Dual Labor Markets and the Equilibrium Distribution of Firms*

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"Towards a New Labour Market?"

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* Includes results from Auciello, Pijoan-Mas, Roldan-Blanco & Tagliati (2022): ***"Dual Labor Markets in Spain: A Firm-Side Perspective"***
The views expressed in this paper are the authors' and may not represent the opinions of Banco de España or the Eurosystem.

Motivation

- Many labor markets are characterized by the co-existence of:
  - Open-ended (OE), or *permanent*, contracts with large termination costs.
  - Fixed-term (FT), or *temporary*, contracts of short duration.

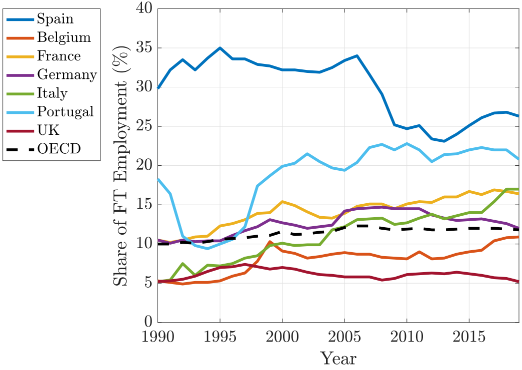

Figure: Share of employment in FT contracts, by year and country.
Source: OECD (stats.oecd.org).

- Effects of duality on workers are widely studied.
- Effects of duality on firms are largely unexplored:
  - What are firm-level determinants of contract choice?
  - What are macro consequences of dual LMs?

This paper:

Study the implications of dual LMs for:

(i) firm dynamics and the size distribution;

(ii) aggregate productivity and unemployment;

(iii) policy design (restricting use of FT contracts).

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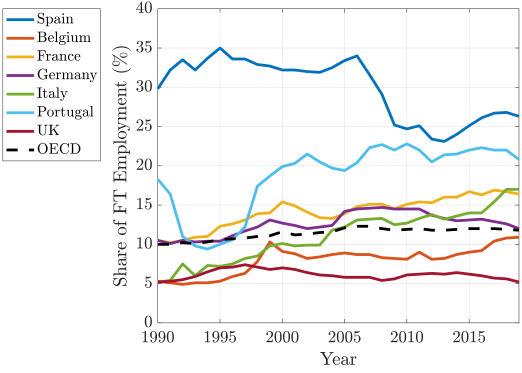

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Roadmap

1. Empirics:

- Micro data on the quasi-universe of Spanish firms (2004-2019).
  1. Large dispersion in usage of FT contracts across firms (FT share distribution is very right-skewed).
  2. Small share (15%) of overall variation in FT share comes from aggregate factors (time, industry, province).
  3. Share of FT workers increases with firm size (even after controlling for firm FE).

2. Firm-Dynamics Search-and-Matching Model:

- *Features* → (i) Multi-worker firms with DRS; (ii) directed search; (iii) long-term contracts; (iv) two-tier LM.
- *Calibration* → Target: (i) EU & UE rates (for both contracts); (ii) FT share across firm sizes.

3. Quantitative Results:

- **Key trade-off** → *High job-filling rates of FTs* vs. *low worker turnover rates of OEs.*
  - With DRS, larger firms have lower MPLs → Low opp. cost of unfilled vacancies → Prefer FT contracts.
- **Policy exercise:** Limit duration of FT contracts.
  - Policy succeeds in temp share ↓ and unemployment ↓ ...
  - ...but at the expense of aggregate productivity ↓ and aggregate output ↓

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Related Literature

1. **Effects of DLMs on labor market outcomes of workers**:

   Blanchard and Landier (2002); Cahuc and Postel-Vinay (2002); Bentolila, Cahuc, Dolado and Le Barbanchon (2012); Sala, Silva and Toledo (2012); Bentolila, Dolado and Jimeno (2008, 2019); Cabrales, Dolado and Mora (2017); Garcia-Louzao, Hospido and Ruggieri (2022).

2. **Co-existence of FT and OE contracts**:

   Dolado, Ortigueira, Stucchi (2016); Caggese and Cuñat (2008); Costain, Jimeno and Thomas (2010); Berton and Garibaldi (2012); Cao, Shao and Silos (2013); Cahuc, Charlot and Malherbet (2016); Dolado, Lalé and Siassi (2021).

3. **Macro-labor models with large firms**:

   Elsby and Michaels (2013); Moscarini and Postel-Vinay (2013); Acemoglu and Hawkins (2014); Kaas and Kircher (2015); Coles and Mortensen (2016); Schaal (2017); Bilal, Engbom, Mongey and Violante (2019); Gouin-Bonenfant (2020); Audoly (2020); Elsby and Gottfries (2021); Roldan-Blanco and Gilbukh (2021).

Contribution: Study dual labor markets from firm-side perspective and quantify macro consequences.

Empirics

Data

- Yearly firm-level data for Spain (2004-2019) from Banco de España's *Central de Balances*:
  - Unbalanced panel, quasi-universe of firms.
  - All non-finance sectors (except public sector and agriculture) at 4-digit NACE Rev. 2 level.
  - After cleaning $\rightarrow$ $\approx$ 7M firm-year observations ($\approx$ 700k unique firms).

- Firm-level information on:
  - Employment and type of employment contract (L_{OE} and L_{FT}).
  - Complete set of balance-sheet items (sales, age, materials, fixed inputs, tangible and intangible assets, ...).

- Temporary share $\rightarrow$ # temp workers within firm as a share of total number of workers:

$$TempSh = \frac{L_{FT}}{L_{FT} + L_{OE}}$$

  - *TempSh* in our firm-level data aggregates well to worker-level data from the Labor Force Survey. ▸ Plot

Stylized Facts

1. Distribution of *TempSh* is very right-skewed. ▸ Plots

	Mean	*p10*	*p25*	*p50*	*p75*	*p90*	*p95*
TempSh	0.183	0	0	0.027	0.294	0.591	0.800

- Distribution is also skewed within firm size & age groups. ▸ Table

2. There is large variation by time, region and sector. ▸ Plots ▸ Maps
   - *TempSh* is pro-cyclical (strong negative correlation with unemployment).
   - Large heterogeneity in *TempSh* across sectors (8% to 43%) and provinces (12% to 39%).

3. Firm-level determinants (details next slide):
   (i) Aggregate factors (time, region, sector) play a limited role → Only 16% of overall variance.
   (ii) Unobserved firm fixed-effects are very important → Nearly half of overall variance.
   (iii) Share of FT contracts is increasing in firm size (both unconditionally and controlling for firm FE).

→ Additional facts: ▸ TempSh and Unemployment by province ▸ TempSh and College Workers by province

Determinants of the Temporary Share

- Quantify the importance of each source of variation:

$$TempSh_{ft} = \alpha_t + \alpha_i + \alpha_r + \alpha_f + \boldsymbol{X}_{ft}^{\top}\boldsymbol{\beta} + \varepsilon_{ft}$$

where

1. $(\alpha_t, \alpha_i, \alpha_r, \alpha_f)$ are time, industry, region and firm FE.
2. $\boldsymbol{X}_{fir,t}$ are size bins in total employment (1-2, 3-5, 6-10, 11-20, etc.), and (possibly) other controls.

	(1)	(2)	(3)	(4)	(5)	(6)
Year FE	✓	✗	✗	✓	✓	✓
Region FE	✗	✓	✗	✓	✓	✓
Industry FE	✗	✗	✓	✓	✓	✓
Size Dummies	✗	✗	✗	✗	✓	✓
Firm FE	✗	✗	✗	✗	✗	✓
N	6,843,672	6,843,672	6,842,273	6,842,273	6,842,273	6,841,042
R^2	0.01	0.05	0.11	0.16	0.18	0.62

Table: Each column corresponds to an OLS regression of the share of temporary workers against several controls. The coefficient for the size dummies in the regressions in columns (5) and (6) are reported in the next slide.

- Other firm-level determinants → Full Regression Tables

Determinants of the Temporary Share

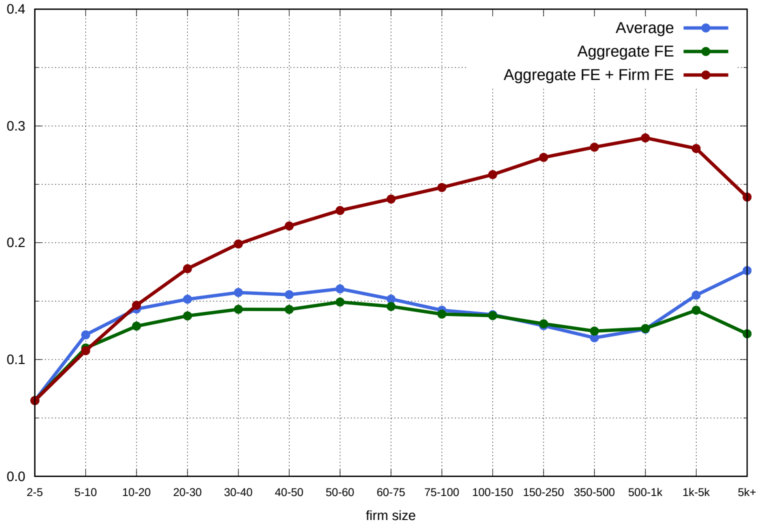

Figure: The blue line is the average of the temporary share across firms of different sizes (employment). The green line reports the coefficients of the size dummies of a regression of temporary share that controls for aggregate variables. The red line corresponds to a regression that additionally controls for firm fixed effects.

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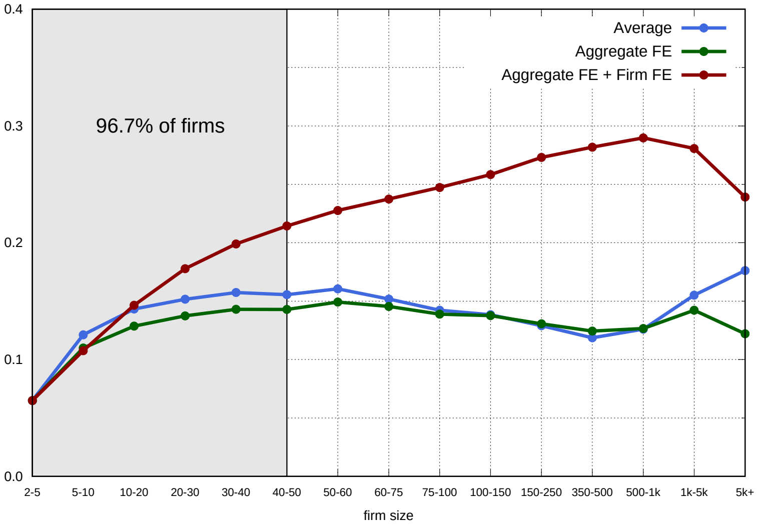

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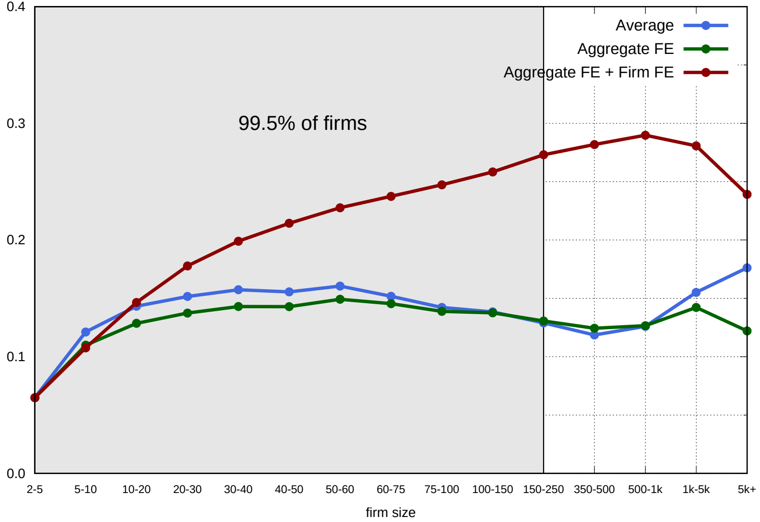

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Model

Environment

- Continuous and infinite time, $t \in \mathbb{R}_+$.
- Two types of agents:

  1. **Workers:** unit measure, ex-ante identical, ex-post:

     (a) *Unemployed* $\rightarrow$ Looking for a job, earning income flow $b > 0$.

     (b) *Employed* $\rightarrow$ Working for different types of firms under FT or OE contracts.

  2. **Firms:** endogenous measure $F > 0$ (free entry):

     (a) *Inactive* $\rightarrow$ No workers, pay fixed entry cost $\kappa > 0$ to get first worker.

     (b) *Incumbent* $\rightarrow$ Productivity $z \sim$ Markov, size $\vec{n} \equiv (n_{OE}, n_{FT}) \in \mathbb{Z}_+^2$, DRS technology:

$$Y(n_{OE}, n_{FT}, z) = \exp(z)\left(\omega n_{OE}^{\alpha} + (1-\omega)\, n_{FT}^{\alpha}\right)^{\nu/\alpha}, \qquad \omega \in (0,1),\ \nu \in (0,1),\ \alpha < 1$$

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Firms

- Worker flows within the firm:

  1. **Hire** workers by posting OE and FT long-term contracts (directed search).

  2. **Lose** workers:

     - Firm exit shock, at rate $s^F > 0$ *(exogenous)*.
     - Worker separation shock, at rate $s_i^W > 0$, $i = OE, FT$ *(exogenous)*.
     - Firing, at rate δ_i *(endogenous)*, with firing cost:

     $$C^F(\delta_i) = \chi_i \delta_i^{\psi_i}, \quad \chi_i > 0, \psi_i > 1$$

     - No on-the-job search.

  3. **Promote** an FT worker to an OE position at rate p, paying cost:

  $$C^P(p) = \chi_p p^{\psi_p}, \quad \chi_p > 0, \psi_p > 1$$

Labor Markets

- Firms attract new workers by posting long-term contracts. Contractual assumptions:
  1. Each contract of type $i = OE, FT$ signed at time t is fully state contingent at each tenure $j > 0$:
  $$c_{i,t,t+j} = c_i(\vec{n}_t^{t+j}, z_t^{t+j})$$
  2. Each contract $c_{i,t,t+j}$ specifies:
     - A wage trajectory, $w_i(\vec{n}_t^{t+j}, z_t^{t+j})$.
     - A per-worker firing rate trajectory, $\delta_i(\vec{n}_t^{t+j}, z_t^{t+j})$.
     - A per-worker promotion rate trajectory (for FT only), $p_{FT}(\vec{n}_t^{t+j}, z_t^{t+j})$.
  3. Full commitment on firm side, no commitment on worker side.
- Submarkets:
  - Indexed by $W_i \equiv$ Worker's expected PDV on the job under contract $i = OE, FT$.
  - Tightness in market segment $W_i \in [\underline{W}_i, \overline{W}_i]$ is $\theta(W_i) = f(W_i)/u(W_i)$.
  - CRS matching function:
  $$\mathcal{M}_i(f, u) = A_i f^{\gamma} u^{1-\gamma}, \qquad A_i > 0,\ \gamma \in (0, 1)$$

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Joint Surplus Problem

- Joint surplus $\rightarrow$ $\Sigma(\vec{n}, z) \equiv \boldsymbol{J}(\vec{n}, z, \vec{W}) + n_{OE} W_{OE} + n_{FT} W_{FT}$. Optimal contract solves:

$$\Sigma(\vec{n}, z) = \max_{p,\{\delta_i, W_i'(\vec{n}_i^+, z)\}} \frac{1}{\rho + s^F} \Bigg\{ \underbrace{\sigma(\vec{n}, z)}_{\text{Flow surplus}} + \underbrace{\sum_{i=OE,FT} n_i(\delta_i + s_i^W)\Big(\Sigma(\vec{n}_i^-, z) - \Sigma(\vec{n}, z)\Big)}_{\text{Type-}i\text{ worker separates}}$$
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subject to the worker-participation constraint:

$$W_i'(\vec{n}_i^+, z) \geq \boldsymbol{U}, \quad \forall i, (\vec{n}', z')$$

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\end{aligned}
$$

subject to the worker-participation constraint:

$$
W_i'(\vec{n}_i^{+},z) \geq \boldsymbol{U}, \quad \forall i,(\vec{n}',z')
$$

Worker/Firm Value Functions | Optimal Policies | Closing the model | Cobb-Douglas matching function

Joint Surplus Problem

- Joint surplus $\rightarrow$ $\boldsymbol{\Sigma}(\vec{n}, z) \equiv \boldsymbol{J}(\vec{n}, z, \vec{W}) + n_{OE} W_{OE} + n_{FT} W_{FT}$. Optimal contract solves:

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▸ Flow surplus

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▸ Worker/Firm Value Functions ▸ Optimal Policies ▸ Closing the model ▸ Cobb-Douglas matching function

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Flow surplus · Worker/Firm Value Functions · Optimal Policies · Closing the model · Cobb-Douglas matching function

Calibration

Model Fit: Moment matching

- Calibration predicts FT and OE workers are strong substitutes ($\alpha \approx 0.9$) and equally productive ($\omega \approx 0.5$).
- ...but FT market is more "liquid" ($A_{FT} \gg A_{OE}$) and FT contracts expire more quickly ($s_{FT} \gg s_{OE}$).

Parameter		**Value**	**Target [Source]**	**Model**	**Data**
Degree of decreasing RTS	ν	0.782	Average employment	6.70	6.72
Substitutability between workers	α	0.898	Agg. labor share	68.6%	61.3%
Relative productivity OE workers	ω	0.490	Average temporary share	17.7%	18.1%
Matching efficiency (FT market)	A_{FT}	1.534	UE rate (FT) Details	19.4%	18.5%
Matching efficiency (OE market)	A_{OE}	0.446	UE rate (OE)	1.4%	2.7%
Separation rate (FT workers)	s^W_{FT}	0.526	EU rate (FT)	13.2%	13.0%
Separation rate (OE workers)	s^W_{OE}	0.049	EU rate (OE)	1.5%	1.4%
Firm exit shock	s^F	0.009	Firm entry rate	0.9%	1.5%
Unemployment benefit	b	0.110	Value of leisure to output	29.1%	40.0%
Firing cost shifter (OE workers)	χ_{OE}	2.965	Temp share by size bin	*Next slide...*	
Promotion cost shifter	χ_p	0.015			

Table: The model period is one quarter. All numbers reported at quarterly frequency. UE and EU rates are averages over HP-filtered quarterly series from the EPA over the period 2005Q1-2018Q4 (data before 2005 is unavailable).

Model Fit: Temporary share by firm size

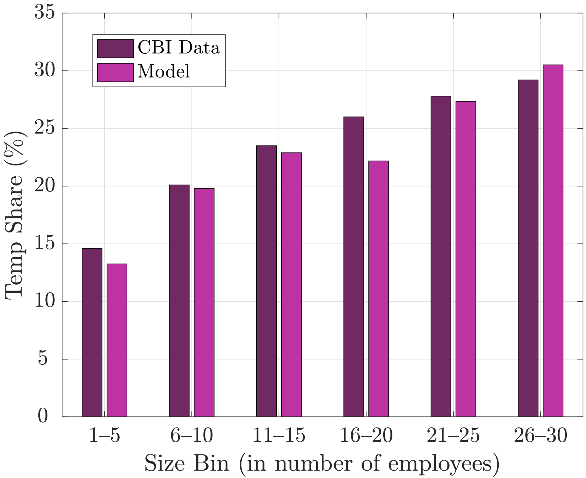

Figure: Each bar plot shows the average temporary share within the corresponding employment size bin, in the CBI data and in the calibrated model.

Trade-off between OE and FT: Key mechanism

- Calibration: matching efficiency is much higher in FT market: $\mathcal{M}_{FT}(f,u)/\mathcal{M}_{OE}(f,u) \simeq 3$
  - Both job-finding $\mu_i(\theta)$ and job-filling $\eta_i(\theta)$ rates are higher in FT market for the same $\theta = f/u$.
  - The calibration needs this to rationalize that $UE_{FT} \gg UE_{OE}$ (and a high FT share) in the data.

- Equilibrium: workers are ex-ante indifferent between contracts, and across firms within a contract.
  - At the same promised value W:
    1. Job-finding rates must be equal in both markets: $\mu_{FT}(\theta_{FT}(W)) = \mu_{OE}(\theta_{OE}(W))$
    2. Labor market tightness must be lower in FT: $\theta_{FT}(W) < \theta_{OE}(W)$
    3. Firms fill FT jobs faster than OE jobs: $\eta_{FT}(\theta_{FT}(W)) > \eta_{OE}(\theta_{OE}(W))$

- Thus, to attract OE workers firms need to promise higher value W! $\rightarrow$ **Key trade-off:**
  - It is harder and more expensive for firms to attract workers to OE positions.
  - But OE workers can be retained for longer (lower turnover rates) $\rightarrow$ Vacancies refilled less often.

- Missing in this discussion is an endogenous *recruiting intensity* or *vacancy posting* margin (in progress...).

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Trade-off between OE and FT: Heterogeneity across firms

- Different firms $(\vec{n}, z)$ resolve trade-off differently at different ratios n_{FT}/n_{OE}. ▸ Policy Functions ▸ Invariant Distribution

  1. At same productivity z, **larger firms**...

     (a) ...face lower opp. costs of unfilled vacancies (lower MPLs) $\Rightarrow$ Force toward higher FT share.

     (b) ...face higher cost of worker turnover (FT have short duration) $\Rightarrow$ Force toward lower FT share.

     $\rightarrow$ First effect dominates as firms get closer to their optimal size.

  2. At same size $\vec{n}$, **more productive firms**...

     - ...face higher opportunity costs of unfilled vacancies (high MPLs) $\Rightarrow$ Target lower share of FT.

- In the calibrated economy, larger firms operate with relatively more FT workers.

Trade-off between OE and FT: The role of matching efficiency

- Counterfactual exercise: Lower A_{FT} s.t. same matching efficiency in both markets ($A_{FT} = A_{OE} = 0.446$).

	Baseline	*Counterfactual*
Average employment per firm	6.70	13.78
Average temporary share	17.71%	1.22%
UE rate (FT)	19.44%	2.28%
UE rate (OE)	1.44%	2.59%
EU rate (FT)	13.23%	13.36%
EU rate (OE)	1.48%	1.45%
Promotion rate	5.50%	47.8%
Unemployment rate	14.5%	24.6%
Output per worker *(Baseline = 1)*	1.00	0.81

- Fast job-filling advantage of FT disappears $\rightarrow$ Firms switch into hiring from OE (as $s^W_{OE} < s^W_{FT}$):
  1. Within firms: Less FT hiring, more promotion $\Rightarrow$ TempSh $\downarrow$ $\Rightarrow$ Less worker separation $\Rightarrow$ Firm size $\uparrow$
  2. Across firms: Job-filling rates $\downarrow$ $\Rightarrow$ Value of being an incumbent $\downarrow$ $\Rightarrow$ Fewer active firms ($F \downarrow$)
  3. $UE_{FT} \downarrow$ sharply $\Rightarrow$ UE rates equalize and EU rates remain unchanged $\Rightarrow$ Unemployment $\uparrow$ (mechanically)
  4. ... but aggregate productivity $\downarrow \Rightarrow$ **Why?**

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Within- and Between-Firm Reallocation

- Let $f(\vec{n}, z) \equiv$ *Share* of firms in state $(\vec{n}, z)$; $E \equiv \#\{\text{employed workers}\}$; $F \equiv \#\{\text{active firms}\}$.
- Aggregate productivity:

$$\frac{Y}{E} = \frac{\sum_{\vec{n}} \sum_{z} Y(\vec{n}, z) f(\vec{n}, z)}{E/F} = \frac{\text{Avg. Output}}{\text{Avg. Firm Size}}$$

- Why does $Y/E \downarrow$? Two effects:
  1. Within-firm effect: Increase in # workers per firm, E/F $\Rightarrow$ $Y/E \downarrow$ force.
  2. Between-firm effect: Change in distribution, $f(\vec{n}, z)$, toward larger firms $\Rightarrow$ $Y/E \uparrow$ force. ▸ Plot

	Baseline	Counterfactual
Output per worker	1.00	0.81
... fixing E/F at baseline (all *between effect*)	.	1.68
... fixing $f(\vec{n}, z)$ at baseline (all *within effect*)	.	0.48

- Within-firm effect ($Y/E \downarrow$ force) dominates:
  - If employment E and # firms F did not change, new mix of workers across firms would $\uparrow$ productivity.
  - Due to DRS $\rightarrow$ Spreading same number of workers across fewer firms reduces productivity.

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Policy

Policy Experiment

- Several countries have recently restricted the use of FT contracts (e.g. Spain, December 2021).
  - Aim: reduce share of temporary workers.
  - Missing in the debate: what happens to firms?

- We capture this type of policy by reducing the average duration of FT contracts, $1/s^W_{FT}$.

- Reducing FT duration makes OE and FT less similar to each other $\Rightarrow$ Duality becomes stronger.

  1. **Policy 1:** Reducing average duration (2 to 1 quarters).
     - ✓ Temp Share $\downarrow$, Unemployment $\downarrow$
     - ✗ # firms $\downarrow$, aggregate productivity $\downarrow$, aggregate output $\downarrow$
  2. **Policy 2:** Increasing average duration (2 to 4 quarters).
     - ✗ Temp Share $\uparrow$, Unemployment $\simeq$
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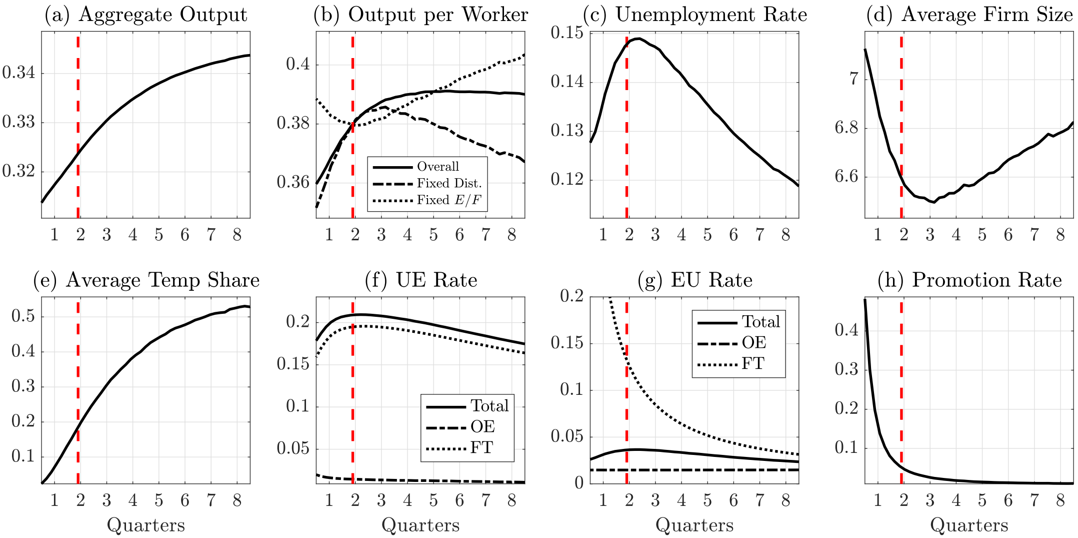
   - ✓ # firms $\uparrow$, aggregate productivity $\uparrow$, aggregate output $\uparrow$

Policy Results: Changing duration of FT contracts

	(A)	(B)	(C)
	Shorter FT duration (1 quarter)	*Baseline calibration* (1.9 quarters)	*Longer FT duration* (4 quarters)
Average employment per firm	6.97	6.70	6.62
Average temp share	6.80%	17.71%	36.91%
UE rate (total)	19.78%	20.84%	20.28%
... UE rate (FT)	18.19%	19.44%	19.04%
... UE rate (OE)	1.63%	1.44%	1.28%
EU rate (total)	3.06%	3.54%	3.28%
... EU rate (FT)	25.00%	13.23%	6.41%
... EU rate (OE)	1.48%	1.48%	1.48%
Promotion rate	16.57%	5.50%	1.97%
Unemployment rate	13.40%	14.51%	13.93%
Output per worker	0.968	1.000	1.027
... fixing avg. firm size E/F (all between effect)	1.008	.	1.016
... fixing distribution $f(\vec{n}, z)$ (all within effect)	0.961	.	1.011

Notes: Column (B) corresponds to the baseline calibration; in column (C), we set $s_{FT}^{W} = 1/4$ so that FT contracts expire on average after 1 year; in column (A) we set $s_{FT}^{W} = 1$, so that FT contracts expire on average after 1 quarter. The last two rows of the table compute output per worker while keeping either the average firm size or the distribution of firms fixed at the baseline calibration.

Policy Results: Changing duration of FT contracts



Notes: For all panels, the horizontal axis represents $1/s_{FT}^{W}$ (the expected duration of FT contracts), and is measured in quarters. The plots shows different stationary solutions of the model, keeping all parameters fixed at their baseline calibration values except for s_{FT}^{W}. The dashed vertical line shows the expected duration of FT contracts in the baseline calibration.

Conclusion

- Study implications of DLM for firm and worker dynamics and aggregate productivity.
- Empirically (Spain, 2004-2019):
  - Large degree of heterogeneity in the usage of FT contracts.
  - Most variation explained by between-firm dispersion.
  - Temporary share increases monotonically in firm size.
- Quantitatively:
  - Firm-dynamics search-and-matching model with DLM structure and long-term contracts.
  - Calibration $\rightarrow$ Trade-off between *fast job-filling rates* (FT) and *high worker retention rates* (OE).
    - Larger firms (lower MPL because of DRS) rely more on FT.
  - Policy $\rightarrow$ Increasing duration of FT contracts.
    - Policy is able to lower temp share, but at the expense of productivity.
    - Effects on unemployment potentially non-monotonic.

Thank you!

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Thank you!

Appendix

Appendix: Distribution of the Temporary Share (1/2)

Back to Empirics

- The distribution of temporary employment within the firm is very right-skewed:
  - The average is 18.1% and the median is 2.7%.
  - A relatively small fraction of firms make a very intensive use of FT contracts.

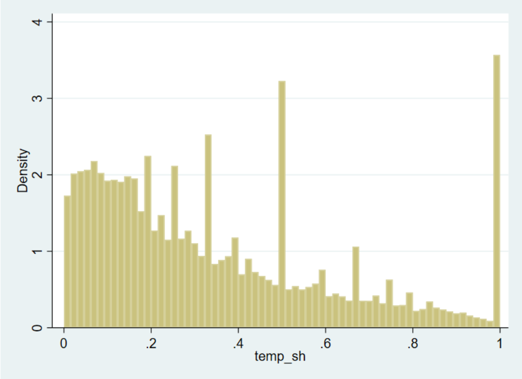
	% firms	Mean	p10	p25	p50	p75	p90	p95
Total	**100.00**	**0.183**	**0**	**0**	**0.027**	**0.294**	**0.591**	**0.800**
Firm size (in number of employees)								
1-9	77.65	0.164	0	0	0	0.250	0.551	0.776
10-49	19.04	0.250	0	0.031	0.163	0.391	0.677	0.825
50-249	2.77	0.249	0	0.032	0.156	0.381	0.684	0.852
+250	0.55	0.232	0	0.032	0.145	0.340	0.626	0.849
Firm age (in years)								
0-5	21	0.248	0	0	0.084	0.448	0.770	1,000
6-10	23	0.199	0	0	0.037	0.333	0.634	0.832
11-15	21	0.175	0	0	0.020	0.280	0.552	0.750
16-20	16	0.152	0	0	0.005	0.232	0.500	0.669
21-30	15	0.134	0	0	0.009	0.198	0.439	0.600
+30	4	0.114	0	0	0.018	0.160	0.358	0.500

Table: Distribution of FT contracts, overall and by size and age bins.

Source: Auciello, Pijoan-Mas, Roldan-Blanco and Tagliati (2022): *“Dual Labor Markets in Spain: A Firm-Side Perspective”*.

Appendix: Distribution of the Temporary Share (2/2)

▸ Back to Empirics



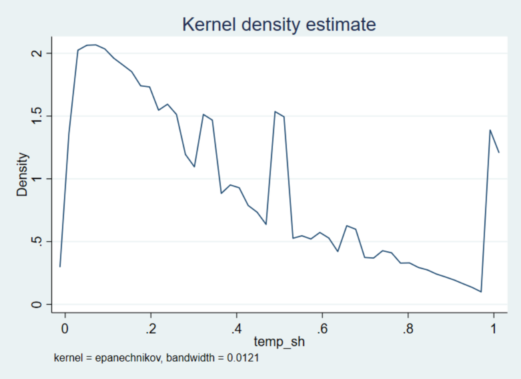


Figure: Histogram and kernel density of the distribution of firm-level temporary share.

Source: Auciello, Pijoan-Mas, Roldan-Blanco and Tagliati (2022): *"Dual Labor Markets in Spain: A Firm-Side Perspective"*.

Appendix: Aggregate Variation in the Temporary Share (1/2)

Back to Empirics

- There is a lot of variation, across time, regions, and sectors.

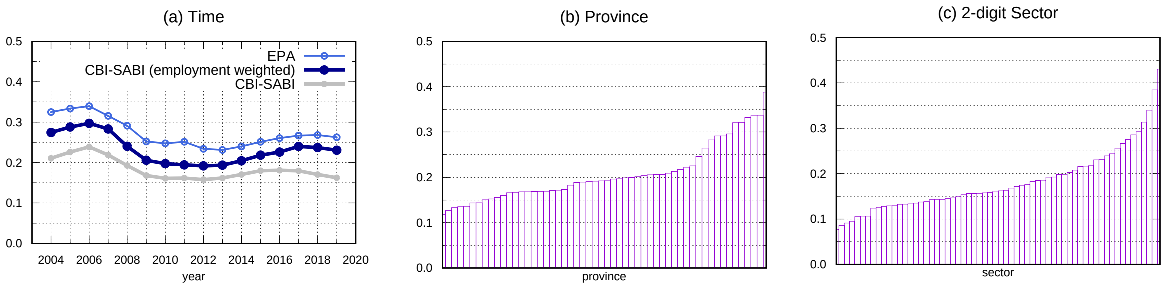


Figure: Panel (a) reports the average share of temporary workers by year. The gray line corresponds to the average across firms; the dark blue line weights each firm by the employment size, thereby providing the share of temporary workers across workers; the light blue line provides the share of temporary workers across workers computed through the Labor Force Survey. Panel (b) reports the average share of temporary workers across firms by province (sorted from smallest to largest). Panel (c) reports the average share of temporary workers across firms by 2-digit sector (sorted from smallest to largest).

Appendix: Aggregate Variation in the Temporary Share (2/2)

Back to Empirics

- Regional variation: simple average (LHS) and province fixed-effects (RHS).

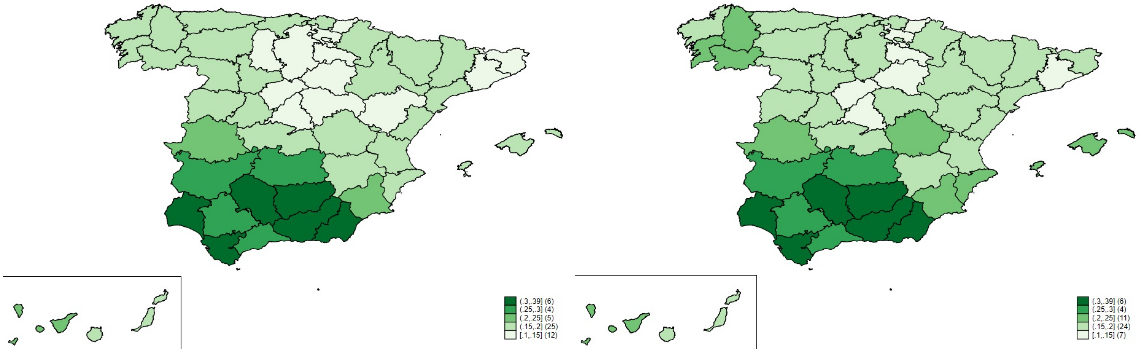


Figure: *LHS:* Simple average of the temporary share within each Spanish province. *RHS:* Province fixed-effects (with Barcelona as the reference region) of a regression of the temporary share against time, sector and province fixed effects.

Source: Auciello, Pijoan-Mas, Roldan-Blanco and Tagliati (2022): *“Dual Labor Markets in Spain: A Firm-Side Perspective”*.

Appendix: Correlation between Unemployment and Temporary Share

Back to Empirics

- At the province level, correlation between unemployment rate and:
  1. Average temporary share (LHS);
  2. Province fixed effect (α_r) (RHS), in a regression $TempSh_{ft} = \alpha_t + \alpha_i + \alpha_r + \varepsilon_{ft}$.

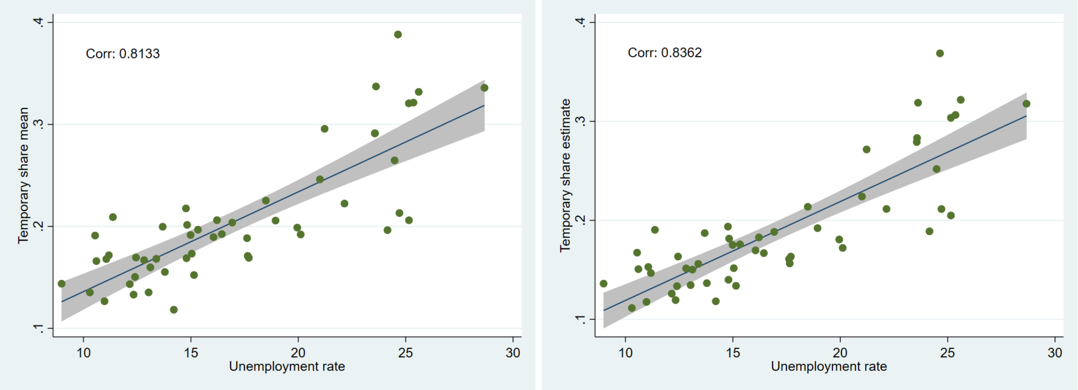


Figure: *LHS:* correlation between the simple average of the temporary share within each Spanish province and the unemployment rate of the province. *RHS:* Correlation between the unemployment rate with province fixed-effects coefficients of a regression of the temporary share against time, sector and province fixed effects.

Source: Auciello, Pijoan-Mas, Roldan-Blanco and Tagliati (2022): *"Dual Labor Markets in Spain: A Firm-Side Perspective"*.

Appendix: Correlation between College Workers and Temporary Share

Back to Empirics

- At the province level, correlation between share of workers with college degree and:
  1. Average temporary share (LHS);
  2. Province fixed effect (α_r) (RHS), in a regression $TempSh_{ft} = \alpha_t + \alpha_i + \alpha_r + \varepsilon_{ft}$.

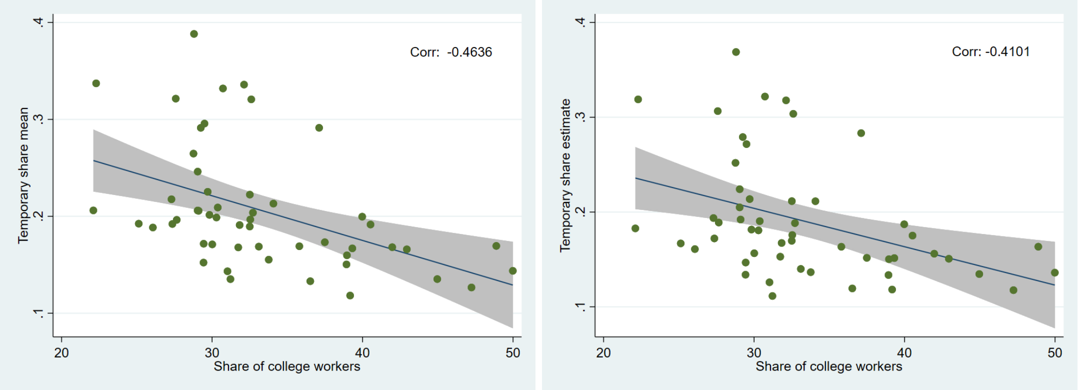


Figure: *LHS:* correlation between the simple average of the temporary share within each Spanish province and the share of college workers in the province. *RHS:* Correlation between the share of college workers with province fixed-effects coefficients of a regression of the temporary share against time, sector and province fixed effects.

Source: Auciello, Pijoan-Mas, Roldan-Blanco and Tagliati (2022): *"Dual Labor Markets in Spain: A Firm-Side Perspective"*.

Appendix: Determinants of the Temporary Share

▸ Back to Empirics

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Empl. (log)	0.0598***	0.0602***	0.0583***	0.0540***	0.0535***	0.0713***	0.0668***
	(0.000101)	(0.000102)	(0.0000991)	(0.000106)	(0.000104)	(0.000271)	(0.000279)
Leverage	7.19e-08	9.21e-08	1.01e-07	7.54e-08	1.24e-07	-1.62e-07	-1.50e-07
	(0.0000003)	(0.0000003)	(0.0000003)	(0.0000003)	(0.0000003)	(0.0000001)	(0.0000001)
Sales p.w. (log)	-0.00577***	-0.00700***	-0.00399***	-0.00115***	-0.00118***	0.00575***	0.00349***
	(0.000158)	(0.000159)	(0.000155)	(0.000164)	(0.000162)	(0.000173)	(0.000175)
Avg wage (log)	-0.0457***	-0.0450***	-0.0307***	-0.0435***	-0.0294***	-0.0183***	-0.0168***
	(0.000283)	(0.000284)	(0.000279)	(0.000285)	(0.000282)	(0.000360)	(0.000362)
Age	-0.00794***	-0.00833***	-0.00793***	-0.00723***	-0.00749***	-0.00767***	
	(0.000025)	(0.000025)	(0.000025)	(0.000025)	(0.000025)	(0.000053)	
Age^2	0.0000700***	0.0000730***	0.0000720***	0.0000663***	0.0000699***	0.000143***	
	(0.00000048)	(0.00000049)	(0.00000049)	(0.00000048)	(0.00000049)	(0.0000013)	
Constant	0.346***	0.352***	0.294***	0.323***	0.282***	0.178***	0.120***
	(0.000749)	(0.000753)	(0.000740)	(0.000755)	(0.000752)	(0.00127)	(0.00124)
Year FE	✗	✓	✗	✗	✓	✗	✓
Province FE	✗	✗	✓	✗	✓	✗	✓
Sector 2dig FE	✗	✗	✗	✓	✓	✗	✓
Firm FE	✗	✗	✗	✗	✗	✓	✓
N	5,300,548	5,300,548	5,300,548	5,299,668	5,299,668	5,284,540	5,283,862
R^2	0.092	0.095	0.135	0.153	0.194	0.672	0.672

Source: Auciello, Pijoan-Mas, Roldan-Blanco and Tagliati (2022): *"Dual Labor Markets in Spain: A Firm-Side Perspective"*.

Appendix: Unemployed Workers

Back to Model

- Unemployed worker searches in ex-post most profitable labor market:

$$\boldsymbol{U}_i = \max_{W \in [\underline{W}, \overline{W}]} U_i(W)$$

where $U_i(W)$ solves:

$$\rho U_i(W) = b + \mu_i(\theta(W)) \max\Big(W - U_i(W), 0\Big)$$

- Ex-ante, workers must remain indifferent between where to search, s.t. $\boldsymbol{U}_{FT} = \boldsymbol{U}_{OE} \equiv \boldsymbol{U}$. Thus:

$$\forall (W, i), \quad U_i(W) \leq \boldsymbol{U}, \text{ with equality if, and only if, } \mu_i(\theta(W)) > 0$$

- This determines the equilibrium market tightness in labor market i:

$$\theta_i(W, \boldsymbol{U}) = \mu_i^{-1}\left(\frac{\rho \boldsymbol{U} - b}{W - \boldsymbol{U}}\right)$$

Appendix: Worker's Value Function

Back to Model

- Value of a worker employed in contract $i = OE, FT$:

$$\rho \boldsymbol{W}_i(\vec{n}, z; \mathcal{C}) = w_i + \underbrace{(\delta_i + s_i^W + s^F)\Big(\boldsymbol{U} - \boldsymbol{W}_i(\vec{n}, z; \mathcal{C})\Big)}_{\text{Worker separates}} + \underbrace{(n_i - 1)(\delta_i + s_i^W)\Big(W_i'(\vec{n}_i^-, z) - \boldsymbol{W}_i(\vec{n}, z; \mathcal{C})\Big)}_{\text{Co-worker type } i \text{ separates}}$$
$$+ \underbrace{n_{-i}(\delta_{-i} + s_{-i}^W)\Big(W_i'(\vec{n}_{-i}^-, z) - \boldsymbol{W}_i(\vec{n}, z; \mathcal{C})\Big)}_{\text{Co-worker type } -i \text{ separates}} + \underbrace{n_{FT}\, p\Big(W_i^p(\vec{n}^p, z) - \boldsymbol{W}_i(\vec{n}, z; \mathcal{C})\Big)}_{\text{Promotions of } FT \text{workers}}$$
$$+ \underbrace{\sum_{j \in \mathcal{I}} \eta_j \Big(W_j'(\vec{n}_j^+, z)\Big)\Big(W_i'(\vec{n}_j^+, z) - \boldsymbol{W}_i(\vec{n}, z; \mathcal{C})\Big)}_{\text{Hiring of type } j \text{ worker}} + \underbrace{\sum_{z' \in \mathcal{Z}} \lambda(z'|z)\Big(W_i'(\vec{n}, z') - \boldsymbol{W}_i(\vec{n}, z; \mathcal{C})\Big)}_{\text{Productivity shocks}}$$

where

$$W_i^p(\vec{n}^p, z) \equiv \begin{cases} \dfrac{1}{n_{FT}}\Big(W_{OE}'(\vec{n}^p, z) + (n_{FT} - 1)W_{FT}'(\vec{n}^p, z)\Big) & \text{for } i = FT \\ W_{OE}'(\vec{n}^p, z) & \text{for } i = OE \end{cases}$$

Notation: $\vec{n}_i^+ \equiv (n_i + 1, n_{-i})$; $\vec{n}_i^- \equiv (n_i - 1, n_{-i})$; $\vec{n}^p \equiv (n_{OE} + 1, n_{FT} - 1)$.

Appendix: Firm's Value Function

Back to Model

- Value of a firm offering menu of contracts $\mathcal{C} \equiv \left(c_{OE}, c_{FT}\right)$ in state $(\vec{n}, z)$:

$$\rho \boldsymbol{J}(\vec{n}, z, \vec{W}) = \max_{\{w_i, \delta_i, p, W_i'(\vec{n}', z')\}_{i \in \mathcal{I}}} \Bigg\{ \underbrace{\exp(z) y(\vec{n})}_{\text{Sales}} - \underbrace{\chi_p p^{\psi_p}}_{\text{Promotion costs}} + \sum_{i \in \mathcal{I}} \Bigg[- \underbrace{w_i n_i}_{\text{Wage bill}} - \underbrace{\chi_i \delta_i^{\psi_i}}_{\text{Firing costs}} + \underbrace{s^F \left(\boldsymbol{J}^e - \boldsymbol{J}(\vec{n}, z, \vec{W}) \right)}_{\text{Firm exits}}$$

$$+ \underbrace{n_i(\delta_i + s_i^W) \left(\boldsymbol{J}(\vec{n}_i^-, z, \vec{W}'(\vec{n}_i^-, z)) - \boldsymbol{J}(\vec{n}, z, \vec{W}) \right)}_{\text{Worker type } i \text{ separates}} + \underbrace{\eta_i \left(W_i'(\vec{n}_i^+, z) \right) \left(\boldsymbol{J}(\vec{n}_i^+, z, \vec{W}'(\vec{n}_i^+, z)) - \boldsymbol{J}(\vec{n}, z, \vec{W}) \right)}_{\text{Firm hires type } i \text{ worker}} \Bigg]$$

$$+ \underbrace{n_{FT} p \left(\boldsymbol{J}(\vec{n}^p, z, \vec{W}'(\vec{n}^p, z)) - \boldsymbol{J}(\vec{n}, z, \vec{W}) \right)}_{\text{Promotion of } FT \text{ workers}} + \underbrace{\sum_{z' \in \mathcal{Z}} \lambda(z'|z) \left(\boldsymbol{J}(\vec{n}, z', \vec{W}'(\vec{n}, z')) - \boldsymbol{J}(\vec{n}, z, \vec{W}) \right)}_{\text{Productivity shock}} \Bigg\}$$

subject to:

$$\forall i: \quad \boldsymbol{W}_i(\vec{n}, z; \mathcal{C}) \geq W_i$$

$$\forall (\vec{n}', z'),\ \forall i: \quad W_i'(\vec{n}', z') \geq \boldsymbol{U}$$

Notation: $\vec{n}_i^+ \equiv (n_i + 1, n_{-i})$; $\vec{n}_i^- \equiv (n_i - 1, n_{-i})$; $\vec{n}^p \equiv (n_{OE} + 1, n_{FT} - 1)$.

Appendix: Flow Surplus

Back to Model

- Flow surplus:

$$\sigma(\vec{n}, z) \equiv \underbrace{\exp(z)\left(\omega n_{OE}^{\alpha} + (1-\omega)\, n_{FT}^{\alpha}\right)^{\nu/\alpha}}_{\text{Firm's profits}} + \underbrace{\sum_{i \in \mathcal{I}} n_i (\delta_i + s_i^W + s^F) \boldsymbol{U}}_{\text{Workers' outside options}}$$

$$- \underbrace{\chi_p p^{\psi_p}}_{\substack{\text{Promotion} \\ \text{costs}}} - \underbrace{\sum_{i \in \mathcal{I}} \chi_i \delta_i^{\psi_i}}_{\text{Firing costs}} - \underbrace{\sum_{i \in \mathcal{I}} \eta_i \Big(W_i'(\vec{n}_i^{+}, z)\Big) W_i'(\vec{n}_i^{+}, z)}_{\text{Commitment costs}}$$

Appendix: Recursive Contracts

Back to Model

- Focus on a Markov Perfect Equilibrium $\rightarrow$ Solve for a recursive equilibrium.
- Firm chooses menu $\mathcal{C} \equiv \left(c_{OE}, c_{FT}\right)$ in state $(\vec{n}, z)$, where each c_i is composed of:

1. A spot wage, w_i.
2. A per-worker firing rate, δ_i.
3. A per-worker promotion rate, p *(for FT contracts only)*.
4. A set of new promised worker values, $\{W_i'(\vec{n}', z')\}$ for each next state $(\vec{n}', z')$, where:

$$(\vec{n}', z') \in \begin{cases} (n_{OE}+1, n_{FT}, z),\ (n_{OE}, n_{FT}+1, z), & \leftarrow \text{hiring} \\ (n_{OE}-1, n_{FT}, z),\ (n_{OE}, n_{FT}-1, z), & \leftarrow \text{worker separation} \\ (n_{OE}+1, n_{FT}-1, z), & \leftarrow \text{promotion} \\ \{(n_{OE}, n_{FT}, z') : \forall z' \in \mathcal{Z}\} & \leftarrow z\text{-shock} \end{cases}$$

Appendix: Value Functions

Back to Model

- **Unemployed Worker:** Earns value $\boldsymbol{U}$ and remain ex-ante indifferent. HJB Equation
- **Employed Worker:** Value $\boldsymbol{W}_i(\vec{n}, z; \mathcal{C})$ while employed in contract $c_i \in \mathcal{C}$. HJB Equation
- **Firm:** Value $\boldsymbol{J}(\vec{n}, z, \vec{W})$, where $\vec{W} \equiv (W_{OE}, W_{FT})$ are the outstanding promises. HJB Equation
  - Firm must choose menu $\mathcal{C} \equiv (c_{OE}, c_{FT})$ under two constraints: Recursive Contracts

1. Promise-keeping $\rightarrow$ $\boldsymbol{W}_i(\vec{n}, z; \mathcal{C}) \geq W_i, \quad \forall i$
2. Worker-participation $\rightarrow$ $W_i'(\vec{n}', z') \geq \boldsymbol{U}, \quad \forall i, \ \forall(\vec{n}', z')$

Proposition: The optimal contract menu in firm state $\boldsymbol{J}(\vec{n}, z, \vec{W})$ maximizes the **joint surplus**:

$$\boldsymbol{\Sigma}(\vec{n}, z) \equiv \boldsymbol{J}(\vec{n}, z, \vec{W}) + \sum_{i=OE,FT} n_i W_i$$

- *Intuition:*
  - The firm pays the lowest w_i that is consistent with promise-keeping $\rightarrow$ $\boldsymbol{W}_i\left(\vec{n}, z; \overbrace{\{w_i, \delta_i, W_i'\}}^{=\mathcal{C}}\right) = W_i$.
  - This makes Σ invariant in $\vec{W}$: payoffs are linear and utilities are transferable.

Appendix: Optimal Policies

▸ Back to Model

Notation: $\vec{n}_i^+ \equiv (n_i+1, n_{-i})$; $\vec{n}_i^- \equiv (n_i-1, n_{-i})$; $\vec{n}^p \equiv (n_{OE}+1, n_{FT}-1)$.

1. Promised value, W_i^+:

$$\underbrace{\underbrace{\frac{\partial \eta_i(W_i^+)}{\partial W_i^+} W_i^+}_{\text{New utility to pre-existing workers}} + \underbrace{\eta_i(W_i^+)}_{\text{Utility to new worker}}}_{\textit{Expected marginal cost}} = \underbrace{\underbrace{\frac{\partial \eta_i(W_i^+)}{\partial W_i^+}}_{\text{Change in job-filling rate}} \underbrace{\left(\boldsymbol{\Sigma}(\vec{n}_i^+, z) - \boldsymbol{\Sigma}(\vec{n}, z)\right)}_{\text{Gain in joint surplus}}}_{\textit{Expected marginal gain}}$$

2. Per-worker firing (δ_i) and promotion (p) rates:

$$\psi_i \chi_i \delta_i^{\psi_i - 1} = n_i \left(\boldsymbol{U} + \boldsymbol{\Sigma}(\vec{n}_i^-, z) - \boldsymbol{\Sigma}(\vec{n}, z) \right) \quad \text{and} \quad \psi_p \chi_p p^{\psi_p - 1} = n_{FT} \left(\boldsymbol{\Sigma}(\vec{n}^p, z) - \boldsymbol{\Sigma}(\vec{n}, z) \right)$$

3. Wage (w_i) $\rightarrow$ Backed out from the promise-keeping constraint:

$$\boldsymbol{W}_i\left(\vec{n}, z; \{w_i, \delta_i, W_i'\}\right) = W_i$$

Appendix: Closing the Model

Back to Model

- The free-entry condition pins down $\theta(n_i^e, z^e)$ for $\vec{n}_i^e \in \{(1,0),(0,1)\}$:

$$\kappa = \max_{\{W_i^e(z^e)\}} \left\{ \sum_{z^e \in \mathcal{Z}} \pi_z(z^e) \left[\sum_{i=OE,FT} \eta_i \Big(W_i^e(z^e) \Big) \Big(\Sigma(\vec{n}_i^e, z^e) - W_i^e(z^e) \Big) \right] \right\}$$

- Distribution and aggregate dynamics:
  - Share of firms $f_t(\vec{n}, z)$ solves a set of flow equations. Details
  - From those, we can obtain:
    1. Dynamics of type-i employed workers using $e_{i,t}(\vec{n}, z) = n_i f_t(\vec{n}, z)$.
    2. EU and UE rates by type of contract. Details
    3. Unemployment rate:

$$U_t = 1 - \sum_i \sum_{\vec{n}} \sum_z e_{i,t}(\vec{n}, z)$$

Appendix: Cobb-Douglas Matching Function

▸ Back to Model

- Cobb-Douglas matching function: $\mathcal{M}_i(f, u) = A_i f^{\gamma} u^{1-\gamma}$.

- Surplus split:

$$\underbrace{W_i'(\vec{n}_i^+, z) - \boldsymbol{U}}_{\text{Surplus for new hire}} = (1-\gamma)\Big(\boldsymbol{\Sigma}(\vec{n}_i^+, z) - \boldsymbol{\Sigma}(\vec{n}, z) - \boldsymbol{U}\Big)$$

$$\underbrace{\boldsymbol{J}\Big(\vec{n}_i^+, z, \vec{W}'(\vec{n}_i^+, z)\Big) - \boldsymbol{J}(\vec{n}, z, \vec{W})}_{\text{Surplus to firm}} = \gamma \underbrace{\Big(\boldsymbol{\Sigma}(\vec{n}_i^+, z) - \boldsymbol{\Sigma}(\vec{n}, z) - \boldsymbol{U}\Big)}_{\text{New joint surplus generated}} + \underbrace{\sum_j n_j \Big(W_j'(\vec{n}, z) - W_j'(\vec{n}_i^+, z)\Big)}_{\substack{\text{Transfer of value between} \\ \text{firm and pre-existing workers}}}$$

- Equilibrium job-filling function:

$$\eta_i(\vec{n}_i^+, z) = A_i^{\frac{1}{\gamma}} \left[(1-\gamma)\frac{\boldsymbol{\Sigma}(\vec{n}_i^+, z) - \boldsymbol{\Sigma}(\vec{n}, z) - \boldsymbol{U}}{\rho \boldsymbol{U} - b}\right]^{\frac{1-\gamma}{\gamma}}$$

Appendix: Distribution Dynamics

▸ Back to Model

- Let $f_t(\vec{n}, z)$ be the measure of firms in state $(\vec{n}, z)$ at time t. Then:

$$\frac{\partial f_t(\vec{n}, z)}{\partial t} = \sum_i \eta_i\big(W_i'(\vec{n}_i^-, z)\big) f_t(\vec{n}_i^-, z) + \sum_i (n_i + 1)\Big(\delta_i(\vec{n}_i^+, z) + s_i^W\Big) f_t(\vec{n}_i^+, z)$$
$$+ (n_{FT} + 1) p(\vec{n}_p^-, z) f_t(\vec{n}_p^-, z) + \sum_{\hat{z} \neq z} \lambda(z|\hat{z}) f_t(\vec{n}, \hat{z})$$
$$- \left[s^F + \sum_i \eta_i\big(W_i'(\vec{n}, z)\big) + \sum_i n_i \Big(\delta_i(\vec{n}, z) + s_i^W\Big) + n_{FT} p(\vec{n}, z) + \sum_{\hat{z} \neq z} \lambda(\hat{z}|z) \right] f_t(\vec{n}, z)$$

- Let F_t^e be the measure of potential entrants at time t. Then:

$$\frac{\partial F_t^e}{\partial t} = s^F F_t + \sum_{z \in \mathcal{Z}} \sum_i \Big(\delta_i(\vec{n}_i^e, z) + s_i^W\Big) f_t(\vec{n}_i^e, z) - F_t^e \left(\sum_{z^e \in \mathcal{Z}} \pi_z(z^e) \sum_i \eta_i\big(W_i'(\vec{n}_i^e, z^e)\big) \right)$$

Appendix: Calibration Strategy

Back to Calibration

- For estimation, restrict sample to firms with $n_{FT} + n_{OE} \leq 50$ ($= 96.7\%$ of firms in our data).
  - Save on state space, keep log-normal productivity innovations (Ornstein-Uhlenbeck process for z).
- Some parameters are set externally:

Parameter	Description	Value	Target/Source
ρ	Discount rate	0.0129	5% annual real interest rate
χ_{FT}	Firing cost shifter (FT)	$+\infty$	Spanish labor market regulation
κ	Fixed entry cost	2,373.05	Measure of active firms in equilibrium
γ	Matching elasticity	0.5	Petrongolo and Pissarides (2001)
(ρ_z, σ_z)	Productivity parameters	(0.2053,0.1700)	Ruíz-Garcia (2020)

- Internally-calibrated parameters set to match key features of Spanish data:
  1. Average temporary share, and relationship between temporary share and firm size.
  2. UE and EU rates for both OE and FT contacts.
  3. Aggregate moments: average firm size, firm entry rate, labor share.

Appendix: UE and EU Rates (1/2)

- **In the data**:
  - Denote by $UE^i_{t,t+1}$ the U-to-E flow from quarter t to $t+1$ into a contract of type $i = OE, FT$.
  - Similarly for $EU^i_{t,t+1}$ (EU flows) and $EE^{FtoO}_{t,t+1}$ (E-to-E flows from an FT into an OE contract).
  - Then, labor market rates are:

$$\widehat{UE}_i^{\text{data}} \equiv \frac{\sum UE^i_{t,t+1}}{\sum U_t}; \qquad \widehat{EU}_i^{\text{data}} \equiv \frac{\sum EU^i_{t,t+1}}{\sum E^i_t}; \qquad \widehat{EE}_{FtoO}^{\text{data}} \equiv \frac{\sum EE^{FtoO}_{t,t+1}}{\sum E^{FT}_t}$$

  where $\sum U_t$ is the #unemployed, and $\sum E^i_t$ is #employed in contract type i.

- **In the model**:
  - Put workers into 3 employment states:
    1. Employed with an OE contract.
    2. Employed with a FT contract.
    3. Unemployed.
  - Then, we write flow equations between these states and look for stationary measures (next slide).

Appendix: UE and EU Rates (2/2)

Back to Model Back to Calibration

- Flow equations:

$$\frac{\partial}{\partial t}\begin{bmatrix} E_{OE} \\ E_{FT} \\ U \end{bmatrix} = \begin{pmatrix} -\lambda_{EU_{OE}} & \lambda_{EE_{FtoO}} & \lambda_{UE_{OE}} \\ 0 & -(\lambda_{EU_{FT}} + \lambda_{EE_{FtoO}}) & \lambda_{UE_{FT}} \\ \lambda_{EU_{OE}} & \lambda_{EU_{FT}} & -(\lambda_{UE_{OE}} + \lambda_{EU_{FT}}) \end{pmatrix} \begin{bmatrix} E_{OE} \\ E_{FT} \\ U \end{bmatrix}$$

where $\lambda_{EU_{OE}} \equiv \frac{EU_{OE}}{E_{OE}}$, $\lambda_{EE_{FtoO}} \equiv \frac{EE_{FtoO}}{E_{FT}}$, $\lambda_{UE_{OE}} \equiv \frac{UE_{OE}}{U}$, $\lambda_{EU_{FT}} \equiv \frac{EU_{FT}}{E_{FT}}$, and $\lambda_{UE_{FT}} \equiv \frac{UE_{FT}}{U}$, with:

$$EU_{OE} \equiv \sum_{\vec{n}} \sum_{z} \left(\delta_{OE}(\vec{n}, z) + s^W_{OE} + s^F\right) e_{OE}(\vec{n}, z) \qquad EE_{FtoO} \equiv \sum_{\vec{n}} \sum_{z} p(\vec{n}, z) e_{FT}(\vec{n}, z)$$

$$UE_{OE} \equiv \sum_{\vec{n}} \sum_{z} \mu_{OE}(\vec{n}^+_{OE}, z) u_{OE}(\vec{n}^+_{OE}, z) \qquad EU_{FT} \equiv \sum_{\vec{n}} \sum_{z} \left(\delta_{FT}(\vec{n}, z) + s^W_{FT} + s^F\right) e_{FT}(\vec{n}, z)$$

$$UE_{FT} \equiv \sum_{\vec{n}} \sum_{z} \mu_{FT}(\vec{n}^+_{FT}, z) u_{FT}(\vec{n}^+_{FT}, z)$$

- Get stationary measures (E_{OE}, E_{FT}, U) by solving the system $\frac{\partial}{\partial t}[E_{OE}, E_{FT}, U]^\top = \vec{0}$.
- Then $\rightarrow$ $\widehat{UE}_i^{\text{model}} = \frac{1-e^{-UE_i \Delta t}}{U}$, $\widehat{EU}_i^{\text{model}} = \frac{1-e^{-EU_i \Delta t}}{E_i}$, and $\widehat{EE}_{FtoO}^{\text{model}} = \frac{1-e^{-EE_{FtoO}\Delta t}}{E_{FT}}$.

Appendix: Hiring, Promotion and Firing Policy Functions

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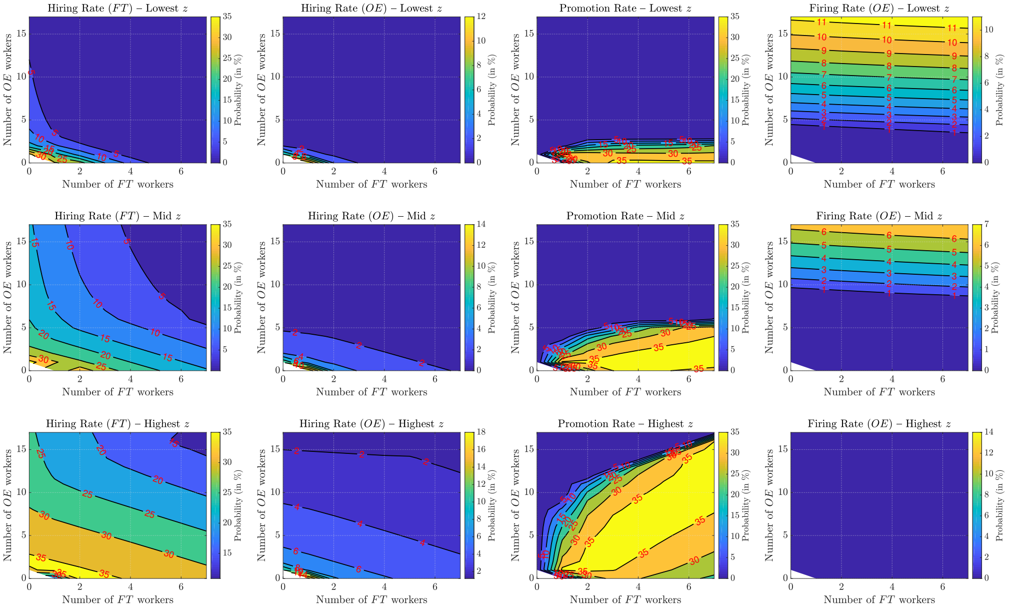

Figure: Hiring, promotion and firing policies, in the (n_{FT}, n_{OE}, z) space.

Appendix: Invariant Distribution

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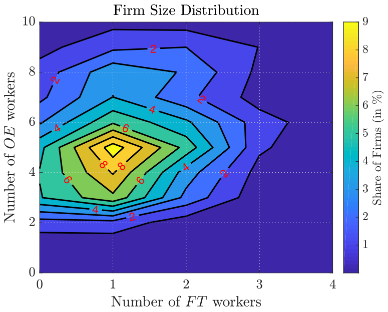
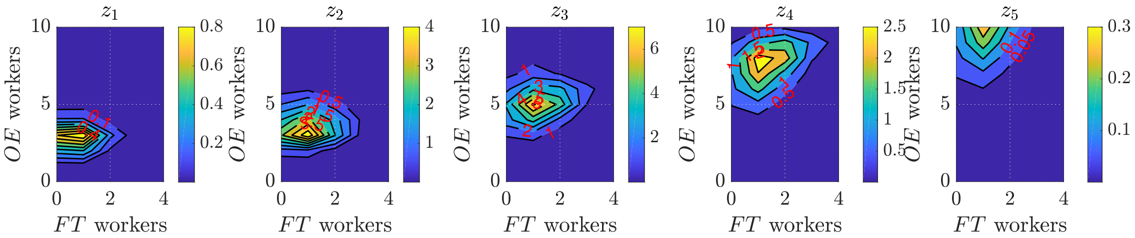
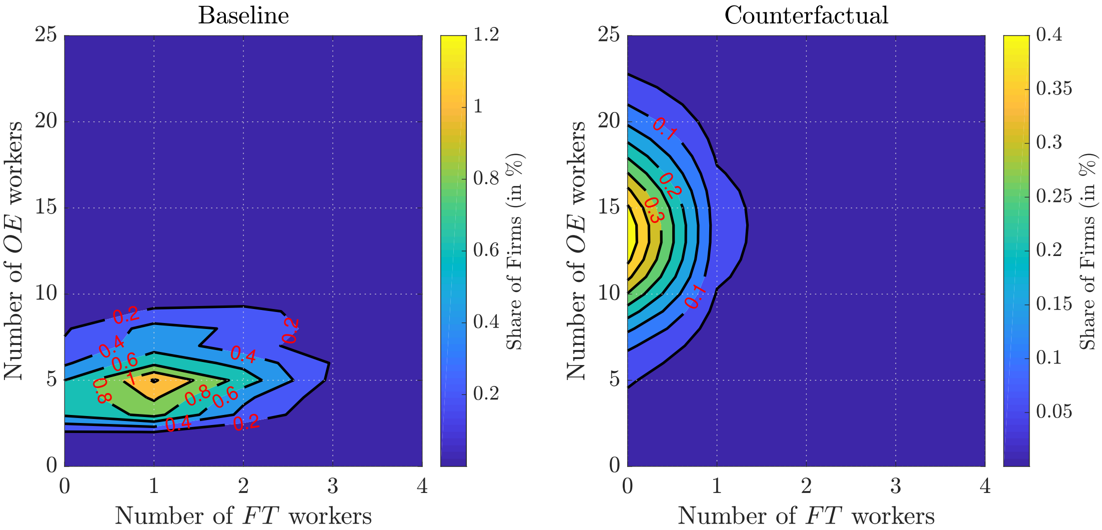




Notes: *Top panel:* Equilibrium distribution of firms in the (n_{FT}, n_{OE}) space, added across productivity states z. *Bottom panel:* Equilibrium distribution in the (n_{FT}, n_{OE}) space, by z-type, where $z_1 < \cdots < z_5$.

Appendix: Change in the Firm Distribution

Back to Counterfactual



Notes: Change in the share of firms in each state (n_{OE}, n_{FT}), aggregated across z states.