{t,i} \tag{10}$$

where α_i are bank fixed effects and γ_t are time fixed effects. $A_{t,i}$ are Bank i's total assets and $F_{t,i}$ are Bank i's earnings from term transformation in the period from $t-1$ to t under the assumption of a certain investment strategy, for instance a portfolio of the passive trading strategy (see Equation (22)). If this assumption is valid, we expect the coefficient β to equal one. However, Chaudron et al. (2022) find that Bank i's net interest margin NIM includes other time-varying components that increase when the earnings from term transformation decrease and vice versa, so that the coefficient β is less than one; in the case of their sample (Dutch banks), it is even close to zero.

In the following, we assume that an interest shock takes place at time $t = t_0$, having an impact on the interest level and on the steepness of the term structure. The variable $T = t - t_0$ gives the period since the shock happened. In addition, we have the assumption of a static balance sheet. This and the modeling above allow us to write the (instantaneous) deviation $C.NIM_i(T)$ of the net interest margin as a consequence of this interest shock (see also Figure 1) where, in the baseline, we assume that the term structure remains unaltered. Using the term structure model of Equation (5) and Equation (37) to relate – in an environment of low interest rates – small changes in the interest level ($\beta_{0,t}$) and in the steepness of the term structure ($\beta_{1,t}$) to the coupon of par-yield bonds ($c_t(m)$) of maturity m, we obtain:

$$\triangle c_t(m) = \beta_{0,t} + \beta_{1,t} \cdot m \tag{11}$$

The parameter values for $\beta_{0,t}$ and $\beta_{1,t}$ given a list of interest rate changes $\triangle r_t(m)$ of different maturities can be estimated using Equation (9).

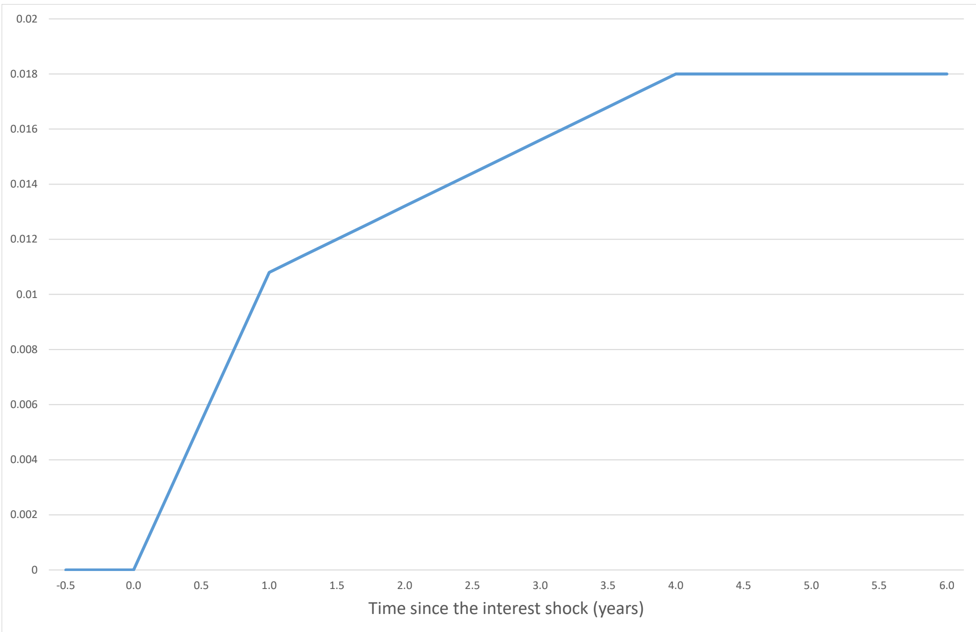
$$\begin{aligned} C.NIM_i(T) \;&=\; \phi_{A,i} \cdot min\left(\frac{T}{M_{A,i}}, 100\%\right) \cdot (\beta_{0,t_0} + \beta_{1,t_0} M_{A,i}) \\ &-\; \phi_{L,i} \cdot min\left(\frac{T}{M_{L,i}}, 100\%\right) \cdot (\beta_{0,t_0} + \beta_{1,t_0} M_{L,i}) \end{aligned} \tag{12}$$

Equation (12), the deviation of the net interest margin from the baseline net interest margin, can be derived as follows: Only new business (not existing business) is affected by the interest rate shock. Due to the revolving manner of the investment strategy, its share (on the asset side) corresponds to $T/M_{A,i}$, capped at 100%. Among the new business, only the share $\phi_{A,i}$ counts, i.e. the fraction that is invested in the strategy with the par-yield

[3] In reality, current accounts are remunerated, but their pass-through is far from 100%; in fact, it is a bit more than 30%. For regional German banks, Busch and Memmel (2021) find that this pass-through is especially low for banks not exposed to strong competition, providing much service and located in rural counties.

[4] The degree of term transformation of the bank in the example is about 2%; the values are roughly in line with the mean values in Table 3.

Figure 1: Deviation of the net interest margin ($C.NIM$) due to an interest shock



This figure shows the deviation of a bank's (instantaneous) net interest margin ($C.NIM$) as a consequence of a shock to the term structure ($\beta_0 = -200$bp and $\beta_1 = 0.008$ in Equation (7), which corresponds to an upward turn of the term structure at a pivot point of 2.5 years of maturity; concerning the bank features, we have $M_{A,i} = 4$, $M_{L,i} = 1$, $\phi_{A,i} = 0.8$ and $\phi_{L,i} = 0.7$ which according to Table 1 and Condition (16) yield positive deviations).

bonds whose coupons have changed by $\beta_{0,t_0} + \beta_{1,t_0} M_{A,i}$. The same is true of the liability side.

Without loss of generality, we assume $M_{A,i} > M_{L,i} > 0$, i.e. that the maturity on the asset-side is larger than the one of the liability-side and that maturities are positive.[5] Concerning period T, i.e. the period since the interest shock has happened, we distinguish three cases:

- Case i): $T \leq M_{L,i}$

$$C.NIM_i(T) = T\left(\frac{\phi_{A,i}}{M_{A,i}} - \frac{\phi_{L,i}}{M_{L,i}}\right)\beta_{0,t_0} + T\left(\phi_{A,i} - \phi_{L,i}\right)\beta_{1,t_0} \tag{13}$$

- Case ii): $M_{L,i} < T \leq M_{A,i}$

$$C.NIM_i(T) = \left(T\frac{\phi_{A,i}}{M_{A,i}} - \phi_{L,i}\right)\beta_{0,t_0} + \left(T\phi_{A,i} - M_{L,i}\phi_{L,i}\right)\beta_{1,t_0} \tag{14}$$

- Case iii): $M_{A,i} < T$

$$C.NIM_i(T) = \left(\phi_{A,i} - \phi_{L,i}\right)\beta_{0,t_0} + \left(M_{A,i}\phi_{A,i} - M_{L,i}\phi_{L,i}\right)\beta_{1,t_0} \tag{15}$$

The results are summed up in Table 1 for different idealized banks, where the short-term effect (the next to last column) is calculated from Equation (13) as Case i) and the long-term effect (last column) is taken from Equation (15) as Case iii). When we look at the short-term effects of an increase in the level of the term structure, we see that the deviation of the net interest margin is negative (for banks that carry out term transformation). However, the long-run effects are often positive. Empirically, the positive effect of an increase in the interest level is often found; the negative short-term effect of an increase in the interest level is less often documented.

An upward-turning of the term structure is said to be beneficial for banks. In the term structure model of Equation (5), this upward-turning is a combination of a decrease in the level, i.e. $\beta_0 < 0$, and an increase of the steepness, i.e. $\beta_1 > 0$ (see Figure 1). Even under the assumption of a positive net long-run pass-through $\phi_A - \phi_L$ and a negative short-run effect $\frac{\phi_A}{M_A} - \frac{\phi_l}{M_L}$, it is unclear whether the long-term effect is positive (see the cell in the last row and in the last column of Table 1). This is only the case if in addition

$$\frac{M_{A,i}\phi_{A,i} - M_{L,i}\phi_{L,i}}{\phi_{A,i} - \phi_{L,i}} > -\frac{\beta_{0,t}}{\beta_{1,t}} \tag{16}$$

Note that the expression $-\beta_{0,t}/\beta_{1,t}$ is, according to Equation (7), the pivotal point m_t^* of a turning in the term structure (provided the two coefficients $\beta_{0,t}$ and $\beta_{1,t}$ have different signs, so that $m_t^* = -\beta_{0,t}/\beta_{1,t}$ is a positive maturity which is, for instance, the case in Figure 1) and that, on the left-hand side of condition (16), there are bank characteristics and, on the right-hand side, there is a term structure characteristic.

Looking at Equation (15), we need two quantities for each bank, namely its long-run pass-through $\phi_A - \phi_L$ and its extent of term transformation $M_A\phi_A - M_L\phi_L$. The term $M_{A,i}\phi_{A,i} - M_{L,i}\phi_{L,i}$ is roughly proportional to the duration of the portfolio to mimic Bank i's interest business.[6] Under the assumption of a low interest level, we obtain for the euro

[5] As long as the maturities M_A and M_L are greater than zero, the effects can be computed in the Equations (13) and (14), only the conditions for the cases have to be altered if $M_A \leq M_L$. As to the simplified central bank in Table 1, the duration on the asset side, M_A, is zero, therefore, the short-term effect cannot be computed.

[6] In the following, we use two concepts of duration, namely the euro duration $D^{€}$ and the modified duration D_{mod}. The euro duration gives the euro amount of the change as a consequence of a small interest change and the modified duration is equal to the euro duration over the present value of the portfolio. As in our model a bank's equity is not explicitly accounted for, the bank's present value is zero, so that we cannot determine the modified duration of the bank, only its euro duration.

Table 1: Impact on a bank's net interest margin (NIM)

No.	Bank charactistic				Term structure	$C.NIM$	
	$\phi_A - \phi_L$	$\frac{\phi_A}{M_A} - \frac{\phi_L}{M_L}$	$M_A\phi_A - M_L\phi_L$	Example		Short-term	Long-term
1	1	n.a.	0	Simplified central bank	pos. shift	n.a.	pos.
2	0	neg.	(pos.)	Commercial bank	pos. shift	neg.	0
					pos. shift + inc. in steep.	neg.	pos.
					neg. shift + inc. in steep.	pos.	pos.
3	pos.	neg.	(pos.)	Traditional bank	pos. shift	neg.	pos.
					Pos. shift + inc. in steep.	?	pos.
					Neg. shift + inc. in steep.	pos.	?

This table shows qualitatively the deviation of the net interest margin ($C.NIM$) for three idealized banks as a consequence of interest rate shocks. The simplified central bank has on its asset-side loans to banks with a negligible maturity ($M_A = 0$) and on the liability-side banknotes ($\phi_L = 0$). The commercial bank has on both sides loans and bonds with maturity $M_A > M_L$ and with complete pass-through. The traditional bank has on its liability-side equity and deposits, so that $\phi_A > \phi_L$ and carries out term transformation, where we assume that the term transformation effect dominates (see the last row, second column with the entry "neg."). Entries in brackets "()" mean that they are derived from the assumptions in the two entries left to them. "?" means that the effect is indeterminate.

duration of this bank's assets A_i minus the euro duration for its liabilities L_i:[7]

$$D_i^{€} := \frac{\partial A_i}{\partial \beta_0} - \frac{\partial L_i}{\partial \beta_0} = \frac{1}{2}\left(M_{A,i}\phi_{A,i} - M_{L,i}\phi_{L,i}\right) \cdot A_i \tag{17}$$

Banks in Germany have to report a similar value, namely for a shock of 200 bp, together with the Basel interest rate coefficient (see Equation (21)). The long-run pass-through $\phi_A - \phi_L$ can be determined by assigning to each balance sheet position (which is available for each bank and at monthly frequency) its pass-through as done in Busch et al. (2021)). The following empirical equation derived from Equation (15) may be estimated:

$$C.NIM_{i,k}(M_{A,i}) = \alpha + \beta_k \cdot (\phi_A - \phi_L)_i + \gamma_k \cdot (M_A\phi_A - M_L\phi_L)_i + \varepsilon_{i,k} \tag{18}$$

where $k = 1, ..., K$ stands for the relevant scenarios. The resulting estimates $\widehat{\beta}_k$ and $\widehat{\gamma}_k$ can be compared with the scenario parameters.

[7] To give an intuition for the duration formula: The modified duration of a par-yield bond corresponds approximately to its maturity, i.e. a par-yield bond with a maturity of five years loses approximately 5% of its value if the interest rate level rises by 1 percentage point (actually, this is only exact at an interest level of 0%). The passive investment strategy $S(m)$ consists in investing in par-yield bonds with maturity m so that this strategy consists at any time of bonds with a residual maturity equally distributed from zero maturity to maturity m, which leads to an average residual maturity of $m/2$, which is approximately equal to the strategy's modified duration (at an interest level of 2% and a maturity $m = 5$, the strategy's duration is $D_{mod} = 2.42$ (instead of 2.5). A portfolio consisting of a long position of $\phi_{A,i} \cdot A_i$ of the strategy $S(M_{A,i})$ and a short position of $\phi_{L,i} \cdot L_i$ of the strategy $S(M_{L,i})$ has the euro duration $D_i^{€} = \frac{1}{2}\left(M_{A,i}\phi_{A,i} - M_{L,i}\phi_{L,i}\right) \cdot A_i$. Note that, in the model, equity is not explicitly accounted for and that, therefore, the euro amount of a bank's assets is equal to its liabilities, i.e. $A_i = L_i$.

Table 2: Summary statistics

Term structure	Model parameter	Unit	Mean	SD	1st perc.	Median	99th perc.
Level	Level	per cent	3.75	3.24	-0.99	3.82	11.65
	Steepness	bp per year	14.09	12.95	-21.39	15.83	39.08
Change (1 month)	Level	bp	-1.66	29.16	-91.49	-1.29	78.26
	Steepness	bp per year	-0.02	2.69	-6.57	-0.16	8.69
Change (3 months)	Level	bp	-5.12	61.88	-194.48	-2.77	172.11
	Steepness	bp per year	-0.07	5.12	-15.64	-0.31	15.04
Change (12 months)	Level	bp	-21.79	145.86	-389.77	-13.58	391.51
	Steepness	bp per year	-0.18	11.79	-32.5	-0.23	30.78

This table shows summary statistics for the level of and changes in the term structure (Period: 1975-01 to 2021-12). "SD", "bp", "1st perc." and "99th perc." mean standard deviation, basis points, first percentile and 99th percentile. The summary statistics are based on the model for the term structure of Equation 5.

3 Data

3.1 Term structure

The interest rates are zero-bond rates, based on German government bonds and derived using the method according to Svensson (1994) with six parameters (see also Schich (1997) for the application to German data). Note that we are not dealing with single bonds, but with an already estimated term structure. The period covers nearly fifty years (1975-01 to 2021-12) and we use monthly data; in the paper, we have $dim(R_t) = 20$ maturities and $T_{Period} = 564$ monthly observations (47 years), yielding 11 280 observations.

In Table 2, we report summary statistics. As to the average steepness, it is 14.09 bp for each year (first column, second row), meaning that a bond with 10 years of maturity yields on average 1.41% p.a. more than the short-term interest rate (first column, second row). The 99th percentile of yearly changes is about 390 bp (fifth column, seventh row), significantly more than the 200 bp of the Basel shock, which was informed by yearly changes. However, the interest rate changes tend to be larger for short maturities and when the interest level is higher, which was the case in the seventies and eighties of the last century.

3.2 Banks' interest business

Balance sheet data of all German banks is used to determine bank-specific weights (w_{ij}) for the different balance sheet positions j. Let w_{ij} be the weight of balance sheet position j of bank i, then

$$\phi_{A,i} = \sum_{j=1}^{J} w_{ij} \cdot \phi_j^A \tag{19}$$

The long-run pass-through of the different balance sheet positions is taken from Memmel (2018).

The term $M_{A,i}$ could be estimated comparable to $\phi_{A,i}$ (see Equation (19)). This estimate would not be as precise as that for $\phi_{A,i}$. For instance, off-balance sheet positions, mainly interest swaps, can be assumed to have a complete pass-through on the asset side and on the liability side, so that they do not alter a bank's net long-run pass-through. By contrast, they are likely to affect the term transformation. Instead of $M_{A,i}$, we could estimate the average fixed interest period on the liability side $M_{L,i}$. However, this would make it necessary to have an estimate for the duration of deposits, which is difficult to obtain (see Kerbl et al. (2019)). To circumvent the problems with the estimation of the maturities, we set $M_A = 5$, the longest maturity available in our survey.

The term $M_{A,i}\phi_{A,i} - M_{L,i}\phi_{L,i}$ is taken from the banks' regular term transformation returns: Let IRR_i be the euro amount of the change in present value due to an interest rate shock of $\triangle r = 200bp$, which is reported quarterly. We approximately obtain:

$$D_i^{€} \approx -\frac{IRR_i}{\triangle r} \tag{20}$$

Together with Equation (17), we derive an expression for $M_{A,i}\phi_{A,i} - M_{L,i}\phi_{L,i}$, namely

$$M_{A,i}\phi_{A,i} - M_{L,i}\phi_{L,i} = -100 \cdot \frac{IRR_i}{A_i} \tag{21}$$

where we use the relationship $100 = 2/\triangle r$.

To determine a bank's exposure to interest rate risk, we use the variable $IRR_{t,i}$ as well, this time to calculate the earnings from term transformation $F_{t,i}$ in Equation (10):

$$F_{t,i} = -50 \cdot \frac{Re_t\left(S(m_1)\right) - Re_t\left(S(m_2)\right)}{D_t^{€}\left(S(m_1)\right) - D_t^{€}\left(S(m_2)\right)} \cdot IRR_{t,i} \tag{22}$$

or in case of an investment in zero-bonds:

$$F_{t,i} = -50 \cdot \frac{r_t\left(m_1\right) - r_t\left(m_2\right)}{m_1 - m_2} \cdot IRR_{t,i} \tag{23}$$

To get yearly data, we calculate (for the Equations (22) and (23) in the last quarter of each year) the sum of the current quarter and of the 3 previous quarters, whereby we make use of the quarterly availability of the interest risk exposure data $IRR_{t,i}$. As Chaudron (2018) rightly states, the net duration (here: in Equation (20)) does not give the durations on the asset or liability side, and hence not the maturities of the passive trading strategies. We proceed as follows to obtain estimators for the durations on the asset and liability side: To find the combination of the trading strategies with the best fit, we try out all combinations of (m_1, m_2) of up to ten years in steps of six months, which yields 190 meaningful combinations[8], and compare the coefficient of determination R^2 of Equation (10). We obtain the best fit (highest R^2 of Equation (10)) for a combination of the maturities (42, 6) for the passive trading strategy (see Equation (22)), yielding a coefficient of 0.66 (which is significantly smaller than the theoretical value of one, but within the expectations). The fit of this combination is better than the fit for all maturity combinations of the strategy of redeploying the whole capital in each period (see Equation (23)). We also challenged the fit against reporting figures of earnings from structural contributions, but here again, the model outperforms the data.[9] We note that most of the explanation is due to the inclusion of time dummies in Equation (10) and that no testing seems possible as a consequence of trying out all combinations.

In Table 3, we report summary statistics for banks' interest business.

[8]The maturities m_1 and m_2 can each be equal to 20 different values, yielding $400 = 20$ x 20 combinations. We subtract the 20 cases where both maturities are equal and exclude the $190 = 19$ x 10 cases where the first maturity is smaller than the second maturity.

[9]Structural contribution may comprise more than just earnings from term transformation, e.g. earnings from own funds. To our knowledge, there is no isolated reporting of the earnings from term transformation.

Table 3: Summary statistics (bank level)

Variable	Year	Unit	Mean	SD	1st perc.	Median	99th perc.	Nobs
IRR	2016	-% per TA	1.96	1.03	-0.54	1.99	4.64	1419
IRR	2018	-% per TA	1.93	1.07	-0.77	1.96	4.76	1383
$\phi_A - \phi_L$	2016	% per TA	25.05	11.85	-11.86	26.43	49.13	1419
$\phi_A - \phi_L$	2018	% per TA	26.07	11.74	-12.67	27.71	48.71	1383

This table shows summary statistics at bank level. The data is from the banks' returns just before the wave of the quantitative survey took place. IRR is a bank's exposure to interest rate risk and the difference $\phi_A - \phi_{L,}$, is its long-run net pass-through. "SD", "1st perc." and "99th perc." mean standard deviation, first percentile and 99th percentile.

Table 4: Scenarios in the LIRES waves

Number k	Scenario	Description	Change in the... level	Change in the... steepness
0	Baseline	Term structure remains constant	0	0
1	Turn	Term structure flattens	125	-11
2	Positive shift	All interest rates increase by 200 bp. The steepness does not change	200	0
3	Negative shift	All interest rates decrease by 100 bp. The steepness does not change	-100	0

This table shows descriptions of the scenarios. "bp" means basis points. All changes take place over night at the beginning of the five year horizon. Values in the two last columns are given in basis points.

3.3 Low-interest rate environment survey

Every other year since 2013, German small and medium-sized banks have been subject to a quantitative survey.[10] The banks have to forecast their interest income and expenses (and other components of their profit and loss statement) for the following 5 years under various interest rate scenarios. We focus on the data from the 2017 and 2019 surveys, as these can be considered as established research data and the reporting was to some extent standardized between these surveys, thus providing comparability.

There are $K = 3$ scenarios that are relevant to us, namely two scenarios with a level shift and one scenario that includes a level shift and a flattening of the term structure. In addition, there is also the scenario of a time-constant term structure, which serves as reference point. All of these scenarios assume a static balance sheet. Let $NIM(T)_{i,k}$ be the net interest margin of Bank i in Scenario k at time T. We calculate the deviation of Bank i's net interest margin as

$$C.NIM_{i,k}(T) := NIM_{i,k}(T) - NIM_{i,0}(T) \tag{24}$$

where $k = 0$ is the baseline scenario of a time-constant term structure (see Table 4).

In Table 5, there are some summary statistics of the quantitative survey.

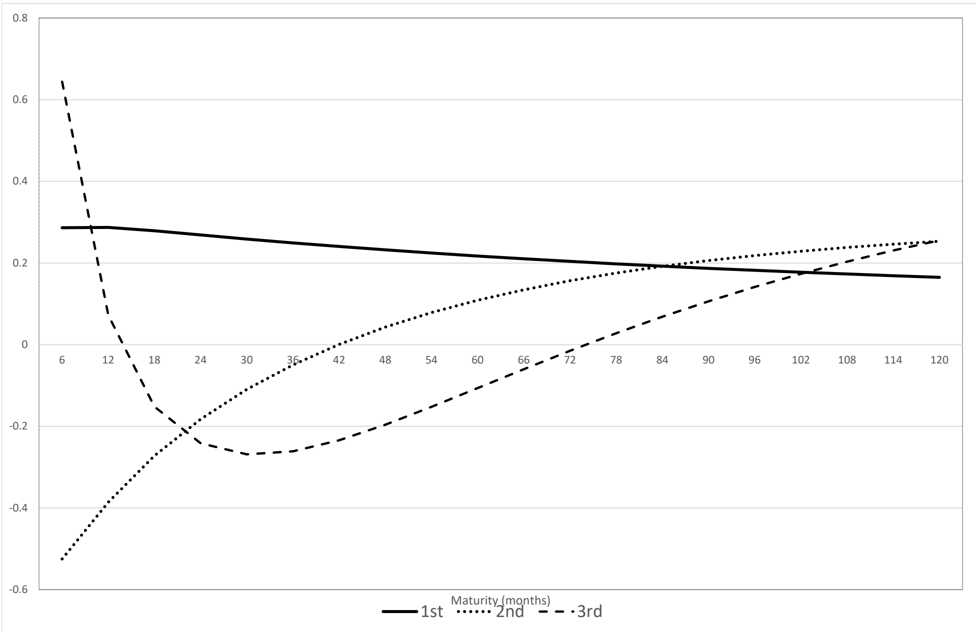
[10] In 2021, no survey wave took place as a consequence of the COVID-19 pandemic.

Table 5: Summary statistics (LIRES)

Variable	Scenario	Wave	Mean	SD	Share >0	Nobs
$C.NIM(1)$	Pos. shift	2017	-10.10	27.93	25.65	1419
$C.NIM(1)$	Pos. shift	2019	-9.93	26.17	27.26	1383
$C.NIM(5)$	Pos. shift	2017	29.08	29.63	90.77	1419
$C.NIM(5)$	Pos. shift	2019	29.32	31.98	88.36	1383

This table shows summary statistics of the deviation of the NIM from the baseline scenario (see Table 4).

Figure 2: PCA: Factor loadings



This figure shows the factor loadings for the first three components of a principal component analysis (PCA) of yearly changes in interest rates of different maturities. German government bonds up to 120 months maturity in steps of 6 months. Monthly data; period: 1975-01 to 2021-12.

4 Results

4.1 Term structure

We investigate the term structure with the help of a principal component analysis (PCA) and determine how much of the variation in the interest rates is explained by the different components.

In Table 6, the shares of explained variances of this PCA are reported. The results are in line with the findings in the literature (see Litterman and Scheinkman (1991), Knez et al. (1994) and Bliss (1997)). The factor loadings of the three first components are displayed in Figure 2. The PCA is a completely statistical method, i.e. it is agnostic about possible structures (live level or steepness shifts) in the data and yet the first component (i.e. the most important one) looks nearly like a parallel level shift (with longer maturities less affected) and the second component resembles a (concave) shift in the steepness.

As to the coefficient of determination R^2 (when regressing the interest rate changes derived from the models on the true interest rate changes, see Subsection 2.1) of the three different models (see Table 7), the following can be noted:

Table 6: Explained Variance of Changes in the Term Structure

Principal component	Change in the term structure					
	1 month		3 months		12 months	
First	81.87%	81.87%	88.18%	88.18%	90.77%	90.77%
Second	11.43%	93.30%	8.56%	96.74%	7.79%	98.56%
Third	5.25%	98.55%	2.50%	99.24%	1.10%	99.66%

This table shows the fraction of the explained variance of the changes in the term structure, derived from a PCA. German government bonds up to 120 months maturity in steps of 6 months. Monthly data; period: 1975-01 to 2021-12. For each of the changes, the additional contribution and the cumulative contribution are reported.

Table 7: Different models: coefficient of determination

Coefficient of determination	Change horizon		
	1 month	3 months	12 months
Parallel shift (see Eq. (6))	81.47%	86.73%	88.14%
Two factors (see Eq.(7))	90.61%	95.49%	97.47%
Three factors (see Eq. (8))	97.13%	97.80%	98.35%

This table shows the coefficient of determination of three different interest rate models. German government bonds up to 120 months maturity in steps of 6 months. Monthly data; period: 1975-01 to 2021-12.

- The parallel shift of the term structure is often applied, for instance in what is known as the Basel interest rate shock (see Basel Committee on Banking Supervision (2004)). One reason is the analytical easiness of a flat term structure shift. There is also an empirical reason: This shift explains up to 88% of the variation (see the first row of Table 7) and is close to the theoretical maximum (see the first row of Table 6). The gap between the theoretical maximum and the parallel shift may be due to the fact that the interest rates of longer maturities are more sluggish and that the loadings for the first factor are not a parallel line, but tend to decrease (see Figure 2).

- The same holds true if we compare the R^2 of the two-factor model (in Table 7, second row) with the theoretical maximum (Table 6, second row, cumulative values). The model (see Equation (7)) may not be an exact fit for the real interest rate changes (see Figure 2, where the second component is not a straight line, but a concave curve).

- The three-factor model has the highest explained variance (see Table 7). However, the additional increase in the explained variance (relative to the two-factor model) seems relatively small, especially for longer change horizons. Note that we are only dealing with a section of the term structure (for insurance companies, longer maturities may be relevant); if we looked at the whole term structure (including longer maturities), the difference in explained variances relative to the two-factor model might be more relevant.

- When we consider shorter horizons of changes in the interest rates (Table 6; 1 month or, to some extent, 3 months), we see that the third principal component makes a substantial contribution to explaining the variance. Looking at the one-year shock, this contribution is only about one per cent.

A comparable result is achieved if use the information criteria AIC and BIC for each month in our sample period 01/1975 to 12/2021 (which yields 564 observations).

Table 8: Information criterion AIC

Model for the Term Structure	Change horizon			
	1 month	3 months	12 months	24 months
Parallel shift (see Eq. (6))	0.5%	0.7%	0.7%	0.4%
Two factors (see Eq.(7))	39.7%	44.5%	49.6%	57.8%
Three factors (see Eq. (8))	59.8%	54.8%	49.6%	41.8%

This table shows how often the respective factor model for the term structure is the best one according to the information criterion AIC (the results for the information criterion BIC are available on request).

Taken together, we choose the two-factor term structure model of Equation (5), which relies on the interest level and the steepness of the term structure in a linear way. Moreover, this modeling of the term structure makes it possible to derive a closed-form solution for the coupon of par-yield bonds $c(m)$ given a term structure strictly with positive slope ($\alpha_1 > 0$) (see Equation (33) in connection with Equation (26)):

$$c(m) = \frac{1 - exp(-\alpha_0 m - \alpha_1 m^2)}{\left(\Phi\left(\sqrt{2\alpha_1} \cdot m + \frac{\alpha_0}{\sqrt{2\alpha_1}}\right) - \Phi\left(\frac{\alpha_0}{\sqrt{2\alpha_1}}\right)\right) \cdot \sqrt{\frac{\pi}{\alpha_1}} \cdot exp\left(\frac{\alpha_0^2}{4\alpha_1}\right)} \tag{25}$$

where $\Phi(\cdot)$ is the cumulative density function of the standard normal distribution.[11]

Equation (25) makes it possible to compare the exact change in the coupon ($\triangle c(m)$) with the approximation in Equation (11). For instance, if we start at $\alpha_0 = 1\%$ and $\alpha_1 = 0.0005$ (which yields a coupon of $c(10) = 1.49\%$ at $m = 10$ years) and if we add around one standard deviation of the yearly changes in the parameters (the exact values are 1.46 and 0.0001179, see Table 2), one obtains $\alpha_{0,+} = 2.5\%$ and $\alpha_{1,+} = 0.0006$ (which yields a coupon according to Equation (25) of $c_+(10) = 3.07\%$). This yields a difference of $c_+(10) - c(10) =$ 1.58 percentage points (pp), while Equation (11) yields $0.015 + 0.0001 \times 10 = 1.60$ pp. Generally speaking, for a low steepness, the approximation works well; however, for a pronounced steepness, the approximation becomes less precise.

It should be mentioned that the time-series correlation between $\beta_{0,t}$ and $\beta_{1,t}$ is strongly negative (for the one-year changes, it is -0.7864 for the period 1975-01 to 2021-12), meaning that an increase in the (short-term) interest level is associated with a decrease in the steepness.

4.2 Banks' interest business

From the summary statistics and Equation (44) in the Appendix A.3, we can make an educated guess about banks' average earnings from term transformation. This equation states that the contribution of term transformation is on average equal to the average steepness minus half of the trend in the interest level multiplied by banks' exposure to interest rate risk. According to Table 2, the average steepness is about 14 bp ("Mean", row 2) and the trend is about -22 bp ("Mean", row 7). According to Table 3, banks' average exposure to interest rate risk (measured as the change in present value due to an interest rate shock of +200 bp, divided by total assets, in percent) was close to 2 in 2016 and 2018; under the assumption that this exposure has been relatively constant through time, we set this exposure to 2. We obtain about 50 bp, which is within the range of the results

[11] For $\alpha_1 = 0$, we obtain a flat term structure and $c(m) = \alpha_0$ (see Equation (33) in connection with Equation (27)).

[12] This study includes also the results for 2013, which are even higher as to term transformation.

[13] This share also includes interest on equity.

Table 9: Studies on earnings from term transformation

Study	Share of NIM	Earnings [in bp per assets]	Sample
Memmel (2011)	12.3%	26.3	German banks, 2005-2009
Busch and Memmel (2016)	33.6%	73.3	German banks, 2012[12]
Chaudron et al. (2022)	8.3%[13]	11.4	Dutch banks, 2008Q1-2020Q4
Approximation in this paper	-	50	German banks, 1975-2021
Study in this paper	10.1%	18.7	German banks, 2014-2020

This table shows studies on earnings from term transformation. "NIM" stands for "net interest margin". "Assets" mean total assets, in the case of Chaudron et al. (2022), it means banking books assets. "bp" means basis points.

of the studies named in Table 9. Note that the average earnings from this strategy are much higher than from the strategy of investing all funds in the then current zero bond and financing this operation by issuing short-term zero bonds. This strategy yields an average steepness of 14 bp. What is more, the risk is doubled, meaning that the average contribution is 14 bp if the risk of the bank is kept constant (compared to 50 bp). However, if we estimate the contribution according to Equation (10) and concentrate on the years where data is available, the contribution is much lower, namely 10.1% of NIM and 18.7 bp (see Table 9).

We estimate Equation (18) by applying a mild outlier correction by removing the first and 99th percentile of the variables. The results in Table 10 show that modeling banks' interest business through the assumed bond portfolios captures several features. In this estimation, "phi" refers to the long-run pass-through $\phi_A - \phi_L$, and "term" to the degree of term transformation $M_A\phi_A - M_L\phi_L$ in Equation (18). The following results can be highlighted:

- All estimates for the coefficient of "phi" have the right sign. However, in absolute terms, the theoretical values are significantly larger than the estimated values. This can be due to noisy values for the long-run pass-through "phi" (see Appendix A.4, Equation (50), something known as attenuation bias).

- The coefficient γ related to the degree of term transformation "term" has the expected sign in the scenario "turn" of a flattening of the term structure. However, the estimates for "term" in the two shift scenarios are often significantly different from zero, the theoretical value in the case of a shift in the term structure (see Table 4). The cause for this may be that the new equilibrium (see Equation 15) is not reached, which leads to a systematic bias. In Appendix A.4, it is shown that the coefficient $\tilde{\gamma}$ (for term) derived from the observed variables is equal to the sum of the true coefficient γ and a component depending on the correlation between the degree of term transformation "term" and the uncompleted change in the net interest margin θ (see Equations (46) and (51)). *Uncompleted change* should be understood in the sense that after the survey horizon of $T_{max} = 5$ years, the change in the net interest margin $C.NIM(5)$ hat not yet attained the end point change $C.NIM(M_A)$. The following considerations may give rise to the belief that this correlation is negative for the scenario of a positive shift. Assume that there are two sorts of banks (indexed by H and L) that differ only in the maximal maturity of the bonds on the asset side M_A. Table

Table 10: Results at bank level

Scenario	Wave	phi	term	R^2	Nobs
Turn	2017	17.40***	-8.46***	13.05	1351
Turn	2017	-4.88		0.05	1351
Turn	2017		-7.93***	12.41	1351
Pos. Shift	2017	85.53***	-7.12***	17.34	1350
Pos. Shift	2017	65.61***		9.00	1350
Pos. Shift	2017		-4.35***	3.43	1350
Neg. Shift	2017	-22.84***	-1.85***	2.72	1346
Neg. Shift	2017	-27.90***		2.02	1346
Neg. Shift	2017		-2.57***	1.48	1346
Turn	2019	23.65***	-10.95***	17.67	1318
Turn	2019	-8.43		0.14	1318
Turn	2019		-10.12***	16.68	1318
Pos. Shift	2019	112.27***	-8.99***	22.72	1317
Pos. Shift	2019	84.64***		12.25	1317
Pos. Shift	2019		-4.94***	3.56	1317
Neg. Shift	2019	-36.91***	-1.10*	4.56	1312
Neg. Shift	2019	-40.22***		4.32	1312
Neg. Shift	2019		-2.42***	1.31	1312

This table shows the results of the regression (18), where the deviation in the net interest margin is taken in the fifth (and last) projection year of the corresponding survey wave (see Table 4). The column R^2 (in per cent) gives the coefficient of determination of Equation (18). "Nobs" gives the sample size. *, ** and *** mean significance at the 10%, 5% and 1% level.

11 in the appendix shows that the correlation between the variables "term" and θ is negative for the positive shift scenario. This may explain the significantly negative coefficients for "term" in the positive shift scenario in Table 10. For a negative shift, the correlation between "term" and θ is positive.

4.3 Robustness checks

If we use the introduction of the euro (1999-01) as the starting point and keep the end point (2021-12), we get qualitatively similar results; the difference between the three-factor model and the two-factor model (in Table 7) becomes smaller. Looking at the low interest rate environment in the euro area (assumed starting point at 2014-06, end point kept at 2021-12), we find that term structure modeling that only relies on a parallel shift performs significantly worse than in other periods, especially for longer change horizons.

As to the modeling of banks' interest business, we challenge our assumptions in Section 3 for deriving the relevant parameters and use other definitions of the long-run pass-through and the exposure to interest rate risk; this leads to similar results as shown above. Moreover, we check whether the investment strategy of redeploying the whole capital in each period yields a better fit for longer maturities (longer than 120 months = 10 years). Indeed, we find that the combination (210 = 17.5 years, 6) yields the best fit. However, this optimal fit for this strategy (measured by the R^2 of Equation (10)) is lower than the best fit of a portfolio using the passive trading strategy $S(m)$ in Subsection 3.2.

5 Conclusion

In our study, we compare different simplified models of the term structure and analyze the impact of an interest rate shock on a bank's net interest margin. Using nearly fifty years of interest rate data for German government bonds of different maturities, we find that changes in the term structure can be described well by two factors, namely by changes in its level and in its steepness. Looking at yearly changes, we find that level changes account for nearly 91% of the variances and changes in steepness for a further 8%.

We also find that the portfolios of bonds can describe the interest business of banks well. The portfolios applying a passive trading strategy allow interest business to be modelled in a parsimonious way and at the same time allow empirical features of German banks' interest business, namely qualitatively different short-run and long-run net pass-through, term transformation and market power, to be reproduced. In addition, the model results fit the results of a quantitative survey and explain the dynamics of banks' net interest margin better than other plausible reference models. While our analysis focuses on the German banking sector, the model and setup could be easily transferred to other banking markets, which might be an interesting extension for future projects.

The modeling described in the paper may be used for stress testing banks' interest business with respect to changes in the term structure. This is especially relevant for banks with a significant exposure to interest rate risk, and can be particularly informative in times where significant changes in the term structure are expected.

A Appendix

A.1 Useful formulae

$$\int_0^m exp\left(-at-bt^2\right)dt = \left(\Phi\left(\sqrt{2b}\cdot m+\frac{a}{\sqrt{2b}}\right)-\Phi\left(\frac{a}{\sqrt{2b}}\right)\right)\cdot\sqrt{\frac{\pi}{b}}\cdot exp\left(\frac{a^2}{4b}\right) \tag{26}$$

where $b>0$ and $\Phi(\cdot)$ is the cumulative density function of the standard normal distribution. The equality of both sides of the equation can be seen by using the following relationship: $\int_f^e \frac{1}{\sqrt{2\pi}}exp\left(-1/2z^2\right)dz = \Phi(e)-\Phi(f)$ and we replace $-at-bt^2$ by $-1/2z^2+a^2/(4b)$ in the left-hand side of Equation (26), and we apply the substitution method, using $z(t)=\sqrt{2b}\cdot t+\frac{a}{\sqrt{2b}}$, so that $e=z(m)=\sqrt{2b}\cdot m+\frac{a}{\sqrt{2b}}$, $f=z(0)=\frac{a}{\sqrt{2b}}$ and $dt=dz\cdot\frac{1}{\sqrt{2b}}$.

For $\delta>0$ and $m>0$, we obtain:

$$\int_0^m exp(-\delta t)dt = \frac{1-exp(-\delta m)}{\delta} \tag{27}$$

$$\int_0^m t\cdot exp(-\delta t)dt = \frac{1}{\delta^2}\left(1-(1+\delta m)\,exp(-\delta m)\right) \tag{28}$$

$$\int_0^m t^2\cdot exp(-\delta t)dt = \frac{1}{\delta^3}\left(2-\left(2+2\delta m+\delta^2m^2\right)exp(-\delta m)\right) \tag{29}$$

For $m>0$ as an integer, we obtain:

$$\sum_{i=1}^m i = \frac{m\,(m+1)}{2} \tag{30}$$

Assume that the vector θ (with dimension N) is multivariate normal $\theta\sim N(\mu;\Sigma)$ and that it is divided into two subvectors with the dimensions N_1 and N_2:

$$\begin{pmatrix}\theta_1\\ \theta_2\end{pmatrix}\sim N\left(\begin{pmatrix}\mu_1\\ \mu_2\end{pmatrix};\begin{pmatrix}\Sigma_{11} & \Sigma_{12}\\ \Sigma_{21} & \Sigma_{22}\end{pmatrix}\right) \tag{31}$$

then

$$E\left(\theta_1|\theta_2=x_2\right)=\mu_1+\Sigma_{21}\Sigma_{22}^{-1}\left(x_2-\mu_2\right) \tag{32}$$

A.2 Coupon of a par-yield bond

Using the definition that the present value of par-yield bonds is equal to one, we obtain

$$1 = c(m) \cdot \int_0^m exp(-r(t)t)dt + exp(-r(m)m) \tag{33}$$

where $r(m)$ is the spot rate, $c(m)$ is the coupon of the par-yield bond and m is its maturity. At a flat term structure, i.e. $r(m) = r \; \forall m$, and using the theorem about implicit functions, we obtain:

$$\frac{\partial c(m)}{\partial \alpha_i} = \frac{r^2 \int_0^m t \cdot \frac{\partial r}{\partial \alpha_i} \cdot exp(-rt)dt + rm \cdot \frac{\partial r}{\partial \alpha_i} \cdot exp(-rm)}{1 - exp(-rm)} \tag{34}$$

with $\frac{\partial r}{\partial \alpha_i} = f_i(\cdot)$ of Equation (3) and r is the flat level of interest, i.e. $r(m) = r = \alpha_0$ meaning that the respective derivatives are determined at $\alpha_1, \ldots, \alpha_n = 0$, and $c(m) = r$. (For the numerator of Equation (34), we apply Equation (27) to Equation (33)). For instance, in the case of Equation (5), we get $\frac{\partial r}{\partial \alpha_0} = 1$ and $\frac{\partial r}{\partial \alpha_1} = m$, yielding (applying Equation (28) to Equation (34)):

$$\frac{\partial c(m)}{\partial \alpha_0} = 1 \tag{35}$$

and (applying Equation (29) to Equation (34))

$$\frac{\partial c(m)}{\partial \alpha_1} = 2\frac{1 - (1 + r \cdot m)exp(-r \cdot m)}{r(1 - exp(-r \cdot m))}, \tag{36}$$

where the derivative is approximately equal to m, i.e. $lim_{r \to 0} \frac{\partial c(m)}{\partial \alpha_1} = m$. For the term structure model in Equation (5), we obtain, as the limiting case for a small steepness:

$$c(m) \approx \alpha_0 + \alpha_1 m \tag{37}$$

A.3 Return of the passive trading strategy $S(m)$

The return of the passive trading strategy $S(m)$ is the moving average of the current and past par-yield coupons (for the notation, see Subsection 2.2):

$$Re_t(S(m)) = \frac{1}{m} \sum_{i=1}^{m} c_{t-i+1} \tag{38}$$

For the linear term structure model of Equation (5) and for a small steepness, we can express the par-yield coupon as in Equation (37) and obtain:

$$Re_t(S(m)) = \frac{1}{m} \sum_{i=1}^{m} \alpha_{0,t-i+1} + \sum_{i=1}^{m} \alpha_{1,t-i+1} \tag{39}$$

In the following, we model the interest level $\alpha_{0,t}$ as a constant μ and a time trend γ, blurred by a noise term $\eta_{0,t}$:

$$\alpha_{0,t} = \mu + \gamma \cdot t + \eta_{0,t} \tag{40}$$

and, for the steepness $\alpha_{1,t}$, we assume that it fluctuates around the average st:

$$\alpha_{1,t} = st + \eta_{1,t} \tag{41}$$

Using the modeling of Equations (40) and (41), we can rewrite Equation (39):

$$\begin{aligned} Re_t(S(m)) &= \mu + m \cdot st + \gamma \cdot \sum_{i=1}^{m}(t-i+1) + \varepsilon_t \\ &= \mu + m \cdot st + \gamma \cdot t - \gamma \cdot \frac{m-1}{2} + \varepsilon_t \end{aligned} \tag{42}$$

where we use Equation (30) to reformulate the sum expression and set $\varepsilon_t = \frac{1}{m}\sum_{i=1}^{m}\eta_{0,t-i+1} + \sum_{i=1}^{m}\eta_{1,t-i+1}$. Often, we look at the return difference of two trading strategies. Using (42), we obtain for the expectation of the return difference:

$$E\left(Re_t(S(M_A)) - Re_t(S(M_L))\right) = \left(st - \frac{\gamma}{2}\right) \cdot (M_A - M_L) \tag{43}$$

The risk of this return difference measured as the euro duration is for a small interest level and for a small steepness $D^{€} = \frac{1}{2}(M_A - M_L)$ and, for a Bank i, it is $D_i^{€} = -50 \cdot IRR_i$ (see Equation (20)). Therefore:

$$E\left(\frac{NII_i^{term}}{A_i}\right) = -100 \cdot \frac{IRR_i}{A_i} \cdot \left(st - \frac{\gamma}{2}\right) \tag{44}$$

i.e. the average contribution from term transformation, the term $E\left(\frac{NII_i^{term}}{A_i}\right)$ is equal to Bank i's standardized interest rate risk multiplied with the difference of the average steepness of the term structure and half of the time trend.

A.4 Biases in the estimation

The true model is given by Equation (18), however we estimate (to keep the notation to a minimum, we drop the index k; we concentrate on the upward-shift scenario):

$$C.NIM(5)_i = \alpha + \tilde{\beta} \cdot phi_i + \tilde{\gamma} \cdot term_i + \tilde{\varepsilon}_i \tag{45}$$

We assume the following relationships between the theoretical values and their empirical counterparts.

$$C.NIM(5)_i = C.NIM(M_{A,i})_i + \theta_i \tag{46}$$

$$phi_i = (\phi_A - \phi_L)_i + \eta_i \tag{47}$$

$$term_i = (M_A\phi_A - M_L\phi_L)_i \tag{48}$$

To facilitate the calculation, the joint distribution is assumed to be normal, namely:

$$\begin{pmatrix} \phi_A - \phi_L \\ M_A\phi_A - M_L\phi_L \\ \varepsilon \\ \eta \\ \theta \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_1 \\ \mu_2 \\ 0 \\ 0 \\ \delta \end{pmatrix}; \begin{pmatrix} \sigma_1^2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 & \sigma_{2\theta} \\ 0 & 0 & \sigma_\varepsilon^2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_\eta^2 & 0 \\ 0 & \sigma_{2\theta} & 0 & 0 & \sigma_\theta^2 \end{pmatrix}\right) \tag{49}$$

Using the conditional expectation in Equation (32), we obtain for the parameters:

$$\tilde{\beta} = \beta \cdot \frac{\sigma_1^2}{\sigma_1^2 + \sigma_\eta^2} \tag{50}$$

$$\tilde{\gamma} = \gamma + \frac{\sigma_{2\theta}}{\sigma_2^2} \tag{51}$$

With the assumptions laid down in Subsection 4.2, one can calculate the covariance between "term" and θ (see Table 11).

Table 11: Banks with differing maturities

Type of bank	H	L
$share$	$p > 0$	$1 - p > 0$
Maturity	$M_A^H > T_{max}$	$M_A^L < T_{max}$
$term$	$M_A^H \phi_A - M_L \phi_L$	$M_A^L \phi_A - M_L \phi_L$
$\theta = C.NIM(T_{max}) - C.NIM(M_A)$	$\theta^H < 0$	$\theta^L = 0$
$share \cdot \theta \cdot \left(term - \overline{term}\right)$	$p\theta^H \left(M_A^H - \overline{M}\right) \phi_A$	0

This table shows components of the covariance between the variables "term" and θ. In this example (which can be easily generalized), there are two sorts of banks that only differ in the maturity of the asset side (the assumptions of Subsection 2.2 are valid).

References

Albertazzi, U. and L. Gambacorta (2009). Bank profitability and the business cycle. *Journal of Financial Stability 5(4)*, 393–409.

Alessandri, P. and B. Nelson (2015). Simple banking: profitability and the yield curve. *Journal of Money, Credit and Banking 47 (1)*, 143–175.

Basel Committee on Banking Supervision (2004). Principles for the management and supervision of interest rate risk. Bank for International Settlements.

Bliss, R. R. (1997). Movements in the term structure of interest rates. Economic Review, Federal Reserve Bank of Atlanta, Fourth Quarter 1997.

Busch, R., C. Drescher, and C. Memmel (2017). Bank stress testing under different balance sheet assumptions. Discussion Paper 07/2017, Deutsche Bundesbank.

Busch, R., H. C. N. Littke, C. Memmel, and S. Niederauer (2021). German banks' behavior in the low interest rate environment. Discussion Paper 23/2021, Deutsche Bundesbank.

Busch, R. and C. Memmel (2016). Quantifying the components of the banks' net interest margin. *Financial Markets and Portfolio Management 30(4)*, 371–396.

Busch, R. and C. Memmel (2017). Banks' net interest margin and the level of interest rates. *Credit and Capital Markets 50(3)*, 363–392.

Busch, R. and C. Memmel (2021). Why are bank rates on deposits so low? *Credit and Capital Markets 54(4)*, 641–668.

Chaudron, R. F. (2018). Bank's interest rate risk and profitability in a prolonged environment of low interest rates. *Journal of Banking and Finance 89*, 94–104.

Chaudron, R. F., L. de Haan, and M. Hoeberichts (2022). Banks' net interest income from maturity transformation and other interest income: Communicating vessels? *Journal of Financial Services Research forthcoming.*

Claessens, S., N. Coleman, and M. Donnelly (2018). "Low-For-Long" interest rates and banks' interest margins and profitability: Cross-country evidence. *Journal of Financial Intermediation 35*, 1–16.

Deutsche Bundesbank (2021). Monthly Report. 09/2021.

Diebold, F. X. and C. Li (2006). Forecasting the term structure of government bond yields. *Journal of Econometrics 130*, 337–364.

Dräger, V., L. Heckmann-Draisbach, and C. Memmel (2021). Interest and credit risk management in German banks: Evidence from a quantitative survey. *German Economic Review 22(1)*, 163–95.

English, W. B. (2002). Interest rate risk in the bank net interest margins. BIS Quarterly Review. December 2002.

Entrop, O., C. Memmel, B. Ruprecht, and M. Wilkens (2015). Determinants of bank interest margins: Impact of maturity transformation. *Journal of Banking and Finance 54* (May), 1–19.

Heckmann-Draisbach, L. and J. Moertel (2020). Hampered interest rate pass-through - a supply side story? Discussion Paper 59/2020, Deutsche Bundesbank.

Ho, T. S. Y. and A. Saunders (1981). The determinants of bank interest margins: Theory and empirical evidence. *Journal of Financial and Quantitative Analysis 16*, 581–600.

Kerbl, S., B. Simunovic, and A. Wolf (2019). Quantifying interest rate risk and the effect of model assumptions behind sight deposits. In OeNB (Ed.), *Financial Stability Report*, Volume 37, pp. 73–85.

Knez, P. J., R. Litterman, and J. Scheinkman (1994). Exploration into factors explaining money market returns. *Journal of Finance 49*, 1861–1882.

Litterman, R. and J. Scheinkman (1991). Common factors affecting bond returns. *Journal of Fixed Income 1*, 51–61.

McShane, R. W. and I. G. Sharpe (1985). A time series/cross section analysis of the determinants of Australian trading bank loan/deposit interest margins: 1962-1981. *Journal of Banking and Finance 9*, 115–136.

Memmel, C. (2008). Which interest rate scenario is the worst one for a bank? Evidence from a tracking bank approach for German savings and cooperative banks. *International Journal of Banking, Accounting and Finance 1(1)*, 85–104.

Memmel, C. (2011). Banks' exposure to interest rate risk, their earnings from term transformation, and the dynamics of the term structure. *Journal of Banking and Finance 35*, 282–289.

Memmel, C. (2014). Banks' interest rate risk: the net interest income perspective versus the market value perspective. *Quantitative Finance 14(6)*, 1059–1068.

Memmel, C. (2018). Why do banks bear interest rate risk? *Schmalenbach Business Review 70* (3), 231–253.

Nelson, C. R. and A. Siegel (1987). Parsimonious modeling of yield curves. *Journal of Business 60*, 473–489.

Oesterreichische Nationalbank (2013). Financial stability report 25.

Schich, S. T. (1997). Estimating the German term structure. Discussion Paper Deutsche Bundesbank, Series 1, 04/1997.

Svensson, L. E. O. (1994). Estimating and interpreting forward interest rates: Sweden 1992 - 94. IMF Working Paper 114.