Priors for the long run

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- Propose a class of prior distributions for VARs that discipline the long-run implications of the model

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Priors for the long run

- Properties
  - Based on macroeconomic theory
  - Conjugate → Easy to implement and combine with existing priors

- Perform well in applications
  - Good (long-run) forecasting performance

Outline

- A specific pathology of (flat-prior) VARs
  - Too much explanatory power of initial conditions and deterministic trends
  - Sims (1996 and 2000)

- Priors for the long run
  - Intuition
  - Specification and implementation

- Alternative interpretations and relation with the literature

- Application: macroeconomic forecasting

Simple example

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- If $\boldsymbol{\rho = 1}$, DC is a simple linear trend: $$DC = y_0 + c \cdot t$$
- Otherwise more complex: $$DC = \frac{c}{1-\rho} + \rho^t \left(y_0 - \frac{c}{1-\rho} \right)$$

Pathology of (flat-prior) VARs (Sims, 1996 and 2000)

- OLS/MLE has a tendency to "use" the complexity of deterministic components to fit the low frequency variation in the data

- Possible because inference is typically conditional on $\boldsymbol{y_0}$
  - No penalization for parameter estimates of implying steady states or trends far away from initial conditions

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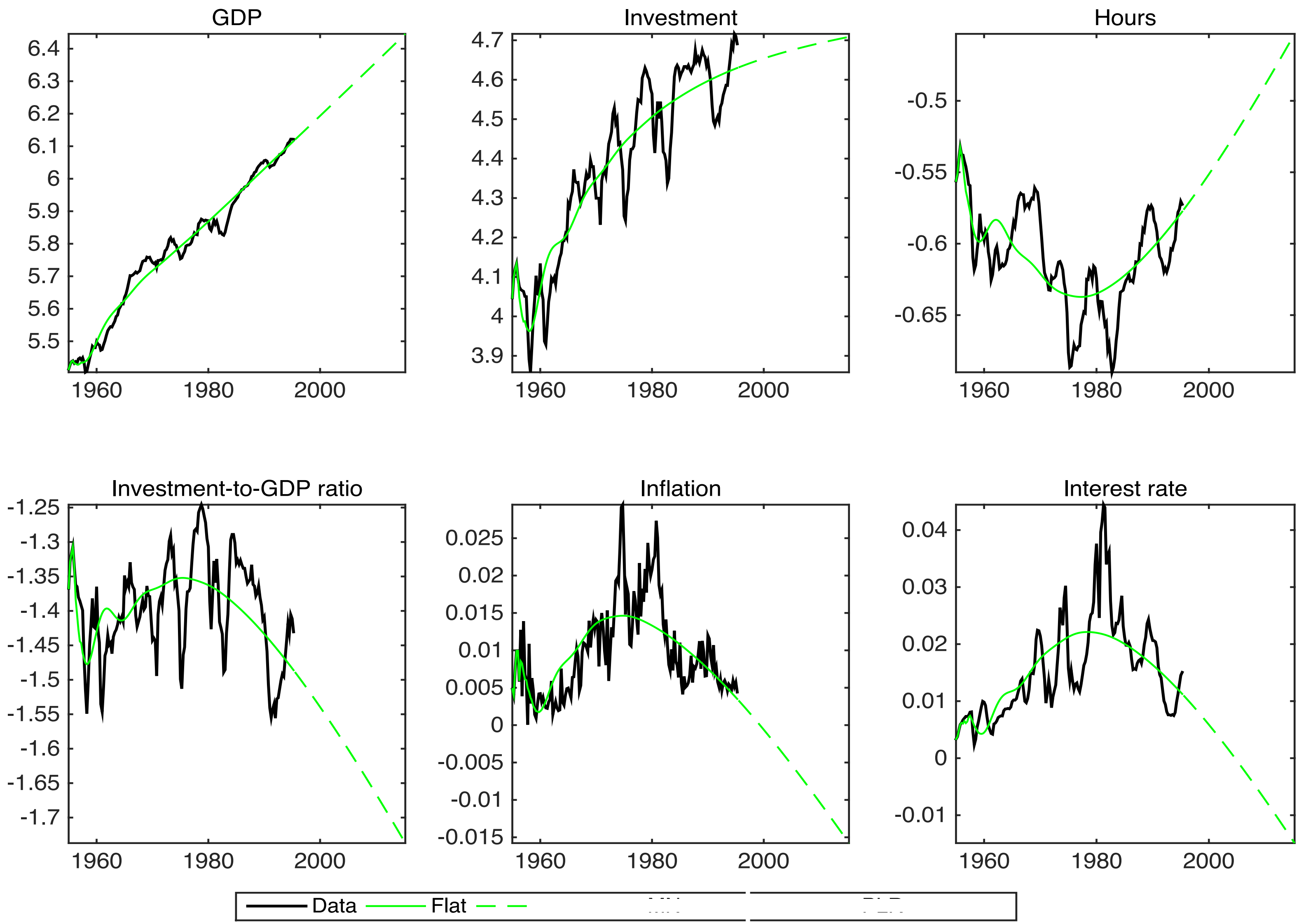
- Example: 7-variable VAR(5) with quarterly data on
  - GDP
  - Consumption
  - Investment
  - Real Wages
  - Hours
  - Inflation
  - Federal funds rate

- Sample: 1955:I – 1994:IV

- Flat or Minnesota prior

“Over-fitting” of deterministic components in VARs



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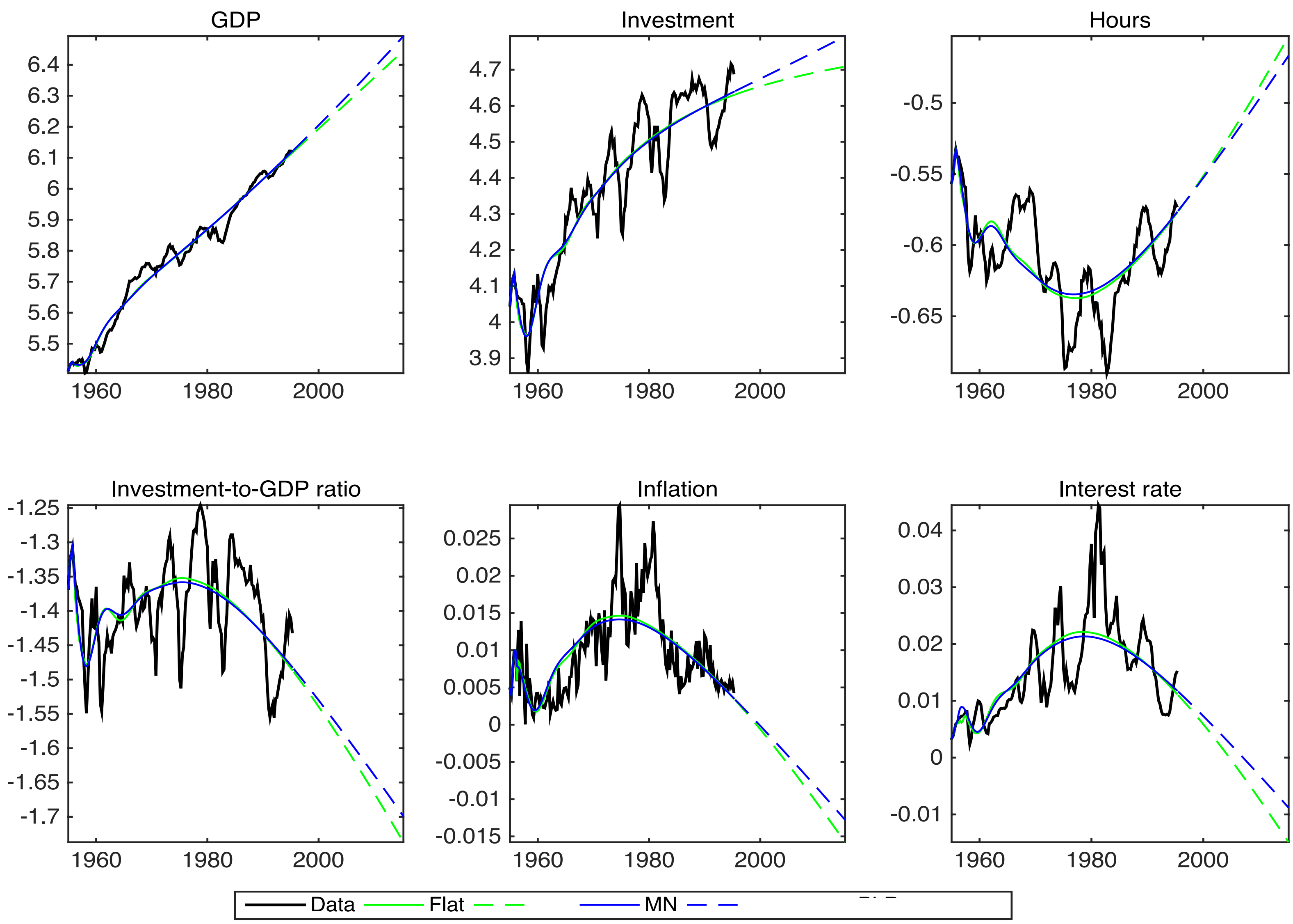

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- Need a prior that downplays excessive explanatory power of initial conditions and deterministic component

- One solution: center prior on "non-stationarity"

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- Standard approach (DLS, SZ, and many followers)
  - Push coefficients towards all variables being independent random walks

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- Loadings associated with these combinations are less(more) likely to be 0

Example: 3-variable VAR of KPSW

$$\Delta y_t = c + \underbrace{\Pi \;\; H^{-1}}_{\Lambda} \;\; \underbrace{H y_{t-1}}_{\tilde{y}_{t-1}} + \varepsilon_t$$

$$\tilde{y}_{t-1} = \overbrace{\begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{pmatrix} \text{Output} \\ \text{Consumption} \\ \text{Investment} \end{pmatrix}}$$

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Common trend: $x_{t-1} + c_{t-1} + i_{t-1}$

Possibly stationary linear combinations: $c_{t-1} - x_{t-1}$, $i_{t-1} - x_{t-1}$

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Common trend

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Prior for the long run: specification and implementation

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  - Can implement it with Theil mixed estimation in the VAR in levels
  - Can be easily combined with existing priors
  - Can compute the ML in closed form
    - Useful for hierarchical modeling and setting of hyperparameters ϕ (GLP, 2013)

Empirical results

- Deterministic component in 7-variable VAR

- Forecasting
  - 3-variable VAR
  - 5-variable VAR
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\\
\\
\longrightarrow \\
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\longrightarrow
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<u>Interpretation of ***H y***</u>

- Real trend
- Consumption-to-GDP ratio
- Investment-to-GDP ratio
- Labor share
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- Real interest rate
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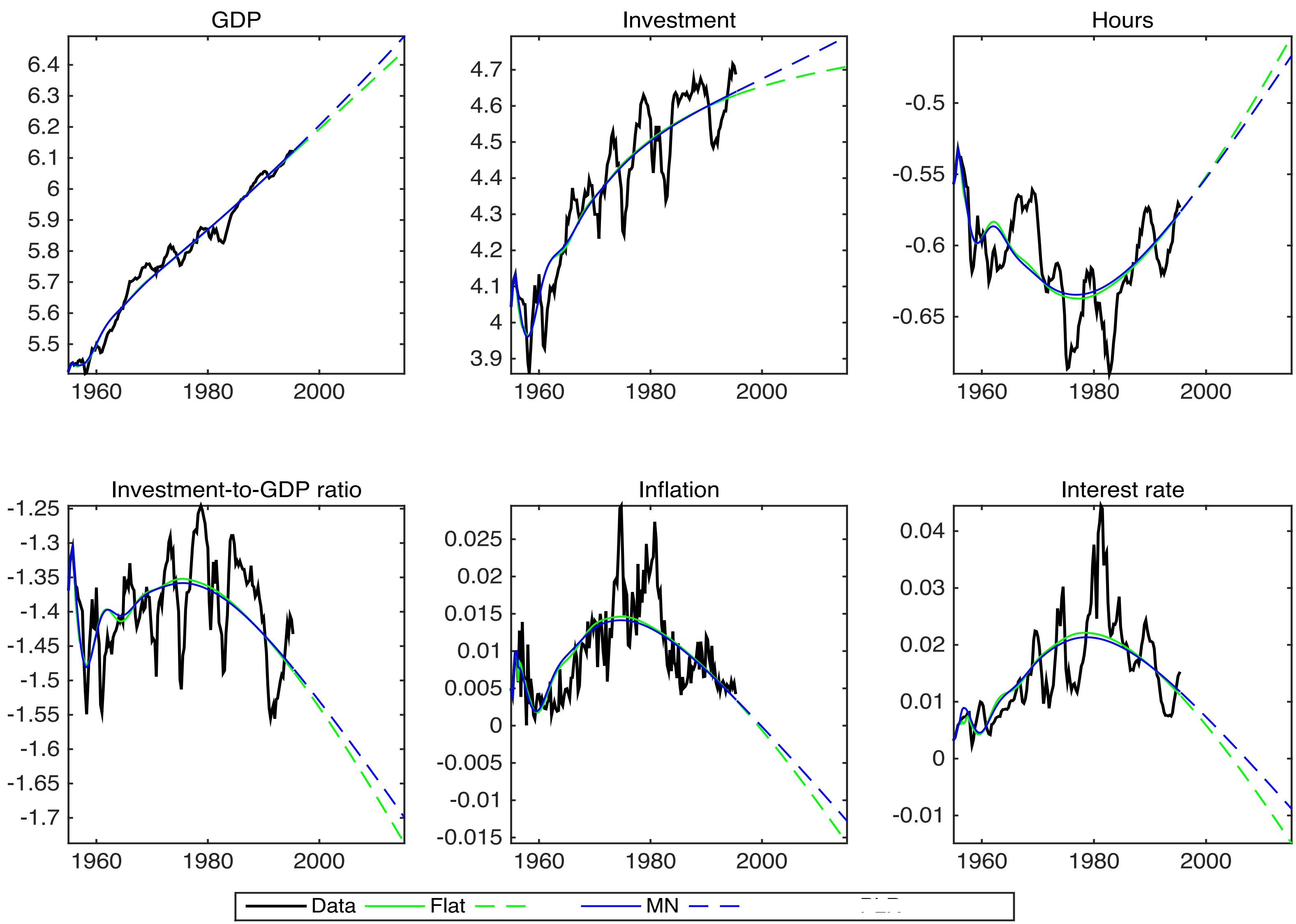
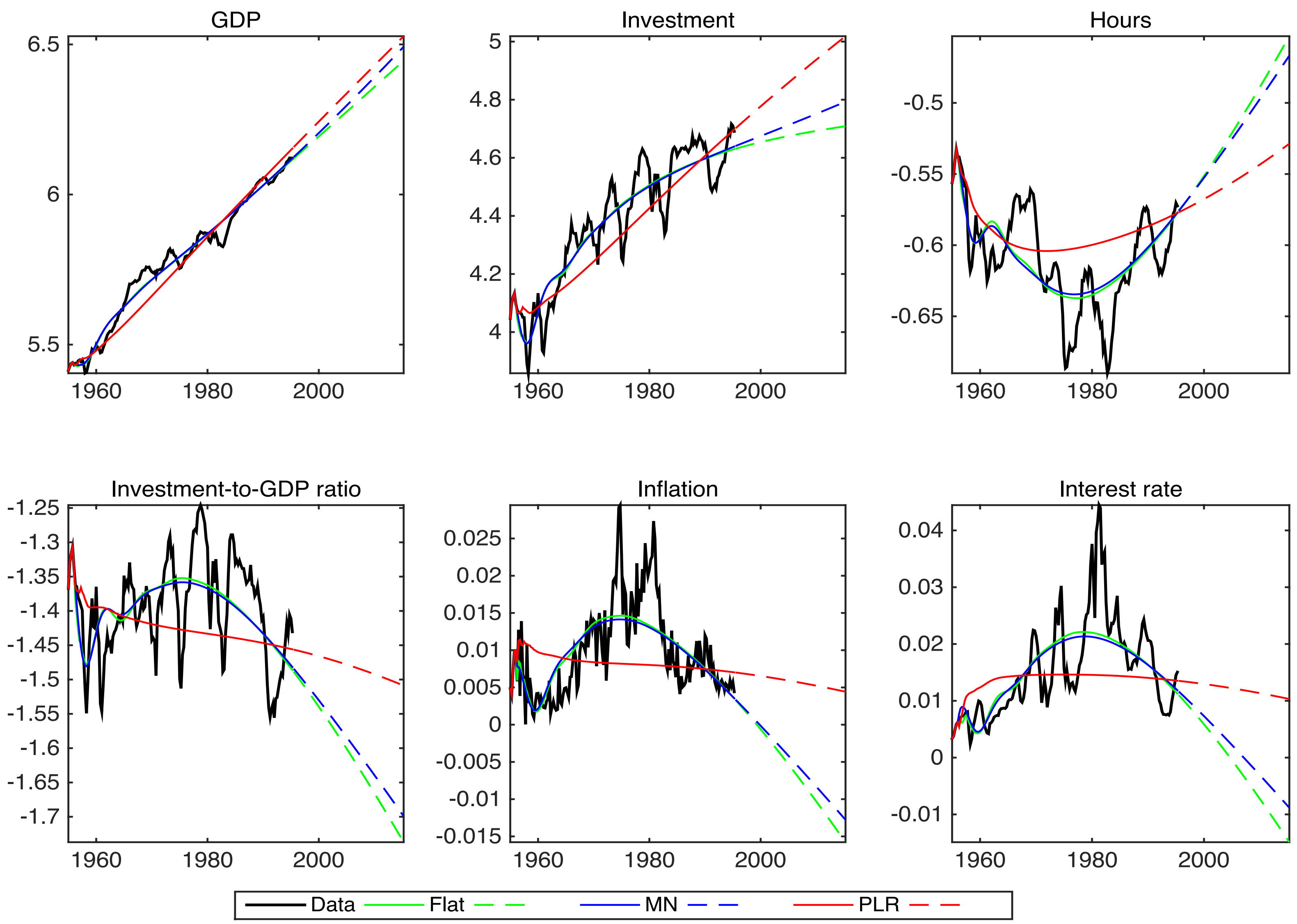


Deterministic components in VARs with Prior for the Long Run



Forecasting results with 3-, 5- and 7-variable VARs

- Recursive estimation starts in 1955:I
- Forecast-evaluation sample: 1985:I – 2013:I

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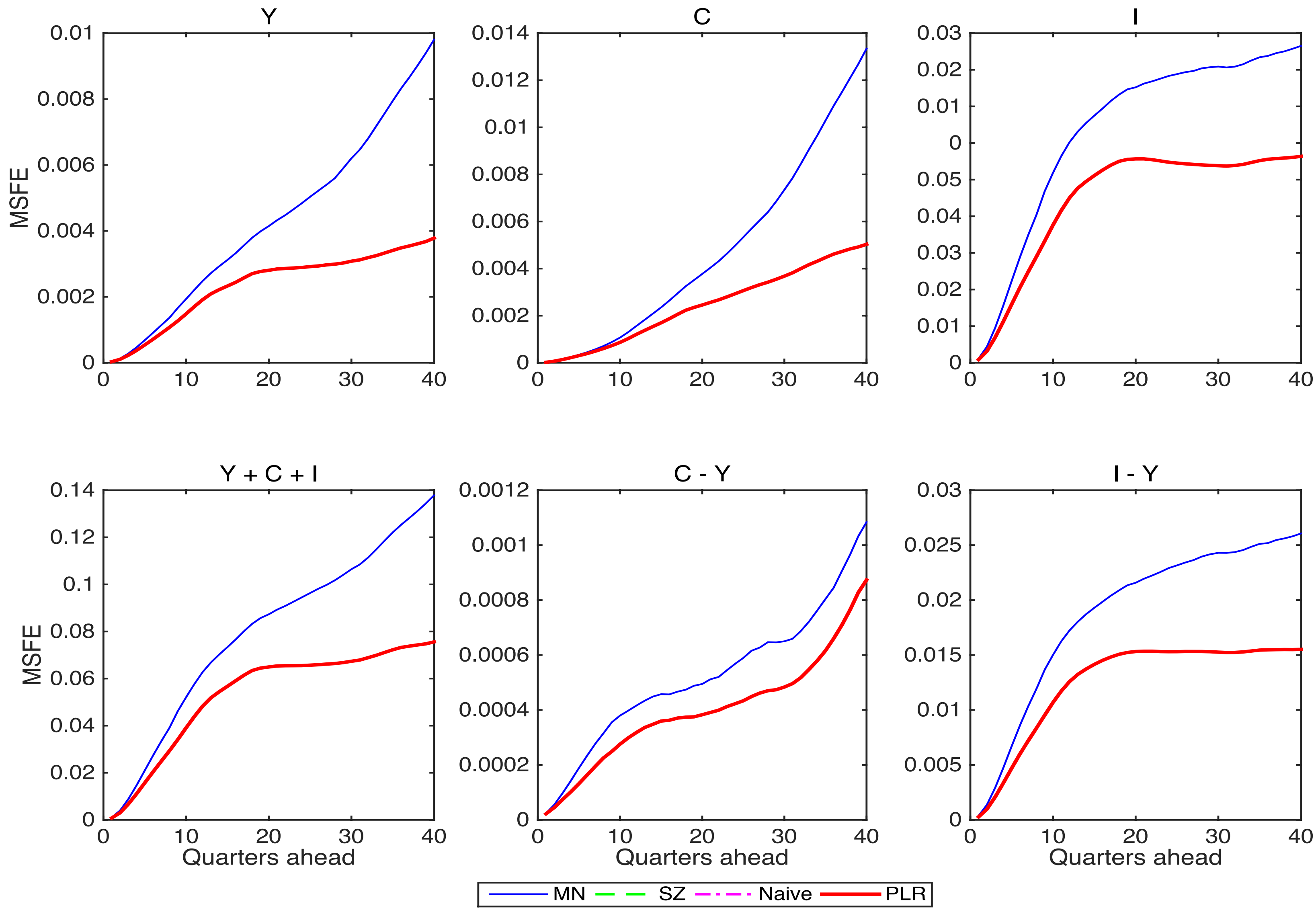

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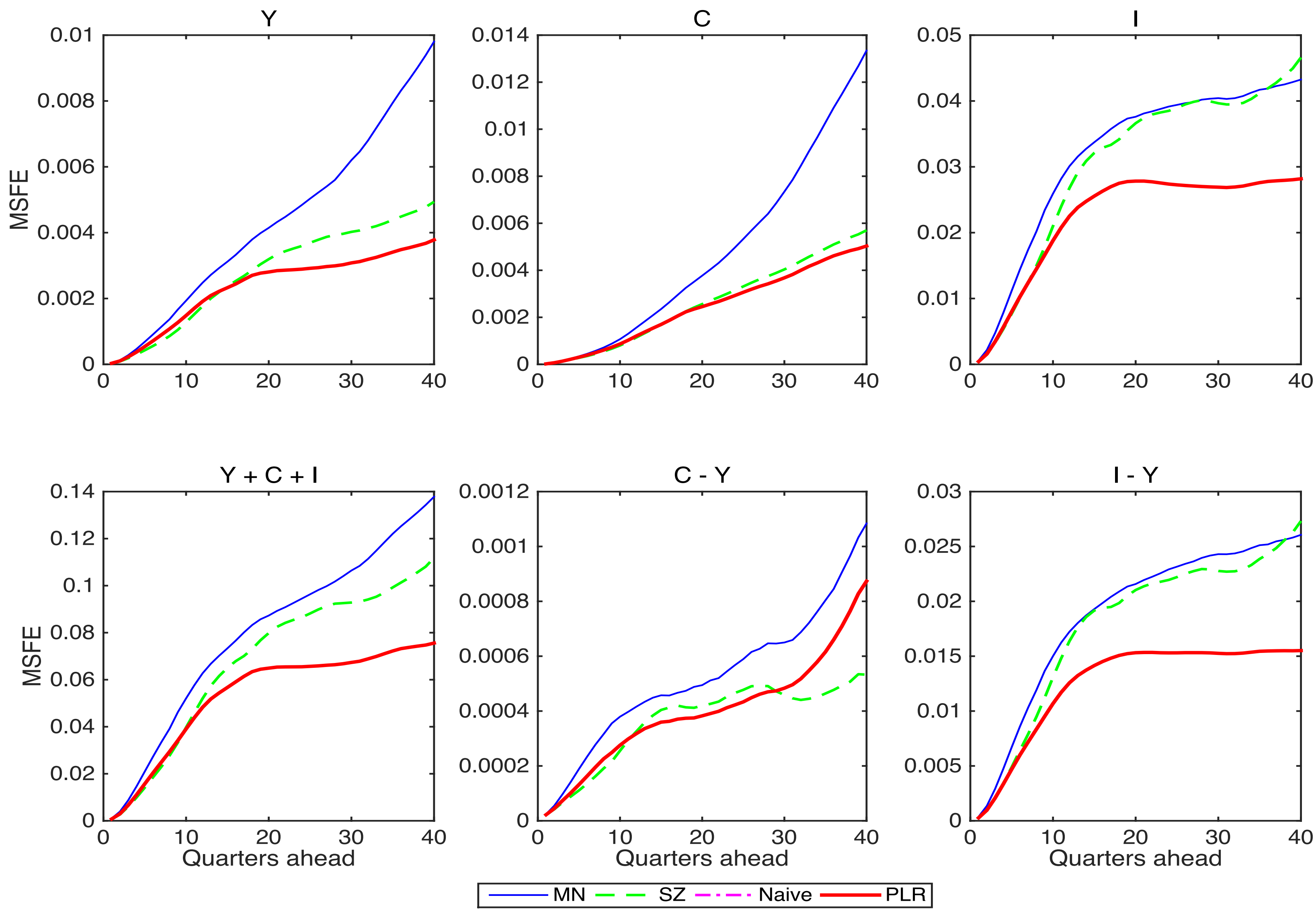
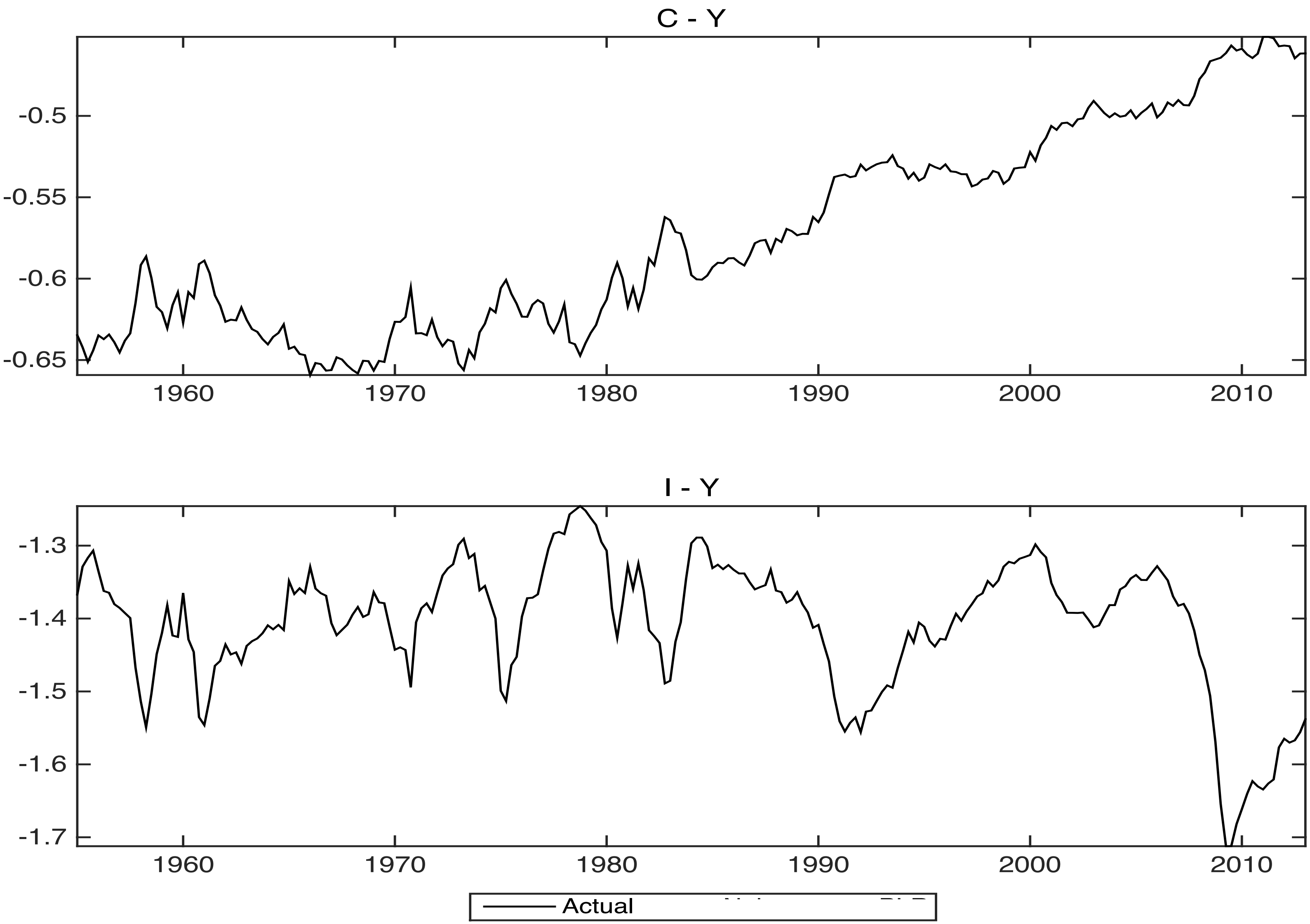


Consumption- and Investment-to-GDP ratios



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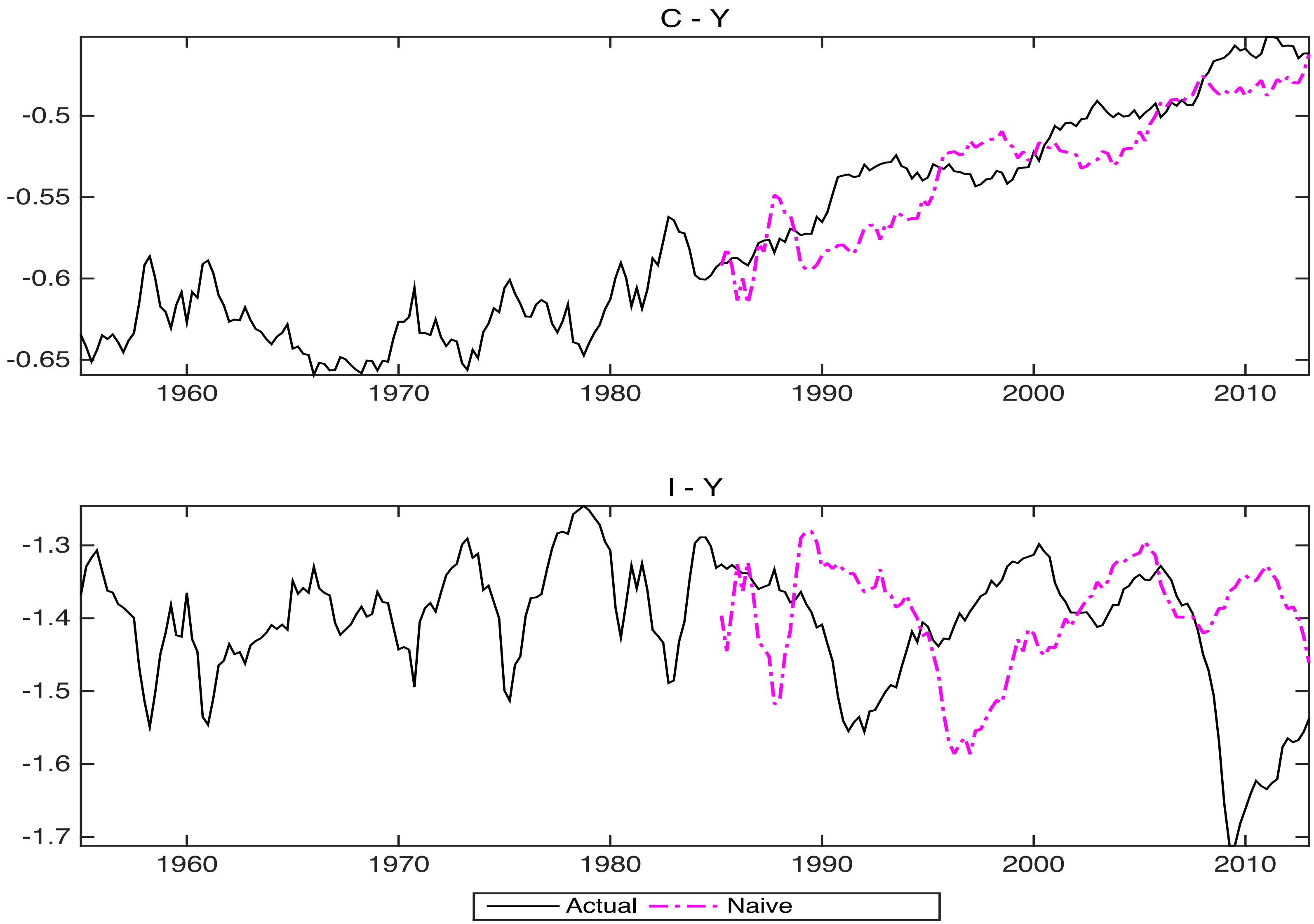

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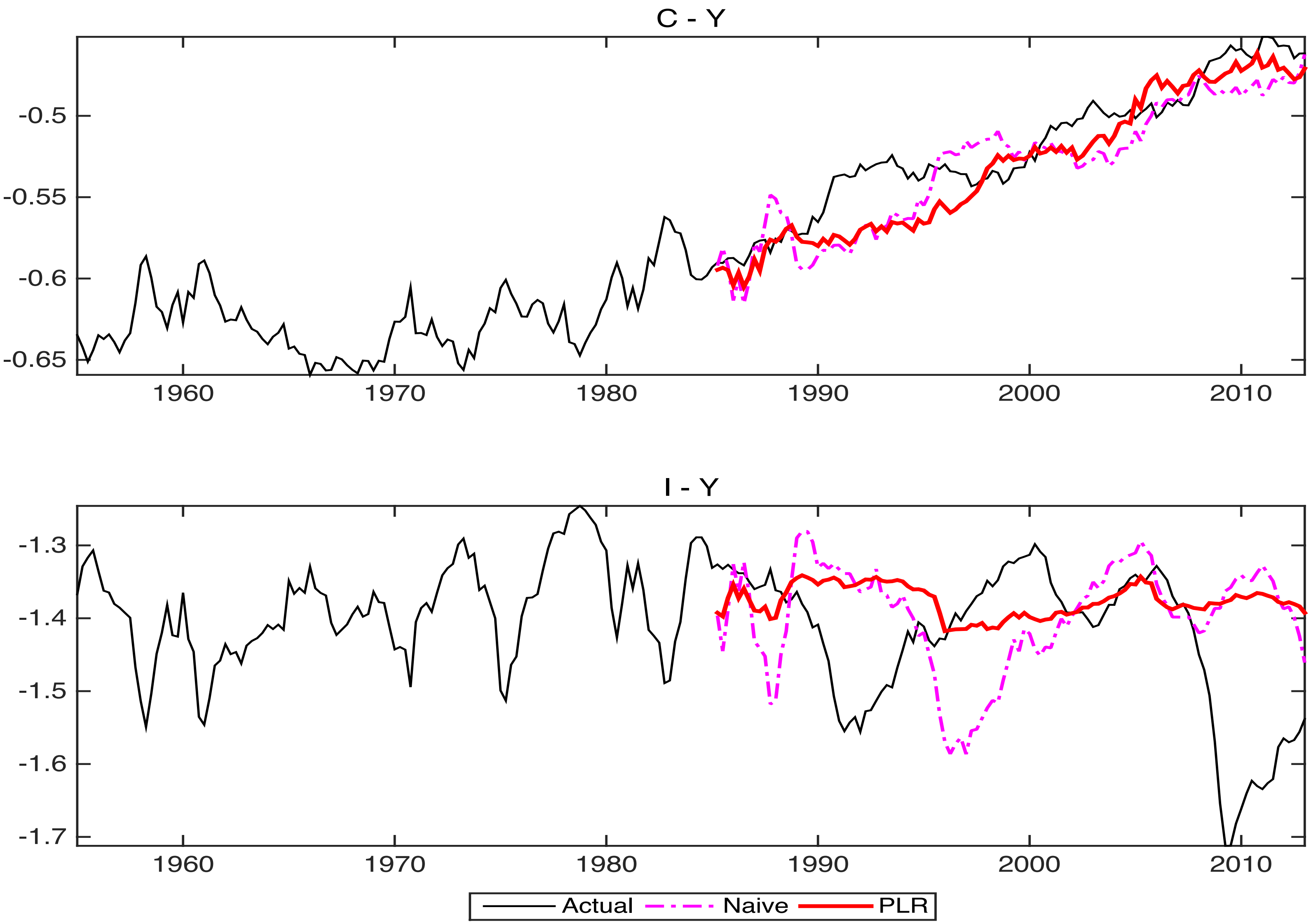
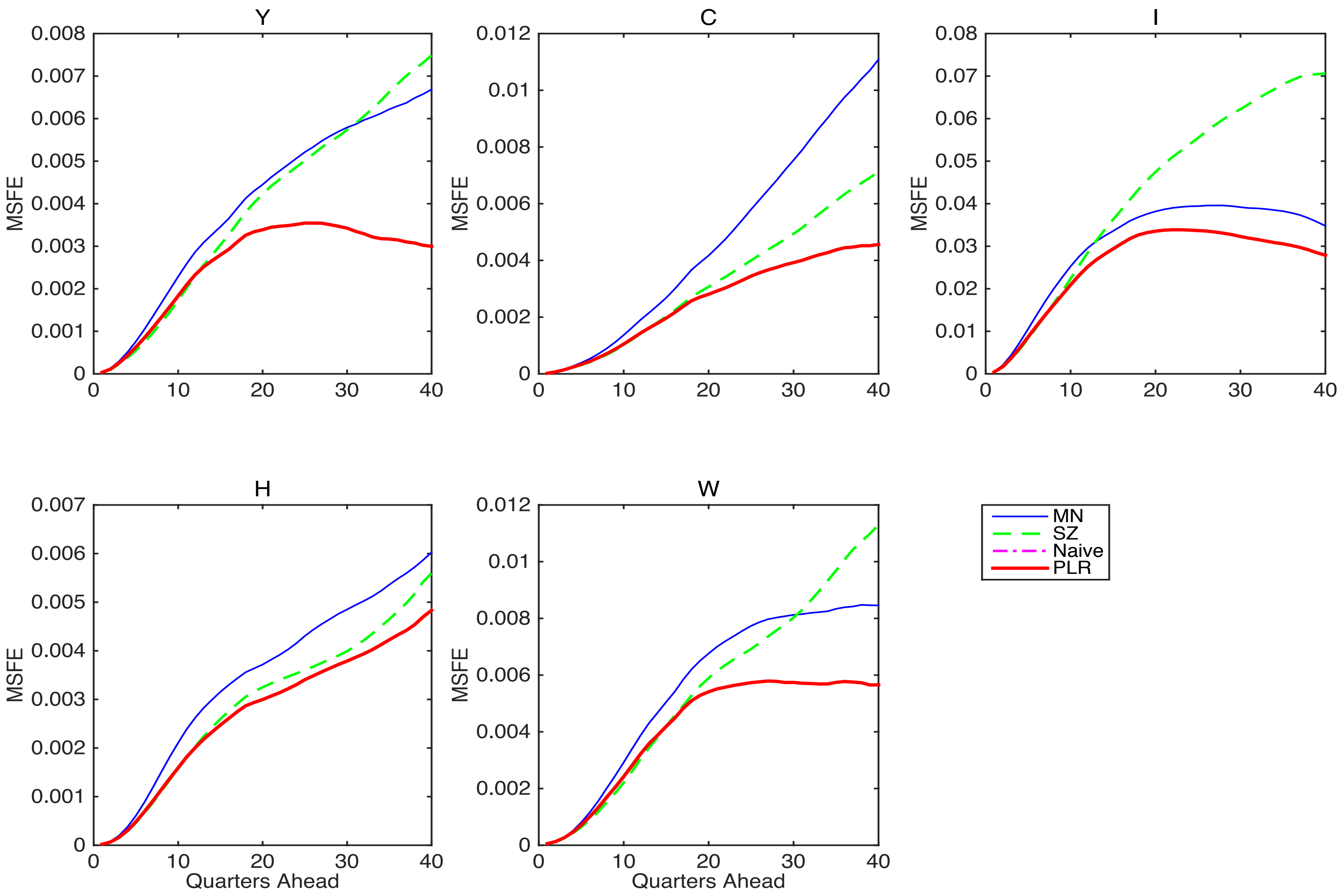


5-variable VAR: MSFE (1985-2013)



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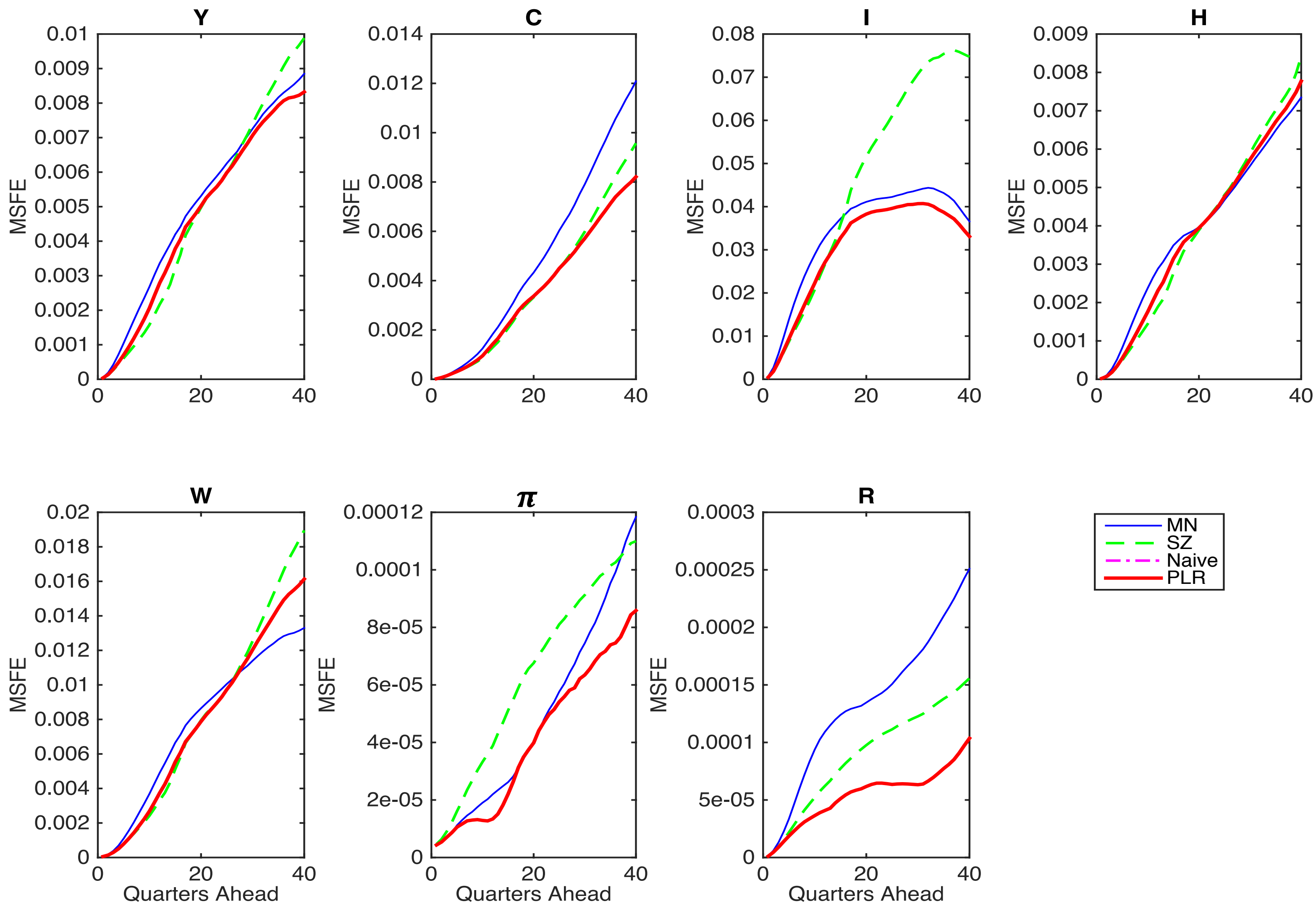

Invariance to rotations of the “stationary” space

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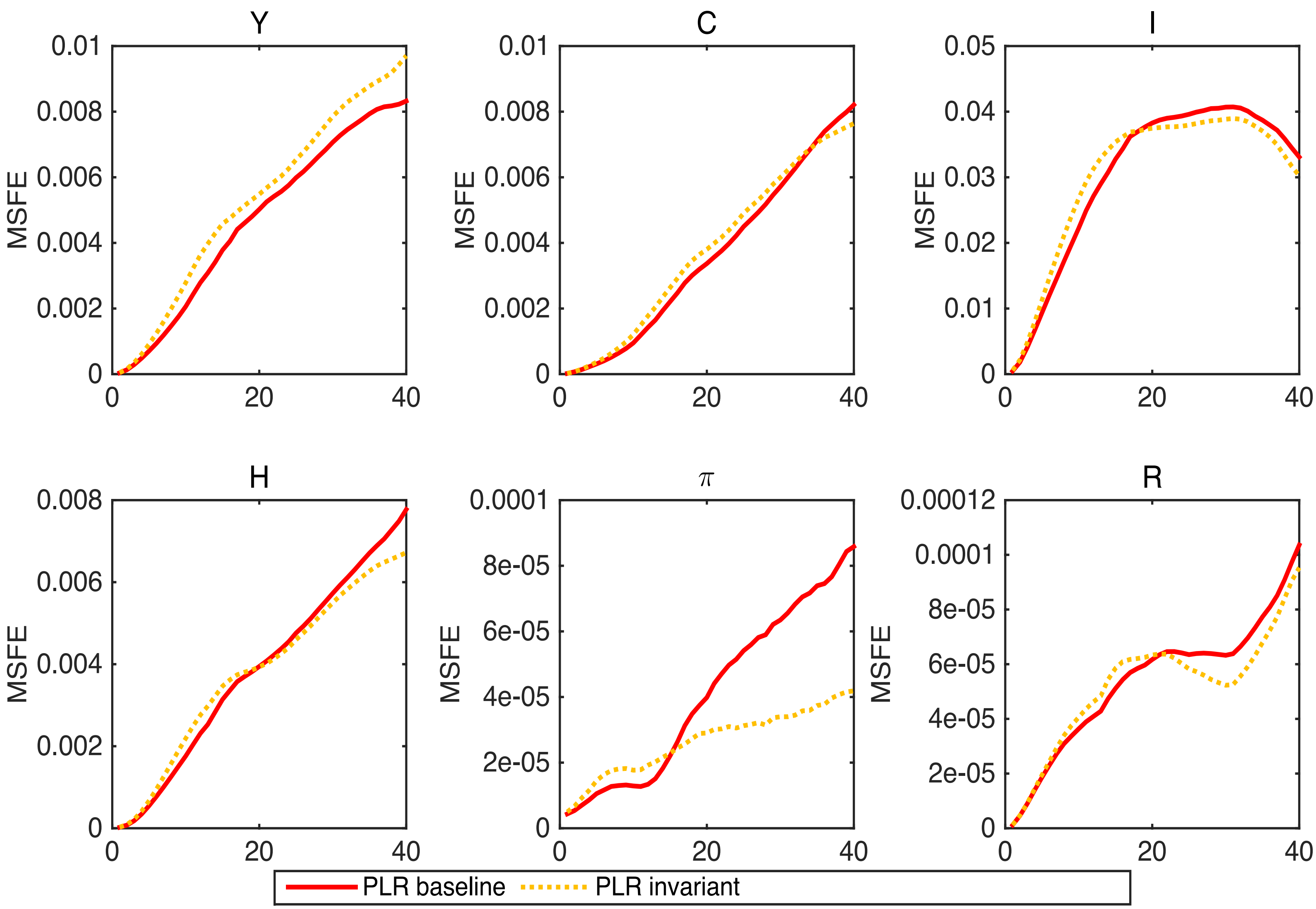
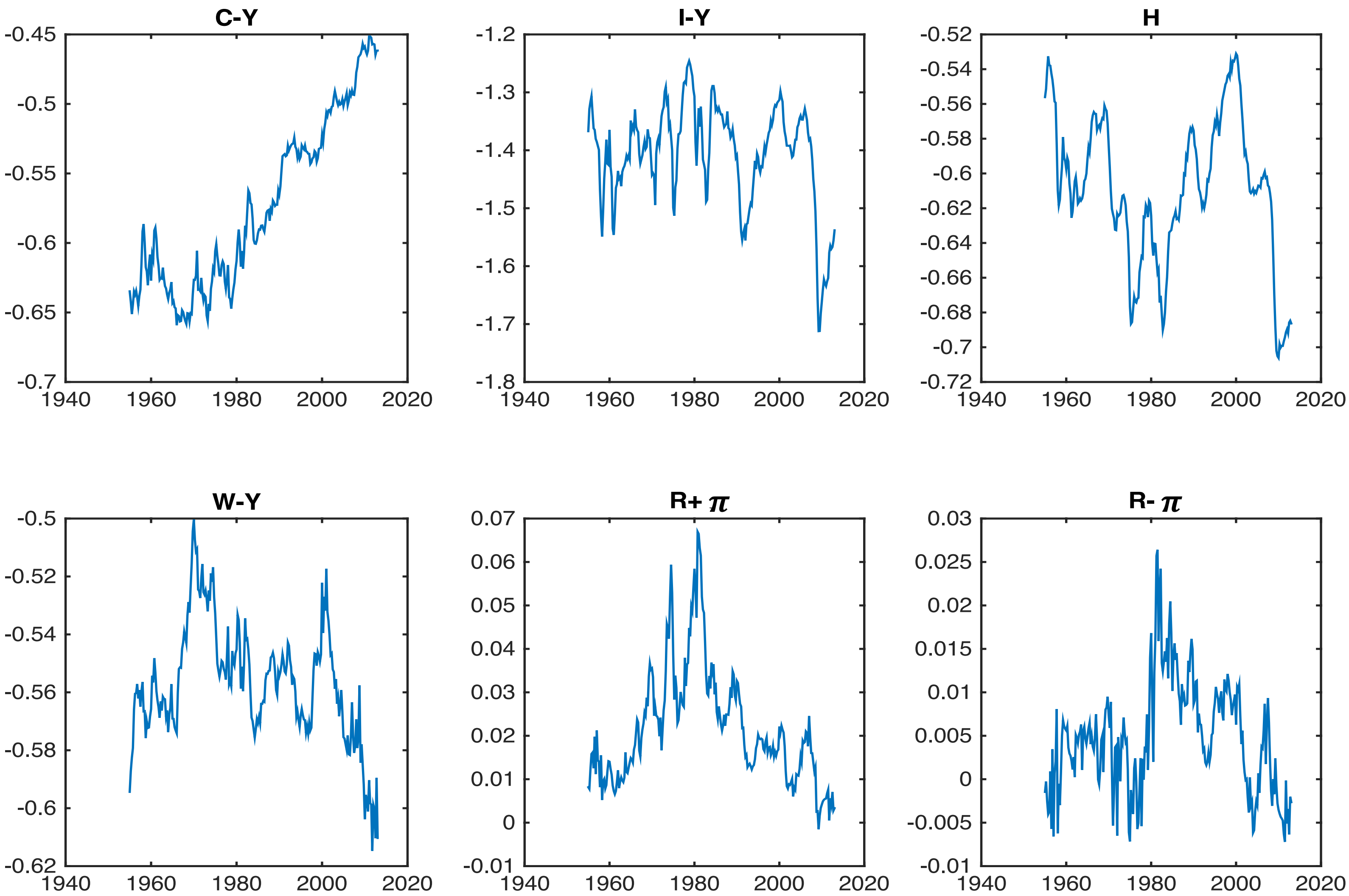
Invariant PLR: $\begin{cases} \Lambda_{\cdot i} \cdot (H_{i\cdot}\bar{y}_0)|H,\Sigma \sim N\left(0, \phi_i^2 \Sigma\right), \quad i = 1, \ldots, n-r \\ \sum_{i=n-r+1}^{n} \Lambda_{\cdot i} \cdot (H_{i\cdot}\bar{y}_0)|H,\Sigma \sim N(0, \phi_{n-r+1}^2 \Sigma) \end{cases}$

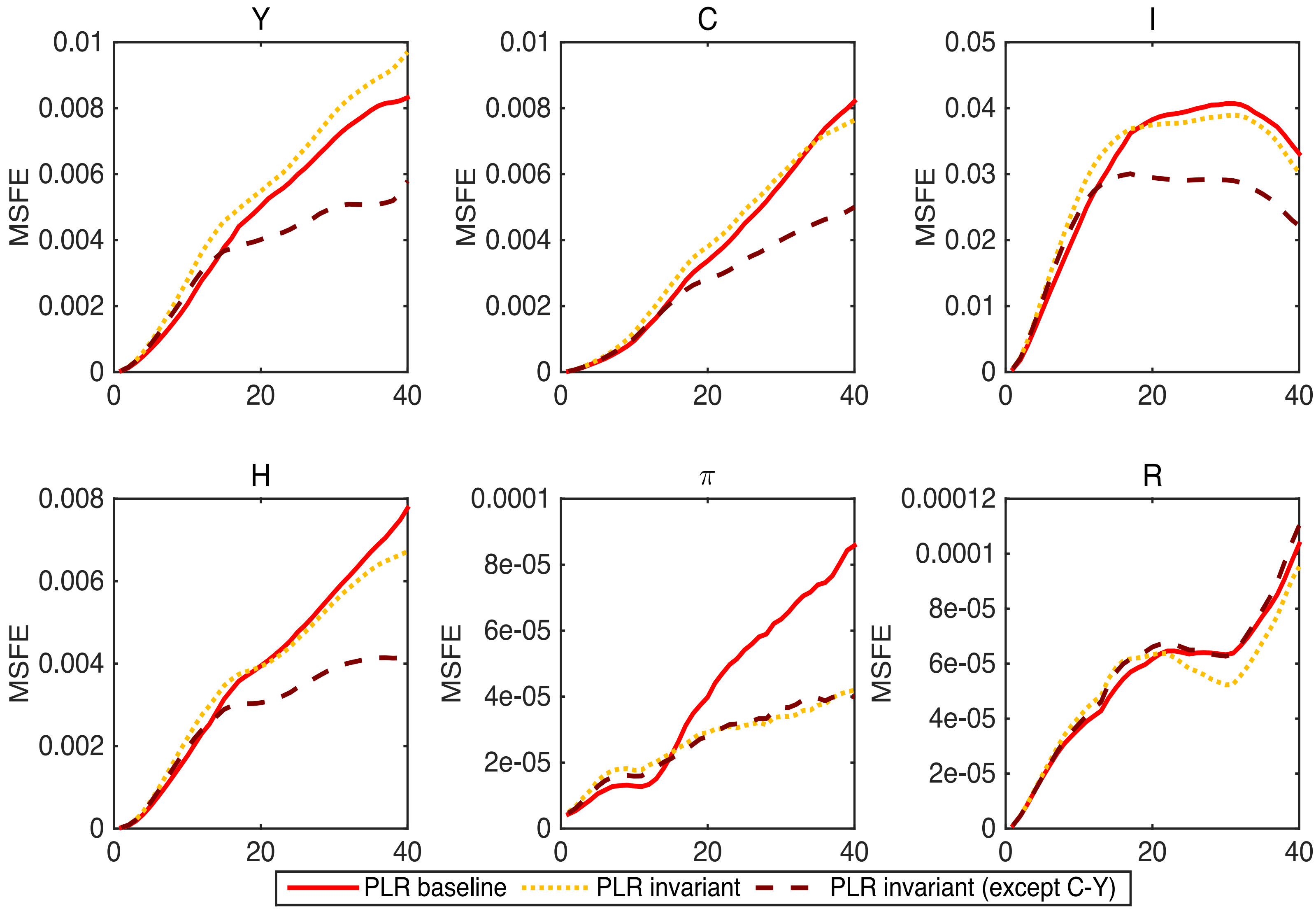
7-variable VAR: Forecasting results with "invariant" PLR



$H y$ in the data



7-variable VAR: Forecasting results with "invariant" PLR



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  - Imposes discipline on long-run behavior of the model
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- “Weak” points
  - Non-automatic procedure → need to think about it
  - Might prove difficult to set up in large-scale models → might require too much thinking

Connections and extreme cases

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- Sum-of-coefficients prior (DLS, SZ)
  - **[β′ β′]′ = *H* = *I***
  - shrink $\boldsymbol{\Lambda_1}$ and $\boldsymbol{\Lambda_2}$ to 0

3-var VAR: Mean Squared Forecast Errors (1985-2013)