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Spillovers in space and time: where spatial econometrics and Global VAR models meet

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Abstract

We bring together the spatial and global vector autoregressive (GVAR) classes of econometric models by providing a detailed methodological review of where they meet in terms of structure, interpretation, and estimation methods. We discuss the structure of cross-section connectivity (weight) matrices used by these models and its implications for estimation. Primarily motivated by the continuously expanding literature on spillovers, we define a broad and measurable concept of spillovers. We formalize it analytically through the indirect effects used in the spatial literature and impulse responses used in the GVAR literature. Finally, we propose a practical step-by-step approach for applied researchers who need to account for the existence and strength of cross-sectional dependence in the data. This approach aims to support the selection of the appropriate modeling and estimation method and of choices that represent empirical spillovers in a clear and interpretable form.

Keywords: Weak and strong cross-sectional dependence, spatial models, GVARs, spillovers.

JEL classification: C33, C38, C51.

Non-technical summary

Cross section and time linkages, as measured in empirical panel data models, are particularly relevant for pan-European policy institutions such as the ECB given the multi-country structure in which spillover effects need to be evaluated. In this context, the objective of this paper is to enhance the understanding of the practical application of two closely related classes of econometric models: spatial models and Global VARs, both well suited for risk and policy spillover analysis.

We bring together these two classes of models by providing a detailed methodological review of where they meet in terms of structure, interpretation, and estimation methods. The starting point for our analytical review is the observation that both frameworks are converging the moment that standard spatial models are expanded to panel setups. Various analytical components of the models are conceptually identical on both sides and have only been labelled differently, such as the derivation to solve the models, or direct and indirect effects in the spatial models versus impulse responses in GVARs. The traditional use of these models (single-equation spatial applications versus multivariate system GVAR applications) do not constitute structural differences and are progressively disappearing in practice as spatial models have been extended to panel and system setups.

A perceived difference may arise with respect to the estimation methods typically employed in the two model classes. We show that the standard choice of maximum likelihood (ML), instrumental variables (IV) or generalized method of moments (GMM) for spatial models, as opposed to OLS for standard GVAR models, stems from the different structures of the connectivity (weight) matrices traditionally employed in these two model classes. The weight structures are often sparse on the spatial model side versus dense in GVAR models (the precise meaning of the two concepts is explained in the paper). However, as these different types of connectivity matrices can actually be used across both classes of models, so can either type of estimation method. Therefore, we uncouple the choice of estimation method from model classification.

Motivated by the continuously expanding literature on spillovers, we define a broad and measurable concept of spillovers and show how both spatial systems and GVARs can be used to measure them, conditional on model limitations. We derive analytical formulas for spillovers, based on estimated coefficients (conditional on the choice of some connectivity matrix W) and a sequence of shocks, and show that these measurements are equivalent across spatial systems and GVAR representations. More precisely, the analytical formalization is based on indirect effects from the spatial literature and impulse responses from the GVAR literature.

Finally, we propose a practical step-by-step approach for applied researchers who need to account for the existence and strength of cross-sectional dependence in the data. The guidance aims to support choosing an appropriate modelling and estimation method as well as presenting spillovers in a clear and interpretable form. We illustrate the guidance with an empirical example involving GDP and credit growth for a sample of European countries and banking systems.

1 Introduction

Cross-sectional dependence has become a major research area in the econometrics literature. It can take two forms: local (spatial) or global (common factors). These two have also been referred to as 'weak' and 'strong' cross-sectional dependence (Chudik et al. (2011)). Local (spatial) dependence has been modelled extensively in the spatial econometrics literature with seminal contributions going back to the 1980s (Anselin (1988)).[1] The literature about modelling global cross-sectional dependence (in particular through the use of Global VARs (Pesaran et al. (2004)) is more recent. The latter is an approach grounded in the more general class of high dimensional and panel VARs (Chudik and Pesaran (2011); Canova and Ciccarelli (2013)). The mainly regional economic and macroeconomic origins of the two strands of econometrics have also resulted in sometimes different concepts used to describe the effects of cross-sectional dependence (connectivity).

To model cross-sectional dependence, both literatures make use of what might be called "cross-sectional interaction effects", where the term interaction denotes that cross-section units are not independent of each other. The standard spatial econometric literature primarily focuses on interaction effects among geographical units reflected by zip codes, neighborhoods, municipalities, counties, regions, jurisdictions, states or countries. Starting from a simple linear regression model, spatial interaction can be best understood if, in addition to the marginal effect of the explanatory on the dependent variable, the behavior of the dependent variable in one unit is co-determined by (i) the dependent variable, (ii) the explanatory variables, and/or (iii) the error term observed in other units. In spatial econometrics these terms are accordingly known as "spatial lags". The GVAR literature also uses spatial lags (known in the literature as "foreign variables") but has traditionally focused mainly on interactions among countries. Moreover, in GVARs there is usually no distinction between different types of variables in the sense of them being all treated as dependent variables in a system of equations.

A key link between the two literatures is the use of connectivity (weight) matrices W to capture the relationships among the units in a sample. If there are N units, W is a matrix of dimension $N \times N$ whose structure of elements indicate whether and to what extent the units affect each other. In the spatial econometrics literature, this matrix W is usually referred to as the "spatial weight matrix", and in the GVAR literature as a "connectivity matrix". One can use these terms interchangeably.

The objectives of this paper are (i) to comprehensively review and compare the two strands of literature, (ii) to integrate them into the general discussion about measuring spillovers, and (iii) to provide a step-by-step guidance to applied researchers in terms of modelling cross-sectional dependence in systems of equations and interpreting the resulting spillover measurements.

Our first contribution is to summarize the key similarities and differences between *conventional* spatial and GVAR models. This will guide the discussion in the remainder of the paper about the source of these differences and the modifications that can be made such that the two classes of models end up representing an identical structure.

Our second contribution is to thoroughly discuss the different assumptions regarding the connectivity (weight) matrix traditionally made in the two literatures, and their implications for the choice of estimation method. We highlight that the standard spatial econometrics literature concentrates on *sparse* connectivity matrices, i.e., on matrices containing many zero off-diagonal elements in each row and column, or elements that quickly converge to zero as the distance separating two units goes to infinity. Conversely, the GVAR literature usually operates with *dense* connectivity matrices, i.e., matrices containing no or hardly any zero off-diagonal elements, in which the off-diagonal elements are rather uniformly distributed or slowly converge to zero (or not at all) as the distance separating two units goes to infinity. This difference with respect to the sparsity or density of the matrix and the speed of convergence of its elements has consequences for the choice of estimation method. If the number of zero off-diagonal elements is high or the speed of convergence fast, either of maximum likelihood (ML), instrumental variables (IV) or generalized method of moments (GMM) are required (the standard approach in spatial models). If the number of zero off-diagonal elements is low

[1] See the references in this book for a more comprehensive review, Anselin (2010) for a recent update, as well as the references in this paper.

or the speed of convergence is slow, ordinary least squares (OLS) can be used (the standard approach in GVAR models). In the paper we relate the concept of sparsity/density to a number of conditions regarding the weights which imply the choice of the corresponding estimation method.

Our third contribution is to carefully relate the concepts of *global and local dominant units* and to again discuss the implications for the choice of estimation method. Units that do have one or more sizeable weight(s) in the matrix W are said to dominate other units either locally or globally, depending on the number of sizeable weights. The concept of global dominant unit has been introduced in the early stages of the GVAR literature (Pesaran et al. (2004)), while the notion of a local dominant unit is more recent (Chudik and Straub (2017)). Given the difficulties they pose for estimation, global dominant units have been addressed either by direct exclusion from estimation (e.g. in early GVAR studies of Pesaran et al. (2004) and Dees et al. (2007)), or by direct modelling via global common factors (e.g. as the infinite dimensional VARs (IVARs) methodology in Chudik and Pesaran (2011, 2013)). While dominant units have not been prevalent in the spatial econometrics literature, we show that this is partly a matter of terminology and in this context the concept of local dominance is indeed relevant. We also show that local dominance structures that are mutual (e.g. two countries affecting each other) imply endogeneity, while non-mutual structures do not.

Our fourth contribution is to propose a general but measurable definition of spillovers that can be formalized analytically via the indirect effects of the spatial literature or, equivalently, impulse responses derived from GVARs. To date, economic theory and econometrics continue to debate various definitions of spillovers and their measurement. Our methodological comparison is therefore primarily motivated by linking the two (structurally equivalent) classes of models to the literature on measuring spillover effects.

Our fifth contribution builds on the work of Bailey et al. (2016a), abbreviated to BHP (2016) and proposes an extended step-by-step guidance to applied researchers who need to account for cross-sectional dependence in systems of equations, including suggestions for model and estimation selection for the appropriate spillover measurement. Practitioners would like to test for weak and/or strong cross-sectional dependence in their data, whether their weight matrix is sparse or dense, and hence whether they need to estimate the parameters of the interaction effects in their model by OLS or by ML/IV/GMM. Wrong choices might lead to biased parameter estimates and consequently to biased estimates of spillover effects.

Our discussion and the guidance draw extensively on the large spatial and GVAR literature to date. The contributions that are most relevant to our review include the spatial econometric studies of Kelejian and Prucha (1999), Lee (2002), Lee (2004), LeSage and Pace (2009), Elhorst (2014), BHP (2016), and Bailey et al. (2016b), abbreviated to BKP (2016) below, and the seminal contributions related to GVAR models by Pesaran et al. (2004), Pesaran and Smith (2006), Pesaran et al. (2006), Dees et al. (2007), Chudik et al. (2011), and Chudik and Pesaran (2013). To anchor our methodological discussion in the literature, we do not provide a separate literature review section but instead cite various additional references as they become relevant throughout the paper.

We organize the discussion of the paper into four areas: (a) model structures and equivalence, (b) connectivity structures and associated estimation methods, (c) the concept of spillovers and (d) the guidance to practitioners (supported by an empirical application). Section 2 sets out the model structures representative for the spatial econometrics and the GVAR literatures and the key distinguishing features of their standard forms. Section 3 defines the ML/IV/GMM and OLS estimators used in these literatures and interprets the conditions on the connectivity (weight) matrix that are associated with different degrees of cross-section dependence. It also discusses the testing procedures for cross-section dependence which in turn inform the choice of the appropriate estimator. Additional topics discussed in Section 3 include the concept of dominant units, the use of exogenous or pre-calibrated versus estimated connectivity matrices, and the model solution. Section 4 introduces a general definition of spillover effects which is then formalized analytically via the indirect spatial system effects and, equivalently, GVAR impulse responses. Sections 5 and 6 distil the key elements of the methodological discussion into a guidance to practitioners regarding model selection, estimation and spillover measurement drawing from the preceding, and provide a supporting illustrative numerical example. Section 7 concludes.

2 Standard model structures—Differences and equivalence options

The starting point for our exposition of where spatial and GVAR models meet in terms of structure is a cross-section of N units observed over T time periods. The model structure representative for the spatial econometrics literature is the *Dynamic Spatial Durbin Panel Data Model* (SDM) which reads, in vector form, as follows.

$$Y_t = \tau Y_{t-1} + \delta W Y_t + \eta W Y_{t-1} + X_t \beta + W X_t \vartheta + \alpha + \lambda_t \iota_N + \epsilon_t \tag{1}$$

where Y_t is an $N \times 1$ vector that consists of one observation of the dependent variable for every unit i $(i = 1, ..., N)$ at time t $(t = 1, ..., T)$, X_t is an $N \times k$ matrix of exogenous explanatory variables and W an $N \times N$ non-negative matrix of known constants describing the linkages of the cross-section units.[2] The terms τ, δ, and η denote the response parameters of, respectively, the time lagged dependent variable Y_{t-1}, the spatially lagged dependent variable WY_t, and the spatially and time lagged dependent variable WY_{t-1}, while β and ϑ are $k \times 1$ vectors of response parameters of the exogenous explanatory variables. The $N \times 1$ vector $\alpha = (\alpha_1, ..., \alpha_N)'$ contains unit specific effects α_i controling for all unit-specific, time-invariant variables whose omission could bias the estimates in a typical cross-section application. Time-specific effects are captured by λ_t $(t = 1, ..., T)$, where ι_N is an $N \times 1$ vector of ones which controls for all time-specific, unit-invariant variables whose omission could bias the estimates in a typical time series application.The error term is represented by the $N \times 1$ vector $\epsilon_t = (\epsilon_{1t}, ..., \epsilon_{Nt})'$ of i.i.d. disturbance terms which have zero mean and finite variance σ^2;[3].

The *standard* Global VAR (GVAR) model structure (Pesaran et al. (2004) and Chudik and Pesaran (2016)) starts from a local equation for each unit i which takes the following form:

$$Y_{it} = \phi_i Y_{i,t-1} + \Lambda_{i0} Y^*_{it} + \Lambda_{i1} Y^*_{i,t-1} + \alpha_i + \Gamma \omega_t + \epsilon_{it} \tag{2}$$

Y_{it} is a vector whose elements consist of observations for k different variables $X_{it,j}$ $(j = 1, ..., k)$ such that $Y_{it} = (X_{it,1}, ..., X_{it,k})'$, for every unit i $(i = 1, ..., N)$ at time t $(t = 1, ..., T)$. Let $X^{tj} = (X_{1t,j}, ..., X_{Nt,j})'$ be an $N \times 1$ vector of all observations on the j-th X variable at time t. Then $X^*_{tj} = W^j X^{tj}$, where W^j is an $N \times N$ non-negative matrix of known constants describing the linkages of the units in the cross-section domain for this j-th X variable $(j = 1, ..., k)$. Consequently, $Y^*_t = \left(W^1 X^{t1}, ..., W^k X^{tk}\right)'$ is a $k \times N$ matrix of weighted foreign variables, and its i-th column Y^*_{it} a $k \times 1$ vector of the foreign variable with respect to unit i and time t. The terms ϕ_i, Λ_{i0} and Λ_{i1} are $k \times k$ matrices of response parameters of the vectors of, respectively, the time lagged dependent variables, the contemporaneous foreign (spatially lagged) variables and the time lagged foreign (spatially lagged) variables. The $k \times 1$ vector α_i contains the intercepts of each variable. ω_t and coefficient matrix Γ denote observed, exogenous common factors which are global from the perspective of all cross-section units, e.g. oil prices.[4] In principle, these variables could also be added to a spatial equation structure as the one in eq. (1). ϵ_{it} a $k \times 1$ is a vector of idiosyncratic shocks (error terms) with mean zero and a nonsingular $k \times k$ covariance matrix Σ.

At their core, linear spatial models and GVARs are based on an explicit modelling of cross-sectional links via connectivity (weight) matrices.[5] However, there are also a number of dimensions across which

[2] The Spatial Durbin Model (SDM), in autoregressive distributed lag panel form, is one of the most general specifications in spatial econometrics. For a broader taxonomy of spatial models, see Elhorst (2014) (sections 2.2, 3.1 and 4.2).

[3] This assumption may be relaxed, i.e., the error terms may also assumed to be spatially autocorrelated, leading to the so-called general nesting spatial model. The parameters of this model are identified when ruling out some rogue special cases (Lee and Yu (2016) (p.143). However, since this extension only affects the efficiency and not the consistency of the parameter estimates, it has been left aside from our discussion.

[4] These can be also endogenized in some subsystems, for example, in the US equation of a multi-country GVAR.

[5] More precisely, GVARs are naturally related to multivariate spatial systems which are also built around exogenous cross-unit connectivity matrices W. GVARs represent one alternative to address the dimensionality problem of multivariate systems with full unit interdependencies and heterogeneous dynamics (Canova and Ciccarelli (2013)).

these models differ, at least in their standard, conventional structures. Table 1 provides an overview and will be used to guide the discussion in the remainder of this paper about the source of these differences and the modifications that can be made such that the two classes of models become equivalent in terms of structure. For this purpose, we start from two simplified versions of the conventional spatial and GVAR model structures in this section and the subsequent sections 3.1-3.4 first, before returning to eqs. (1) and (2).

Table 1: Key distinguishing features of standard spatial and GVAR representations

Feature	**Spatial**	**GVAR**
Typical focus	Single equation, univariate dependent variable ($k = 1$)	Simultaneous equation system, multivariate ($k \geq 1$)
Cross-section/time dimension	N large, T small ($N > T$)	N sufficiently large, T larger ($N < T$)
Slope coefficients	Homogenous across units for each explanatory variable	Heterogeneous, unit-specific coefficients
Treatment of spatially lagged ("foreign") variable (RHS)	Endogenous (WY_t)	Weakly exogenous ($Y_t^* = WY_t$)
Strictly exogenous variables	Included	Observed global common factors
Link (weight) matrix W	Usually location-based and exogenous, reflecting time-invariant neighbor structures, and one matrix for all variables.	Usually macro-financial empirical, can be time variant, and potentially different matrices for each variable.
Cross-sectional dependence	Usually "weak", i.e., related to a limited number of neighbors with relatively large weights (referred to as a sparse connectivity matrix W).	Usually "strong", i.e., related to a large number of neighbors with rather evenly distributed weights (referred to as a dense connectivity matrix W).

These simplified versions take the form of, respectively, a *univariate spatial autoregressive (SAR) panel model* with *unit-specific spatial fixed effects* (α):

$$Y_t = \delta W Y_t + \alpha \left(+X_t \beta\right) + \epsilon_t \tag{3}$$

along with a *univariate GVAR mode*l without time autoregressive lags of Y_t or additional lags of the foreign variable vector WY_t, written in stacked format, i.e. combining the equations of all cross-section items:

$$Y_t = \Psi W Y_t + \alpha + \epsilon_t \tag{4}$$

where $\Psi = \text{diag}\left(\delta_1, ..., \delta_N\right)$. The term $X_t\beta$, inserted in eq. (3) in parentheses, denotes that this equation may contain an exogenous explanatory variable which in eq. (4) is endogeneized as a dependent variable. Both models contain the right hand-side variable WY_t as well as unit-specific intercepts. The difference is that the slope coefficients Ψ are unit-specific in the GVAR model, whereas δ is a scalar in the spatial model. The two model structures would therefore become equivalent when these coefficients in the spatial model would also be allowed to be heterogeneous across units.[6] Given the increased availability of observations over time for many economic variables in spatial studies (Elhorst (2014)), coefficient heterogeneity can easily

[6] Aquaro et al. (2015) considered such an extension, whereby in a spatial panel the slope coefficients of WY_t are also different for different units in the sample.

be tested for. We employ a standard likelihood ratio (LR) test to assess whether either a (restricted) homogenous or (unrestricted) heterogeneous model version is better suited and apply it to our empirical example in Section 6.

Spatial methods have been developed with a focus on univariate dependent variable vectors, and involving spatial dependence between a large set of cross-sectional units (LeSage and Pace (2009)). GVAR models introduce cross-unit dependence in otherwise standard VARs where the traditional focus is on the time dimension. However, due to the availability of cross-section data observed over multiple periods, both spatial panels (Elhorst (2014)) and multivariate spatial systems (Kelejian and Prucha (2004); Vega and Elhorst (2014); Baltagi and Deng (2015); Yang and Lee (2017)) have become more common, thus bridging the gap with GVARs. Traditionally, empirical studies employing spatial econometric techniques are based on relatively large numbers of cross-sectional units over a relatively short period of time, denoted by N large, T small in Table 1. More recent studies are based on spatial panels where T is also large and consider heterogeneous coefficients, such as in Aquaro et al. (2015). GVAR studies tend to be based on relatively more observations over time than across space. For example, in Pesaran et al. (2004)'s original application $N = 11$ and $T = 161$. However, it would be wrong to regard this N immediately as small since it is sufficiently large in view of their specification of the connectivity matrix to justify the estimation technique proposed in their paper. This is denoted by N sufficiently large, T larger in Table 1.

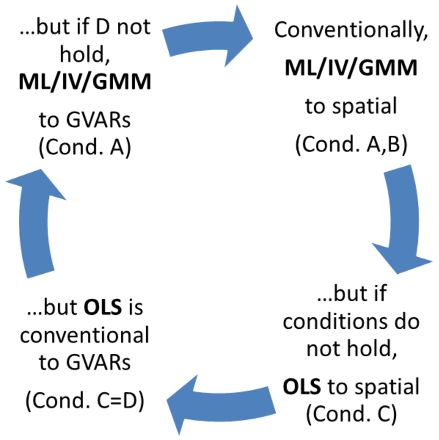
Importantly, the spatially lagged variable WY_t is usually treated as endogenous in spatial applications while GVAR models treat the same variable as weakly exogenous, both under certain conditions. This treatment is stemming from different connectivity assumptions between cross-section units which in turn is associated with the traditional use of mainly ML/IV/GMM estimators in spatial econometrics and OLS in GVARs. Nonetheless, as we shall explain in Section 3.2, especially Lee (2002) and Mutl (2009) have discussed the connectivity assumptions under which either of the estimation methods can in fact be used for both spatial and GVAR models, resulting in model similarity also in terms of estimation method.

Spatial models have also been developed to include strictly exogenous explanatory variables, while GVARs are designed to tackle full endogeneity (the only strictly exogenous variables are usually observed global common factors, such as the oil price).

Finally, the kind of cross-section connectivity traditionally modelled in standard spatial econometrics is of a local nature, reflecting weak cross-sectional dependence (as defined in Chudik et al. (2011)) with a limited number of neighbors with sizeable weights. In practice, this has been implemented through sparse link (weight) matrices, usually location-based, time-invariant and sometimes according to a particular functional form (Elhorst (2014)). Conversely, the GVAR literature tends to use weight matrices which are dense (i.e., reflecting a large number of neighbors with mostly evenly distributed small weights) and which represent strong cross-sectional dependence. However, in practical applications the connectivity matrix always tends to stand somewhere along a continuous range between fully sparse and fully dense forms. As we shall see in the following section, the location of W within this range requires an identical treatment in either the spatial or GVAR representation, as a result of which the distinction between models is mainly semantic.

We can therefore conclude that, from a structural point of view, the conventional spatial econometric model is a special case of the conventional GVAR model, i.e., $k = 1$ and homogenous coefficients vs. $k \geq 1$ and heterogeneous coefficients. We argue that the historical distinction between the two classes of models is mainly semantic and the result of the original difference in focus. More recently, studies began to acknowledge their similarities and our paper aims to contribute to this trend by discussing their equivalence in terms of structure, estimation and spillover measurement.

Figure 1: Estimation conditions and the equivalence of the spatial and GVAR framework



Note: The chart shows the transition in the conditions under which OLS/ML/IV/GMM are equally applicable to both spatial and GVAR models.

3 Connectivity structure and estimation methods in spatial and GVAR models

3.1 Conditions for consistent estimation

In standard spatial econometrics it is assumed that the spatially lagged term WY_t is endogenous under certain conditions, hence ML (Lee (2004)) or IV/GMM are the commonly used estimation methods (Kelejian and Prucha (1999)), while OLS is a special case (Lee (2002)). Conversely, the standard GVAR literature assumes that WYt is weakly exogenous under certain conditions, hence OLS is the commonly used estimation technique (Pesaran et al. (2004)), while ML/IV/GMM is a special case (Mutl (2009)).

The discussion below first introduces these estimators for the model depicted in eq. (3) and then reviews the connectivity conditions (denoted as conditions A, B, and C) under which the coefficients of spatial models can be consistently estimated by either ML/IV/GMM or OLS, as illustrated in Figure 1. Next, the correspondence with the GVAR framework will be discussed; focusing on condition D and its relationship with condition C.

Let $Z_t = [WY_t \; X_t]$ be the regressor matrix of the simple spatial autoregressive panel in eq. (3) including $X_t\beta$, and $\varsigma = \left(\delta, \beta'\right)$ the corresponding vector of parameters. A crucial step to get the ML, IV/GMM and OLS estimators of ς and σ^2 is that the unit-specific effects $\alpha = (\alpha_1, ..., \alpha_N)'$ are eliminated from the regression by demeaning Y_t and Z_t (Baltagi (2005)). This transformation takes the form $Y_t^d = Y_t - T^{-1}\sum_{t=1}^{T} Y_t$ and $Z_t^d = Z_t - T^{-1}\sum_{t=1}^{T} Z_t$.[7] Then the log-likelihood function of the model in eq. (3) based on the demeaned data takes the following form.

$$\ln L = -\frac{NT}{2}\ln\left(2\pi\sigma^2\right) + T\ln|I - \delta W| - \frac{1}{2\sigma^2}\sum_{t=1}^{T}\left(Y_t^d - Z_t^d\varsigma\right)'\left(Y_t^d - Z_t^d\varsigma\right) \tag{5}$$

where the second term on the right hand-side represents the Jacobian term of the transformation from ϵ_t to Y_t^d accounting for the endogeneity of WY_t^d. A detailed derivation of the ML estimators of ς and σ^2

[7] One complication of this transformation in spatial models is that, due to the contemporaneous term WY_t and time-invariant W in most applications, the resulting error terms are linearly time dependent. One alternative transformation has been proposed in Lee and Yu (2010), leading to a minor correction of the estimate for σ^2 which is irrelevant for the scope of this paper.

and the corresponding variance-covariance matrix is provided by Elhorst (2014) (Section 3.3.1).

The IV/GMM estimator of ς takes the following form.

$$\hat{\varsigma}_{IV/GMM} = \left(\hat{Z}'\hat{Z}\right)^{-1}\hat{Z}'\hat{Y}^d \tag{6}$$

where $\hat{Z} = [\hat{WY}^d\ X^d]$, $\hat{WY}^d$ is obtained as the prediction of WY^d on a set of instrumental variables so as to account for the endogeneity of WY. The $NT\times 1$ vectors Y^d, WY^d, $\hat{WY}^d$ and $NT\times k$ matrix X^d are stacks of T successive cross-sections of N observations of the corresponding $N\times 1$ vectors Y_t^d, WY_t^d and $\hat{WY}_t^d$, and the $N\times k$ matrix X_t^d. As instruments for WY^d, Kelejian et al. (2004) suggest $[X^d\ WX^d ... W^gX^d]$, where g is a pre-selected constant.[8] Typically, $g = 1$ or $g = 2$, dependent on the number of regressors and the type of model.

The OLS estimator of ς is:

$$\hat{\varsigma}_{OLS} = \left(Z^{d'}Z^d\right)^{-1}Z^{d'}Y^d \tag{7}$$

where $Z^d = [WY^d\ X^d]$, as a result of which

$$\hat{\varsigma}_{OLS} = \varsigma_{OLS} + \left(Z^{d'}Z^d\right)^{-1}Z^{d'}\epsilon^d \tag{8}$$

Following Lee (2002) for cross-section data, the consistency of $\hat{\varsigma}_{OLS}$ depends on the limiting behavior of the expected value of the component $N^{-1}WY_t^{d'}\epsilon_t^d$, representing the correlation between the spatial lag WY_t^d and the disturbance term ϵ_t^d in a particular time period t, averaged over the corresponding sample size N. This yields

$$E\left(N^{-1}WY_t^{d'}\epsilon_t^d\right) = \frac{\sigma^2}{N}\text{trace}\left[W(I-\delta W)^{-1}\right] \tag{9}$$

In general, the right hand-side of this expected value is non-zero, provided that $\delta \neq 0$, except in some specific cases. The value of the variable Y_t at each point in time is determined jointly across neighboring units. This endogeneity, i.e., the mutual local dominance of a limited number of neighbors, calls for the use of ML/IV/GMM methods.[9] If the expectation in eq. (9) does not go to zero and the true $\delta \neq 0$, the OLS estimator is inconsistent. This can be easily seen from the log Jacobian term $|I - \delta W|$ in eq. (5) which then will not converge to zero.

However, OLS can be used for spatial models under the condition that a significant portion of neighboring units influence another unit in the aggregate, despite the weak bilateral influence of each individual unit (Lee (2002)). We will show that this is precisely the standard assumption in the GVAR literature which assumes that WY is weakly exogenous based on the 'smallness' condition which ensures that all units receive small and equal enough weights to not imply a dominant unit structure (Pesaran et al. (2004)).[10] For this purpose, we turn to the detailed conditions under which these two distinctive estimation methods yield consistent estimators.

[8] Lee (2003) introduces the optimal instrument IV/GMM estimator, but Kelejian et al. (2004) show that the IV/GMM estimator based on this set of instruments has quite similar small sample properties.

[9] Other options are Bayesian MCMC and Lasso estimators, but these estimators have much overlap with the ML estimator.

[10] An extreme case is $k = 1$, i.e., the spatial econometric or GVAR model does not contain any exogenous explanatory or other dependent variable, only unit-specific intercepts $\alpha = (\alpha_1, ..., \alpha_N)'$. The presence of α avoids the purely autoregressive model for which the OLS estimator is inconsistent under all circumstances (Lee (2002) (Section 6). Furthermore, since the model also does not contain time-specific effects (λ_t), we also stay away from near perfect multicolinearity problems pointed out in Kelejian et al. (2006).

The spatial literature employs a number of key assumptions and conditions to prove consistency of the ML/IV/GMM estimators.[11] To avoid confusion it is important to make a distinction between spatial weight matrices W in raw form that are *non-normalized*, and matrices W that are *normalized* such that the row or column elements sum up to one or the largest eigenvalue of W equals one. Assuming that (i) W is a nonnegative, non-normalized matrix of known constants with zero diagonal elements (i.e., cross-unit connectivity exists but a particular unit cannot influence itself) and (ii) the matrix $I_N - \delta W$ is non-singular, where I_N represents an N-dimensional identity matrix, either of the following **additional assumptions** are needed for the consistency of the ML/IV/GMM estimators:

A. Boundedness: the row and column sums of the non-normalized matrix W are uniformly upper bounded in absolute value[12] as N goes to infinity (Kelejian and Prucha (1999)).

$$0 < \lim_{N \to +\infty} \sum_{j=1}^{N} |w_{ij}| \leq K \tag{10}$$

where K is a constant independent of N. That is, in the limit, each additional cross-section unit has *no connection* to the initial set of units.

Or:

B. Weak divergence: the row and column sums of the non-normalized W diverge to infinity at a rate slower than N (Lee (2004)).

$$\lim_{N \to +\infty} \frac{\sum_{j=1}^{N} w_{ij}}{N} = 0 \tag{11}$$

That is, in the limit, each additional cross-section unit has a *decaying connectivity* to the initial set of units.

Both conditions "localize" the cross-section correlation, i.e., the average correlation between any two spatial units converges to zero as infinitely many additional units are added to the sample.[13] Although Lee's (2004) weak divergence condition reads like a normalization procedure, note that this "normalization" is by the sample size N and not by the row sums of W. Further note that condition A in this respect is not so much different from condition B; if the non-normalized elements of W would be divided by the sample size N, then its row sums should also go to zero, just as under condition B.

When the cross-unit connectivity does not die out in the limit, i.e., assumptions A or B are not satisfied, Lee (2002) (theorem 1) shows that there exists an alternative condition in the cross-section domain under which the OLS estimator will be consistent:

C. Strong divergence: the row and column sums of the non-normalized W diverge to infinity at a rate faster than $\sqrt{N}$:

$$\lim_{N \to +\infty} \frac{\sum_{j=1}^{N} w_{ij}}{\sqrt{N}} = \infty \tag{12}$$

That is, in the limit, each additional cross-section unit has a *weak bilateral connectivity* to the initial units yet, taken together, a *significant aggregate impact*.

[11] See Kelejian and Prucha (1999) and Lee (2004), and the discussion in Elhorst (2014) (Section 2.3).

[12] The same assumption is needed for $(I_N - \rho W)^{-1}$. Even though the elements of W are generally non-negative (from an economic viewpoint), negative values do not have to be excluded. Furthermore, even if all elements of W are non-negative, this does not guarantee that all the elements of $(I_N - \rho W)^{-1}$ are also non-negative. This explains the terminology uniformly bounded in absolute value.

[13] The stationarity assumptions imposed on the spatial weights matrix W also extend to spatial panel data (see Lee and Yu (2010)).

Lee (2002) interprets condition C as follows: "It rules out cases where each unit has only a (fixed) finite number of neighbors even when the total number of units increases to infinity". This implies that it rules out the case discussed in Kelejian and Prucha (1999) where each unit is neighbor to no more than a given number, say q, of other units (condition A), and the case put forward by Lee (2004) where the weights are formulated such that they decline as a function of some measure of distance between neighbors (condition B). By contrast, it does cover economic spatial environments where each unit can be influenced aggregately by a significant proportion of units in the population.

OLS is the method of choice used in GVARs because the standard conditions for consistency relate to a small and rather equally distributed cross-unit connectivity, i.e., a matrix of dense bilateral connections. The consistency of the OLS estimator is ensured by the following assumption (in addition to standard VAR system stability):

D. Smallness: The *normalized* weights (denoted by a tilde) used in the construction of the foreign-variables, $\tilde{w}_{ij} \geq 0$, are small, being of order $1/N$, such that

$$\lim_{N \to +\infty} \sum_{j=1}^{N} \tilde{w}_{ij}^2 = 0 \tag{13}$$

Condition D[14] is sufficient for $(WY_t, \epsilon_t) \xrightarrow{N \to \infty} = 0$ and for the OLS estimator in eq. (8) to be consistent ("weak exogeneity").

There is also a model residual-related condition linking to the *weak dependence of shocks*, which should be sufficiently small such that

$$\lim_{N \to \infty} \frac{\sum_{j=1}^{N} \sigma_{ij,ls}^2}{N} = 0 \tag{14}$$

where σ is the covariance of the i-th variable in country i with the s-th variable in country j. This condition is satisfied when the unit-specific shocks are purely idiosyncratic but it is also for a certain degree of dependence across the idiosyncratic shocks. For example, the condition is met if there exists an ordering j seen from the viewpoint of unit i for which $\sigma_{ij,ls}$ decays exponentially with $|i - j|$. It is not necessary for this ordering to be known and it need not be the same for other cross-section units. This is, however, a secondary assumption in that it relates to the efficiency rather than the consistency of the OLS estimator (cf. footnote 3).

Importantly, all asymptotics are cross-sectional in nature (with implications for the consistency of the associated estimators). Taking the GVAR model as point of departure, Mutl (2009) argues that for small samples, i.e., a limited or finite number of cross-section units N, the smallness assumption may not hold since the weights, even if rather equally distributed, become too large. In the extreme case of $N = 2$, the two off-diagonal elements of the connectivity matrix are both unity and the impact of the neighbor large. Consequently, the OLS estimator also ceases to be consistent. In turn, this calls for ML/IV/GMM methods which, according to Mutl (2009), should be the default choice in GVAR applications. More specifically, he proposes a consistent and asymptotically normal IV/GMM estimator built on replacing condition D by an assumption which allows some cross-sectional dependence to remain in the limit:

$$\sum_{j=1}^{N} |w_{ij}^m| \leq K < \infty, \forall i, m \tag{15}$$

[14] The corresponding granularity conditions of condition D for a non-normalized W are set out in Chudik and Pesaran (2015) (p.5), among others, and take the form: (i) $||W_{.j}|| = O\left(N^{-\frac{1}{2}}\right)$ and (ii) $w_{ij}/||W_{.j}|| = O\left(N^{-\frac{1}{2}}\right)$ for all i and j, where $||W_{.j}|| = ||(w_{1j}, ..., w_{Nj})|| = \sqrt{\sum_{j=1}^{N} w_{ij}^2}$.

This assumption is equivalent to assumption A.[15] Yet if condition A is satisfied, condition C is not and the OLS estimator applied to GVARs inconsistent. Hence the IV/GMM estimator (or alternatively, the ML estimator) would be needed just as in any standard spatial model.

To summarize, conditions A and B are underpinning the use of ML/IV/GMM estimators in the standard spatial representation. However, when conditions A or B do not hold, conditions C or D allow the use of OLS in either of the structurally equivalent spatial system or GVAR representation. Conversely, when condition D does not hold in a standard GVAR, ML/IV/GMM estimation is required, which closes the circle depicted in Figure 1. To highlight the unified framework behind both spatial and GVAR models, we note that condition D is equivalent to condition C; both rule out cases where each unit has only a finite number of neighbors even when the total number of units increases to infinity, and both focus on economic cross-sections where each unit can be influenced aggregately by a significant proportion of units in the population.

To demonstrate the equivalence of conditions C and D, we consider a parameterized inverse distance matrix, which is a matrix that has not been considered in this context in the literature before. The diagonal elements of this matrix equal 0, while the off-diagonal elements equal $1/(x_{ij}^{\gamma})$, where x_{ij} denotes the distance between two units i and j, and γ is an unknown parameter to be determined or estimated. The term *distance* should be understood in the broadest sense of the word. This might be the geographical but also the economic distance between two units.

Further, consider an infinite number of N equally spaced nodes on a circle and assume that one can travel from one node to another only along this circle and that the distance between each pair of neighboring nodes takes a uniform value of x. Then the distance of each node to its first neighbor in one of the two directions is x, to its second neighbor the distance is $2x$, and so on. When W is a parameterized inverse distance matrix, the corresponding row or column sum of this series in a discrete space is $\frac{1}{x}\left(\frac{1}{1^{\gamma}}+\frac{1}{2^{\gamma}}+\frac{1}{3^{\gamma}}...\right)$. In a continuous rather than a discrete space (to ease calculations), this sum can be calculated as the surface below the function $f(x) = 1/x^{\gamma}$ over the interval $[1, N]$, represented by the integral $\int_1^N \frac{1}{x^{\gamma}}dx$, which yields: (i) $1/(1-\gamma)(N^{1-\gamma}-1)$ for $\gamma > 0$ and $\gamma \neq 1$, (ii) $N-1$ for $\gamma = 0$, and (iii) $\ln N$ for $\gamma = 1$ (note: negative values of γ are irrelevant). If N goes to infinity, these expressions show that condition A is satisfied, provided that the distance decay parameter is greater than 1 ($\gamma > 1$). Condition B is satisfied if $\gamma > 0$ which can be seen by dividing the outcome under (i) by N, to obtain $1/(1-\gamma)(N^{-\gamma}-1/N)$. This implies that condition B relaxes condition A. Condition C can similarly be verified by dividing the outcome under (i) by $\sqrt{N}$. From this it follows that condition C is not satisfied for $\gamma > 1/2$. Consequently, the ML/IV/GMM estimator when the spatial weights matrix is specified as an inverse distance matrix with a distance decay effect greater than or equal to 0.5 is consistent, whereas the OLS estimator is not. In this case, the OLS estimator will be asymptotically biased and its limiting distribution degenerate (Lee (2002), theorem 4). However, the OLS estimator is consistent if $0 < \gamma < 1/2$, i.e., if the distance decay effect is small and distant locations keep having impact as a result.

To verify whether condition D produces the same outcome as condition C is difficult since the matrix W then needs to be normalized first. Therefore, we consider the corresponding granularity conditions $\sqrt{\sum_{j=1}^{N} w_{ij}^2} = O\left(N^{-\frac{1}{2}}\right)$ and $w_{ij}/\sqrt{\sum_{j=1}^{N} w_{ij}^2} = O\left(N^{-\frac{1}{2}}\right)$ for all i and j. $\sum_{j=1}^{N} w_{ij}^2$ can be calculated by the integral $\int_1^N \frac{1}{x^{2\gamma}}dx$. This in combination with the square root yields $\sqrt{\sum_{j=1}^{N} w_{ij}^2} = \sqrt{\frac{1}{1-2\gamma}\left(N^{1-2\gamma}-1\right)}$, an expression that is defined and will be of order $O\left(N^{-\frac{1}{2}}\right)$ only if $0 < \gamma < 1/2$.[16] This shows that condition C and the granularity conditions as an alternative to condition D are equivalent for a general connectivity matrix such as the parameterized inverse distance matrix.

As previously noted, we relate the concept of sparsity/density of connectivity matrix directly to conditions A-D above. The conditions that require ML/IV/GMM (OLS) are associated with sparse (dense) matrices.[17]

[15] This condition can be best compared to a p-order binary contiguity matrix, where p is small, saying that no spatial unit is assumed to be a neighbor to more than a fixed number of other units.

[16] $\sqrt{1/p\,(N^p-1)}$ is defined and of order $O\left(N^{-1/2}\right)$ if $0 < p < 1$. This implies $0 < 1-2\gamma < 1$ or $0 < \gamma < 0.5$.

[17] Our definition of sparsity excludes an extreme type of a reduced rank connectivity matrix in which only one row of the connectivity matrix has all off-diagonal elements rather uniformly distributed. In such a case, OLS would produce consistent

3.2 Testing for cross-sectional dependence

Given the structural equivalence of the spatial system and GVAR representations and the implication of the limiting behavior of cross-section connectivity for the estimation method, testing for the existence and strength of cross-sectional dependence in the data is the obvious starting point ahead of estimation. BHP (2016) present a two-step procedure to distinguish between weak and strong cross-sectional dependence and subsequently model each type through either standard common factor or standard spatial models, i.e., a conventional spatial model with homogeneous slope coefficients. Starting from their procedure, we extend it to include the appropriate estimation method and spillover measurement, building on the de-facto equivalence between "augmented" spatial and GVAR models.

The two-step procedure in BHP (2016) is based on the CD-test developed in Pesaran (2004, 2015a), and the α-exponent estimator developed in BKP (2016). Let x_{it} denote the individual observation of (one of) the dependent variable(s) of unit i at time t ($i = 1, ..., N$; $t = 1, ..., T$). Then the CD test statistic is defined as:

$$CD = \sqrt{2T/N(N-1)} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \hat{\rho}_{ij} \tag{16}$$

where $\hat{\rho}_{ij}$ denotes the sample correlation coefficient between x_{it} and x_{jt} of two units i and j observed over time. The test verifies the degree of cross-sectional dependence in terms of the rate at which the average (over all $N-1$ unit pairs) pair-wise correlation coefficient varies with N as N goes to infinity. BHP (2016, Section 2.3) show that the average correlation coefficient has the order property of

$$\bar{\rho}_N = \frac{2}{N(N-1)} \sum_{i=1}^{N} \sum_{j=i+1}^{N} \rho_{ij} = O\left(N^{2\alpha-2}\right) \tag{17}$$

where α is a parameter that can take values on the [0,1] interval. For $0 < \alpha < 1/2$, $\bar{\rho}_N$ tends to go to zero very fast, pointing to weak dependence.[18] If $\alpha = 1$, $\bar{\rho}_N$ converges to a non-zero value and strong dependence (common factors) needs to be accounted for.[19] Note that α will retain this value of unity if the number of ρ's tend to infinity at the same rate as N^2. The range $1/2 \leq \alpha < 3/4$ is considered to represent moderate and $3/4 \leq \alpha < 1$ quite-strong cross-sectional dependence. These four distinct cases are listed in Table 2 and will be linked to the type of spatial weight/connectivity matrix (sparse or dense) and the required estimation method. From eq. (17), it follows that the order of convergence of the average cross-section correlation coefficient is $N^{-1/2}$ consistent with estimation conditions C and D, provided that $\alpha = 3/4$. Following BKP (2016), α can be estimated in empirical applications by

$$\alpha = 1 + \frac{\ln \sigma_{\bar{x}}^2}{2\ lnN} - \frac{\ln u_v^2}{2 \ln N} - \frac{c_N}{2N \ln N \sigma_{\bar{x}}^{2\prime}} \tag{18}$$

where $\sigma_{\bar{x}}^2$ in the second right hand-side term is defined as $\sigma_{\bar{x}}^2 = 1/T \sum_{t=1}^{T} (\bar{x}_t - \bar{x})^2$ and $\bar{x} = 1/T \sum_{t=1}^{T} \bar{x}_t$. The latter formulas state that, first, the cross-section average ($\bar{x}_t$) needs to be determined in each time period, secondly, the time average $\bar{x}$ over these T cross-section averages and finally, the variance $\sigma_{\bar{x}}^2$ of this time average. The terms u_v^2 and c_N are small sample bias-correction terms obtained by running separate regressions of x_{it} on a constant and $\bar{x}_t$ for each unit i, such that each of these regressions is based on T observations.

estimates. Note that extreme sparsity does not reflect a leader (column) structure as encountered in standard spatial models (requiring ML).

[18] For example, a p-order binary contiguity matrix W in which each unit has only a limited number of neighboring units can represent this weak (local, spatial) dependence.

[19] Global observed common factors may augment standard GVARs as outlined in Chudik and Pesaran (2016) or Dees et al. (2007) or can be added to dynamic spatial panel data models as in Vega and Elhorst (2016).

Table 2: Interplay between cross-section dependence, weight structure and estimation method

α	Cross section dependence	Weight structure	Estimation
$0 < \alpha < 0.5$	weak	sparse: local, mutually dominant units	ML/IV/GMM
$0.5 < \alpha < 0.75$	moderate	still quite sparse	
$0.75 < \alpha < 1$	quite strong	dense	OLS
$\alpha = 1$	strong	CS averages or PC (no weights involved)	

The reason for BHP's (2016) two-step procedure is that the parameter α can be estimated consistently only for $1/2 < \alpha \leq 1$. Therefore, we first need to find out whether α is smaller or greater than 1/2. This can be tested with the CD test, since Pesaran (2015a) showed that the hypothesis of $0 \leq \alpha < 1/2$ corresponds to the hypothesis of weak cross-sectional dependence underlying his CD test.[20,21]

Relating this two-step procedure to the discussion about conditions for estimation in Section 3.1, the following test strategy may be followed. If the null of weak cross–sectional dependence cannot be rejected by the CD-test ($0 < \alpha < 1/2$), the cross-section connectivity is one between local, mutually dominant units represented by a sparse connectivity matrix W with at most a fixed or a rapidly declining number of neighbors (for example a locational matrix based on sharing country borders) associated with conditions A and B. In case this happens, practitioners should proceed by directly modeling the data with a spatial model structure. If the null of weak cross–sectional dependence is rejected by the CD-test ($\alpha > 1/2$), then α should be estimated by eq. (18). For outcomes pointing to moderate dependence ($1/2 \leq \alpha < 3/4$), the cross-unit connectivity becomes denser but correlations still decay sufficiently fast ($> \sqrt{N}$) such that after a certain distance the impact of neighboring units becomes negligible.[22] Consequently, endogeneity still has to be accounted via ML/IV/GMM estimation. This is the domain of standard spatial models focusing on ML/IV/GMM estimation to account for local endogeneity, as indicated in the upper part of Table 2. When $\alpha > 3/4$ ("quite strong" dependence) the average correlation coefficient tends to go to zero slowly (i.e., slower than $\sqrt{N}$), such that each unit affects all other units almost equally (represented by a dense connectivity matrix W), associated with conditions C and D. Although the bilateral impact of each individual unit is small, since it concerns many units, the aggregate effect of all these units can be large. Under this circumstance, the spatially lagged term WY_t may be treated as "weakly exogenous" and the OLS estimator is consistent (Pesaran et al. (2004)).[23] This is the setup of a standard GVAR estimated by OLS, as indicated in the bottom part of Table 2.

Should the hypothesis of weak dependence be rejected and α not be significantly different from 1, BHP (2016) propose to first model the data via a standard common factor model and then to re-test the residuals.[24] Should evidence of remaining weak dependence in the residuals be found, one should proceed by modelling

[20] More specifically, if N and T go to infinity both with $T = O(N^\epsilon)$ for ϵ in the range (0,1], then the relationship between the null of weak cross-sectional dependence underlying the CD-test and α takes the form $0 < \alpha < ((2-\epsilon))/4$. In other words, we have $\alpha < 1/2$ only if ϵ is small (T is almost fixed and N goes to infinity). If ϵ increases, α decreases, with a minimum of $\alpha = 1/4$ for $\epsilon = 1$ (N and T go to infinity at the same rate).

[21] Moscone and Tosetti (2009) survey a number of additional tests of cross-sectional independence. They also review statistics that can test the null of independence against the alternative of "spatial (local)" correlation (i.e., conditional on a pre-specified spatial (local) connectivity matrix) as a specific form of weak cross-sectional dependence. An example is a local version of Pesaran's CD test computed using a connectivity matrix but whose asymptotics do not depend on the structure of the connectivity matrix. The local test converges to its global variant as the number of neighbors for each unit increases towards N.

[22] For example, this can be represented by an inverse distance matrix ($W = 1/d$) or a parameterized inverse distance matrix ($W = 1/d^\gamma$ with $\gamma \geq 1/2$ (see Section 3.1).

[23] However, if N is small, the OLS estimator can be biased despite the fact that the weights are rather evenly distributed.

[24] In this context, BHP (2016) refers to a GVAR as a standard approximation (via taking cross-sectional averages) of a common unobserved factor model, as derived in Dees et al. (2007) and improved in Chudik and Pesaran (2011, 2013). Another possibility would be the use of principal component analysis.

the residuals (called "defactorized" observations by BHP) by a standard spatial model. BHP (2016) also point out that, in principle, for a pre-determined connectivity matrix, the two steps can be combined in a meta approach that addresses both types of cross-sectional dependence simultaneously. This approach is considered in Vega and Elhorst (2016). Alternatively, one might adjust the covariance matrix of the OLS coefficient estimates for the presence of common factors, as suggested by Andrews (2005), in combination with a spatial econometric model accounting for weak cross-sectional dependence, as set out in Kuersteiner and Prucha (2015).

3.3 Dominant units

From a GVAR perspective, a globally dominant unit can be defined as a unit that can potentially have a large impact on any of the other units. This definition of 'globally dominant' unit allows the presumably dominant unit (according to the latter criterion) to potentially be influenced directly by a single other unit from the world. In terms of N asymptotics, the definition of a globally dominant unit implies that its weight from the perspective of all other units does not vanish to zero if N goes to infinity. One example taken from the spatial econometrics literature is the leader matrix. If unit i represents the leader and all units are assumed to follow the leader, all elements in column i of the W matrix are 1, except the diagonal element w_{ii}.

Based on a connectivity (weight) matrix W one can identify–in a judgemental manner–potential globally dominant units if a sizable weight (from the perspective of other units) is assigned to the presumably dominant unit. A formal yet indirect way of detecting globally dominant units is to test for weak exogeneity of the foreign variable vectors in all GVAR equations (Pesaran et al. (2004); Dees et al. (2007)).[25] In empirical applications, usually for units (e.g. countries) such as the US and sizable European economies such as the UK, France, and Germany, some or all of the equations in their unit block (the set of variable specific equations in each unit) do not satisfy the weak exogeneity assumption, suggesting their globally dominant unit feature. A formal and more direct method to identify dominant units, put forward in Konstantakis et al. (2015), is built around the product of the weight and data matrices of the model. The eigenvalue distribution of the product matrix is used to compute the ratio of the modulus of subdominant eigenvalues to the most dominant (largest) one. Subsequently, a globally dominant unit is identified by choosing a threshold for that ratio.

The notion of strongly and weakly dominant units has also been discussed in Pesaran and Yang (2016), who develop a set of general conditions under which network structures (micro sectoral shocks) contribute to aggregate fluctuations. The authors illustrate that at most a finite number of units can be strongly dominant and that the number of weakly dominant units may rise with N. They develop, moreover, a nonparametric estimator of the *degree of pervasiveness* of micro shocks, which is identical to the exponent of cross-sectional dependence introduced in BKP (2016) (see Section 3.2).

The consequence of the presence of a globally dominant unit in the equation system is that the estimates of the dominant unit equations might be biased due to endogeneity (specifically due in turn to reverse causality). In the early GVAR papers this aspect was circumvented by excluding the foreign variable vectors from the dominant unit equations (Pesaran et al. (2004); Dees et al. (2007); among others). In Dees et al. (2007), for example, weighted foreign equity prices, long- and short-term interest rates, and the oil price (global factor) were excluded from the US equation system, while weighted foreign GDP growth and price inflation were kept in the model. In Pesaran et al. (2004) also the foreign GDP and price inflation variables

[25] This condition can be examined also if short and long-run dynamics are both explicitly accounted for. The required reduced-rank estimations of the corresponding error-correction models of spatial systems like GVAR are pioneered by Johansen (1988) and Johansen (1995) for endogenous I(1) variable sets and modified by Pesaran et al. (2000) for weakly exogenous I(1) variables sets. To test for weak exogeneity, Pesaran et al. (2004) suggest regressing the foreign variables reformulated in first-differences on the error correction terms and to test whether their coefficients are significant. This method also relates to the notion of "long-run causality" and "long-run forcing" discussed in Pesaran et al. (2000). For the analytical treatment presented in this paper, the extension to error correction models is not needed, since the relevant concepts such as smallness of the cross-section items and dense versus sparse weight matrices can indeed be understood entirely from models that operate on stationary data inputs.

were excluded from the US block, as apparently found to be suffering from endogeneity. To the extent that a dominant unit itself may, however, be influenced by the aggregate of the rest of the world, such a circumvention can cause omitted variable biases (Elhorst et al. (2013)).

The global dominant unit concept has come back to fore as part of the infinite dimensional VAR (IVAR) models (Chudik and Pesaran (2013)) where their presence is explicitly addressed instead of being circumvented, based on the notion of weak and strong cross-sectional dependence discussed in Section 3.2. The IVAR equation system distinguishes between a small number of neighbors and a potentially larger number of non-neighbors. The effects of the direct neighbors can be consistently estimated, provided that the impact from the aggregate of non-neighbors vanishes if N goes to infinity, in which case they can be ignored and dropped from the equation, or if their impact is explicitly accounted for. The latter is possible by treating the dominant unit as a dynamic common factor in the equations of the non-dominant units, and conversely, by estimating the resulting infinite order distributed lag relationship between dominant and non-dominant units by augmented least squares (ALS) estimators (Chudik and Pesaran (2013)).

Local dominant units (Chudik and Straub (2017)) help to bridge the gap between dominance structures considered in GVARs to those in standard spatial system representations. Local dominant units generally arise in spatial models when the connectivity (weight) matrices W are based on geographical borders. Each unit (country) can in this case be seen as having its own locally dominant neighbors in a locational sense, since the number of bordering neighbors tends to be small. Moreover, such local dominance patterns tend to be two-way (mutual), since the border a country A shares with a country B is also the border country B shares with country A, and the number of bordering countries is upper limited. This mutual local dominance induces the endogeneity which, in turn, rationalizes the standard use of ML/IV/GMM in spatial model applications (see Section 3.2).

A global dominant unit, on the other hand, induces endogeneity in principle only in the equation of the dominant unit itself (one-way relationship). By contrast, a local dominant unit that appears in a GVAR in a non-mutual way does not necessarily cause endogeneity and thus not necessarily require the use of ML/IV/GMM. If country B has a large impact on country A locally, i.e., not on any other countries, and A has hardly any impact on country B [$O(N^{-1/2})$], then country A's equation block can be consistently estimated by OLS as it is not potent enough to influence country B due to its negligible feedback effect. Similarly, country B's equation block can be consistently estimated by OLS since country A's impact on B compared to other countries is limited.

3.4 Connectivity (weight) matrices

Virtually all spatial model applications in the literature employ pre-specified Ws among which binary contiguity and inverse distance-based specifications are the most popular. It has been argued that theory should be the driving force for specifying W (Corrado and Fingleton (2012)). While this suggestion has some merit, it does not help in cases when theories for suggesting structures and a quantification of W are missing. Moreover, even if theories would be available, they might be flawed and suggest a W that is significantly deviating from the "true" W. In this respect, LeSage and Pace (2014) demonstrate that it should be tested whether the direct and indirect effects estimates (to be introduced in Section 4) are robust to small changes in W. If the chosen weights are close to the true ones, the estimates of direct and indirect effects (to be distinguished from the coefficient estimates)will hardly change. Conversely, small changes to grossly misspecified weights might still induce significant changes to direct and indirect effects. Several studies also developed statistics to test different weight specifications against each other (see the emerging literature on Bayesian posterior model probabilities and the J-test, pioneered respectively by LeSage and Pace (2009) (ch. 6) and Kelejian (2008)).

The common practice to date in GVAR studies has been to employ pre-specified connectivity matrices which are mostly based on trade flows, given that most applications tend to focus on standard macroeconomic variables whose relation across countries is thought to be well captured by the intensity of trade. In selected applications, a specification search was conducted based on a small set of pre-specified weight sets from which the chosen ones implied the best predictive performance of the model (see e.g. Eickmeier and Ng (2015)). The Mixed-Cross-Section GVAR methodology developed in Gross et al. (2016) and Gross et al. (2017) employs bank loan exposure-based weights to link consolidated bank variables with macroeconomic variables, as well as unit weighting schemes to link countries to their central banks.

In the spatial econometrics literature, on the other hand, it has been argued that geographical and distance-based measures are often being too readily used to calibrate weights (Partridge et al. (2012)). An attempt to unclench the otherwise restrictive structure implied specifically by inverse distance-based Ws has been discussed in Vega and Elhorst (2015) who suggest parameterizing the inverse distance W. In Section 3 of their paper they also provide several other parameterizations as well as approaches (including references) that have been considered to endogenize the W matrix.[26] The proposed parameterization is a power function with a distance decay parameter that is estimated along with the remaining model coefficients using nonlinear least squares. It is shown that this parameterization outperforms its non-parameterized counterpart, as well as the binary contiguity-based specification.

In the GVAR model literature there has been an attempt to estimate the weights along with all other GVAR model parameters (Gross (2013)). The identifying assumptions for the weights to be estimable include one stating that a significant relation between a unit and the 'rest of the world' must exist, as otherwise they become nuisance parameters which may be altered without changing the likelihood of the model. The weights are estimated along with the other coefficients using a constrained numerical optimization method. The imposed constraints are that the weights are bounded within the [0,1] interval and that they sum to unity in each column of the matrix. While the estimation of the weights goes—at first glance—against the rationale of GVARs and their objective to address the curse of dimensionality by pre-parameterizing the weights, the argument is that if it is possible to detect a significant deviation of the estimated from some predefined weights (based on informed judgement), the pre-defined weights might not be correctly specified.

Finally, a relevant link is Bailey et al. (2014). The authors of this paper develop a regularization method for the estimation of large covariance matrices, using elements from the multiple testing (MT) literature. Next, they first test for the significance of pairwise correlations and then set those that are insignificant to zero; an approach that compares favorably to thresholding approaches as developed in Karoui (2008), in particular when $N > T$.[27] The covariance matrix estimation topic does relate to the weight estimation method, albeit it does not consider a dynamic model structure "around" the connectivity matrix/weights, but instead a covariance matrix of the data directly.

3.5 Solving the models

The notion of "solving the model" has been set out in both the GVAR literature (Pesaran et al. (2004) (Section 3) and the spatial econometrics literature (LeSage and Pace (2009), Section 2.7). It denotes the steps needed to rewrite the simultaneous equation structure in time autoregressive form which would then allow the computation of direct and indirect (spillover) effects estimates, the impulse responses derived from GVARs, and the models' forecasts.

In the spatial model literature, the model solution step is referred to as a "rewriting" step before deriving the partial derivatives with respect to the exogenous model variables. Based on the dynamic SDM model in eq. (1), the solution (rewriting) step implies moving the contemporaneous WY_t term to the left and then pre-multiplying the equation by the so-called spatial multiplier matrix $(I - \delta W)^{-1}$, yielding

$$Y_t = (I - \delta W)^{-1} (\tau Y_{t-1} + \eta W Y_{t-1} + X_t \beta + W X_t \vartheta + \alpha + \lambda_t \iota_N + \epsilon_t) \quad (19)$$

[26] Recent related econometric-theoretical contributions are of Kelejian and Piras (2014) and Qu and Lee (2015).

[27] Karoui (2008) relates also to the notion of sparse connectivity structures. He examines the theoretical properties of the idea of thresholding the entries of a sample covariance (or correlation) matrix when it is sparse (which complicates the estimation step in particular when N and T are large) and proposes an alternative definition of sparsity, based on properties of the graph corresponding to the adjacency matrix of the covariance, and being "compatible" with spectral analysis.

In the GVAR literature, the exposition of the solution step consumes more space, involving a sequence of several equations (depending on the level of detail that is chosen to be presented). The reason is that the exposition of the model always starts from a unit specific equation; see eq. (2) representing a GVAR equation or equation system for a cross-section unit i, while the spatial model in eq. (1) was already stacked for all cross-section units. The GVAR solution steps, starting from eq. (2), can be summarized as follows.

Step 1: Generate A-matrices. Stack the time t cross-section unit variable vectors along with the weighted foreign variable vectors, to get

$$m_{it} = (Y_{it}\ Y_{it}^{*})' \tag{20}$$

which can be rewritten as

$$\underbrace{(I_k \ \ -\Lambda_{i0})}_{\equiv A_{i0}} m_{it} = \alpha_i + \underbrace{(\phi_i \ \ \Lambda_{i1})}_{\equiv A_{i1}} m_{i,t-1} + \Gamma_i \omega_t + \epsilon_{it} \tag{21}$$

Step 2: Generate L-matrices ("link" matrices). With a global, stacked variable vector $s_t = (Y_{1t}, Y_{2t}, ..., Y_{Nt})'$, the cross-section-specific variables vectors m can be linked. The link matrices L are used to map the local cross-section variables into the global vector, which involves the weights from the connectivity matrices W [28]:

$$m_{it} = L_i s_t \Rightarrow A_{i0} L_i s_t = \alpha_i + A_{i1} L_i s_{t-1} + \Gamma_i \omega_t + \epsilon_{it} \tag{22}$$

Step 3: Generate G-matrices. Stack the unit-specific equation systems into a global system:

$$G_0 = \begin{bmatrix} A_{10}L_1 \\ ... \\ A_{N0}L_N \end{bmatrix}, G_1 = \begin{bmatrix} A_{11}L_1 \\ ... \\ A_{N1}L_N \end{bmatrix}, ..., \Gamma = \begin{bmatrix} \Gamma_1 \\ ... \\ \Gamma_N \end{bmatrix}, \alpha = \begin{bmatrix} \alpha_1 \\ ... \\ \alpha_N \end{bmatrix} \tag{23}$$

Step 4: Generate H-matrices. The global system can be solved by pre-multiplying the stacked system by the inverse of G_0, to obtain:

$$s_t = \underbrace{G_0^{-1}\alpha}_{\equiv H_0} + \underbrace{G_0^{-1}G_1}_{\equiv H_1} s_{t-1} + \underbrace{G_0^{-1}\Gamma}_{\equiv H_\Gamma} \omega_t + G_0^{-1}\epsilon_t \tag{24}$$

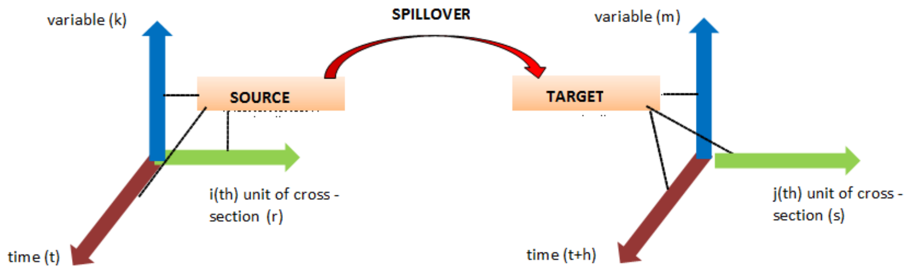
The first three steps involve "reformatting and stacking" which are not necessary for a spatial model since it is already written in stacked form. The last step represents the actual GVAR solution step which is mathematically equivalent indeed to the solution of the spatial system as shown in eq. (19). The spatial multiplier matrix $(I - \delta W)^{-1}$ in the spatial model corresponds to the G_0^{-1} matrix in the GVAR model (though impulse responses from one variable to another variable are not accounted for yet, see Section 4 for the extension).

4 The concept of spillovers

Spillovers and contagion are the buzzwords of the day among many academic macro-economists and policy institutions. Loosely based on a general concept of interconnectedness, they have received increasing attention following the international effects of the late 1990s Asian and Russian crises and considerably more

[28] For a detailed exposition of how the GVAR link matrices in Step 2 are constructed see Pesaran et al. (2004) (Section 3) for a standard GVAR and Gross et al. (2016) (Section 2.2) for a mixed cross-section GVAR.

Figure 2: The dimensions of measurable spillovers



Note: The chart depicts the different dimensions along which a shock can be defined, to spill from a source over to target.

so in the wake of the first global financial crisis of the 21st century, which fundamentally brought to the fore the importance of cross-border effects of shocks and policies. The quest for modelling and measuring interconnectedness has since then spawned an extensive literature, enriched more recently also by borrowing tools and theories from other branches of economics (e.g. regional and spatial economics) as well as social sciences (e.g. social networks) and mathematics (e.g. graph theory). Traditional macroeconomic research has significantly refocused on the cross-border interconnections between economies and policies. In addition, given that financial shocks and institutions were identified as both originators as well as key transmission channels in the latter crisis, much analysis concentrates on the structural and policy links between the real and financial sides of economies.[29]

Despite the wealth of research and policy discussions, a commonly accepted definition of spillovers and contagion remains elusive. This is hardly surprising given the complexity of modelling, let alone measuring, the size and transmission potential of multidimensional links between diversified economies that aggregate heterogeneous agents (individuals, financial and non-financial entities and public policymaking institutions). In addition, because an ultimate identification of cause and effect is needed in order to state something meaningful about the importance of a connection, definitions are also to a certain extent bound to be partly model/method specific. Our paper is chiefly motivated by the above considerations and discusses the practical application of two very closely related empirical methodologies that have been designed to incorporate interconnections. We therefore need a general but measurable definition of spillovers and the absence of a broad agreement allows for some flexibility.

We build on definitions from the post-Asian crisis contagion literature and then generalize the concept to any possible triplet of time/cross-section(s)/economic variable(s). Kaminsky et al. (2003) define spillovers as "gradual and protracted effects that may cumulatively have major economic consequences" (p.55) while contagion implies that such effects are "immediate and excessive" (p.55) (relative to an equilibrium). Rigobon (2001) initially proposed a similar definition of contagion which implies an unexpectedly large effect over an underlying transmission channel ("spillover"), but in later work he acknowledged the difficulties of conceptually separating spillovers from contagion, settling for a more general definition as "the phenomenon in which a shock from one country is transmitted to another" (Rigobon (2016), p.3).

In our view, a general definition of a measurable spillover represents an effect that is spread from a source to a target over an identified transmission channel, where each source and target is defined along three dimensions: (i) cross-section(s), (ii) economic variable(s), and (iii) time (see Figure 2).

A spillover is a possibly time-varying effect—policy induced (endogenous) or exogenous—that transmits from source to target over a clearly identified channel. This definition is cross-sectional in nature because even the static measurement of an economic variable must be done across a number of units located in space.

[29]See, for example, the seminars "Rethinking Macroeconomic Policies I,II and III" (IMF 2011-2015), "Interconnectedness: Building Bridges between Research and Policy" (IMF 2014), "Cross-border spillovers" (IMF 15th Jacques Polack research conference 2014), "Global Financial Interconnectedness" (BIS Research Network meeting 2015), "Spillover Report Series" (IMF 2011), and "First annual ECB macroprudential policy and research conference" (ECB-IMF 2016).

It will often be time-varying multivariate to capture interesting real-world empirics, but in principle it may also affect the same variable in another unit (along the horizontal cross-section axis in Figure 2), or another variable in the same unit (along the vertical cross-variable axis in Figure 2), both at the same moment in time. The definition is agnostic about the speed of transmission and the type of channels as well as the structure and content of the underlying links between units in each cross-section (bilateral, exposure to a common unit or multilateral; the content of links between units can be physical, economic, statistical correlations, informational, etc.[30]). The definition also accommodates common and idiosyncratic (source specific) effects, as long as the empirical method applied identifies the source, as well as multiple cross-sections at once (e.g. individuals, countries, banks, firms, jurisdictions, etc.).

Finally, we believe it is important to make a distinction between a spillover effect and the underlying interconnections between units in each cross-section. Both spatial and GVAR methods are centered on the concept of a connectivity (weight) matrix W which can (i) be pre-defined empirically (informed by physical or economic distances, or statistical correlations), (ii) take a functional form, or (iii) be estimated subject to constraints (see Section 3.4). As we shall see in the following section, spillover effects under either spatial or GVAR representations are a function of shocks, coefficient estimates, and the specification of W. The spatial system and GVAR representations are intuitively and equivalently designed to tackle the cross-sectional nature of spillovers. They are nonetheless both linear and as such subject to the caveats of approximating the measurement of complex, non-linear phenomena using linear methods.[31] They are also related to a broader class of empirical linear methods designed to address various dynamic heterogeneities across multiple dimensions.[32]

Spillovers defined as above can be measured in either spatial systems or GVARs through spatial indirect effects analogous to impulse responses from GVAR models. In standard spatial models, *indirect effects* are interpreted as *spillover effects* from changes in the exogenous variables in one particular unit to the dependent variable in other units as represented by the horizontal cross section axes in Figure 2 (see also Elhorst (2014), Section 2.7). The link to the GVAR representation is straightforward and consists of: (i) changing to a spatial system representation, (ii) the computation of responses to shocks in the error term rather than a change in an exogenous variable, (iii) a greater focus on pairwise responses rather than only row or column aggregation.

The coefficient estimates of a standard linear regression equation reflect the *direct (marginal) effects* of exogenous variables X on the dependent variable Y. By contrast, the coefficient estimates in a spatial econometric equation do not. The marginal effects in a spatial model, denoted as direct and indirect effects of X on Y, which in turn are conditional on the cross-unit connectivity (weight) matrix W, can be derived by considering the reduced (solved) form of the model.

For this exposition, we consider a two-variable (e.g. Y=GDP and C=credit), two-equation spatial system based on eqs. (1) and (2), later extended for purposes of our companion empirical application in Section 6:

$$Y_{it} = \tau_{yi} Y_{i,t-1} + \delta_{yi} W_{1i} Y_{it} + \theta_{yi} W_{2i} C_{jt} + \epsilon_{y,i,t} \tag{25}$$

$$C_{jt} = \tau_{cj} C_{j,t-1} + \delta_{cj} W_{3j} C_{jt} + \theta_{cj} W_{4j} Y_{it} + \epsilon_{c,j,t} \tag{26}$$

The cross-section item counters for GDP and credit are $i = 1, ..., N$ and $j = 1, ..., M$, respectively. Having two separate counters means that the two cross-sections can be treated independently, to allow the

[30]See Corrado and Fingleton (2012) for a thorough discussion about the information content of cross-section connections used to inform the connectivity (weight) matrix W.

[31]Rigobon (2016) argues that the empirical assessment of spillovers and contagion is one of the most complicated applied questions, essentially because of time varying biases resulting from a combination of endogeneity and omitted variable bias and heteroscedasticity. Moreover, distinguishing between spillover and contagion is tenuous because the definitions are model-dependent. The paper discusses biases present in linear models, as well as non-parametric techniques and limited dependent models.

[32]See the survey of spatial panel methods in Elhorst (2014), and of panel VARs in Canova and Ciccarelli (2013) and Pesaran (2015b) (chapter 30).

number and order of items in the cross-sections to be different, potentially (e.g. countries and banks/banking systems). They could also be set to a common counter i in which case the cross-section numbers and orders would be the same in the two equations, cross-sections respectively. Note, moreover, that the weight matrices W_1 and W_3 are square and having zero diagonal elements, while the matrices W_2 and W_4 may not be square (unless $N = M$) and are in general having non-zero diagonal elements. For the sake of simplifying the notation in the following, we do however assume that $N = M$. To derive direct and indirect effect estimates, the model first has to be solved along the lines set out in Section 3.5, which for eq. (25) in stacked format yields:

$$Y_t = (I - \text{diag}(\delta_y)W_1)^{-1} (\text{diag}(\tau_y)Y_{t-1} + \text{diag}(\theta_y)W_2C_t + \epsilon_{yt}) \tag{27}$$

The matrix of partial derivatives of the expected value of Y_t with respect to the explanatory variable C_t in unit 1 up to unit N (say c_{jt} for $j = 1, ..., N$ respectively) yields the full $N \times N$ matrix of marginal effects:

$$\begin{aligned}
\begin{bmatrix} \frac{\partial E(Y_t)}{\partial c_{1t}} & ... & \frac{\partial E(Y_t)}{\partial c_{Nt}} \end{bmatrix} &= \begin{bmatrix} \frac{\partial E(y_{1t})}{\partial c_{1t}} & ... & \frac{\partial E(y_{1t})}{\partial c_{Nt}} \\ ... & ... & ... \\ \frac{\partial E(y_{Nt})}{\partial c_{1t}} & ... & \frac{\partial E(y_{Nt})}{\partial c_{Nt}} \end{bmatrix} \\
&= (I - \text{diag}(\delta_y)W_1)^{-1} \begin{bmatrix} w_{2,11}\theta_{y,1} & w_{2,12}\theta_{y,1} & ... & w_{2,1N}\theta_{y,1} \\ w_{2,21}\theta_{y,2} & w_{2,22}\theta_{y,2} & ... & w_{2,2N}\theta_{y,2} \\ ... & ... & ... & ... \\ w_{2,N1}\theta_{y,N} & w_{2,N2}\theta_{y,N} & ... & w_{2,NN}\theta_{y,N} \end{bmatrix} \\
&= (I - \text{diag}(\delta_y)W_1)^{-1} \text{diag}(\theta_y)W_2
\end{aligned} \tag{28}$$

The diagonal elements in eq. (28) measure the direct effects of C on Y, whereas indirect effects are measured by the off-diagonal elements. Due to the impact of the spatial multiplier matrix, direct effects are *not* the same as the standard marginal linear effect θ_y, just as indirect effects are *not* the same as δ_y; both direct and indirect effects incorporate the relationships across units resulting from cross-sectional dependence.

To better understand the direct and indirect effects that follow from this model, the infinite series expansion of the spatial multiplier matrix is considered:

$$(I - \text{diag}(\delta_y)W)^{-1} = \left(I + \text{diag}(\delta_y)W + (\text{diag}(\delta_y)W)^2 + (\text{diag}(\delta_y)W)^3 + ...\right) \tag{29}$$

Since the *diagonal* elements of the identity matrix are all ones and the off-diagonal all zeros, the *first* term represents a *first-order direct* effect of C on Y. Conversely, since the *diagonal* elements of the *second* term in the expansion are zero by assumption, this term represents a *first-order indirect (spillover)* effect of C on Y. All other terms on the right hand-side of eq. (29) represent *second- and higher-order direct and indirect* effects.

First-order direct effects reflect the impact of C on Y in the same cross-section unit, not accounting for feedback effects via neighboring units. Second and higher-order direct effects refer to the impact of changing C onto Y in the same unit, but also accounting for feedback effects. An indirect effect from unit i to unit j causing another indirect effect from unit j to unit i, is a feedback effect on unit i itself. Feedback effects are represented by the non-zero diagonal elements of higher order terms in eq. (29), beginning with W^2.

First-order indirect effects refer to the impact of changing C in one unit onto Y in another unit, accounting only for bilateral connections. Second and higher-order indirect effects account for feedback through ever more distant neighbors.

Importantly, even if the slope coefficients in eq. (25) would be homogenous, the direct and indirect effects will be unit-specific, hence the presentation of both effects may consume much space and be impractical for that reason. With N units and k explanatory variables, it is possible to obtain k different $N \times N$ matrices of direct and spillover effects. For homogenous coefficient spatial models, LeSage and Pace (2009) propose to report a single direct effect measured by the average diagonal element and a single indirect effect measured by the average row or column sum of the off-diagonal elements of the matrix expression in eq. (28). We recall that this indirect effect is also interpreted as a spillover effect. When taking averages over all row or column sums, the numerical magnitudes of these two calculations of the indirect effect are the same, but when indirect effects are reported for each single unit, i.e., at the observational level, they will be different. Also note that the indirect (spillover) effects computed for models including a spatially autoregressive lag WY are global in nature (the power terms in the infinite expansion ensure that effects are transmitted across all units, even those that do not share a bilateral connection in W). See also the discussion in Vega and Elhorst (2015) about local versus global spillovers and some of the limitations, among which the proportionality of direct and indirect effects, of the standard SAR model.

For heterogeneous models, LeSage and Chih (2016) suggest to separate indirect effects between *spill-in* and *spill-out* effects. Spill-in effects are row-specific sums of the off-diagonal elements in eq. (28) and represent the *sensitivity* (vulnerability, response) of variable Y in unit i to changes in variable C in all other units. Conversely, *spill-out* effects are column-specific sums of the off-diagonal elements and represent the *impact* (impulse) of the change in variable C in unit i on changes in variable Y in all other units.

In contrast to LeSage and Chih (2016), we compute the spill-in/out measures as the *average* over all off-diagonal elements in a row or column rather than their sums, because the latter have no economic interpretation as they increase with the number of units in the cross-section. The average spill-in and spill-out measures across countries are illustrated in the upper left matrix of Figure 3 for the first variable (V1) and in the bottom right matrix for the second variable (V2).[33] The shock originating country is displayed along the columns and the respondents along the rows. The spill-out (impact) measure is the average off-diagonal element along the rows in any column, and the spill-in (vulnerability) measure is the average off-diagonal element along the columns for any given row, so in both cases excluding the diagonal itself. This is closely related to the spillover measurement proposed in a series of papers by Diebold and Yilmaz (2009) and Diebold and Yilmaz (2014) which are based on share of the forecast error variance in $y(i)$ explained by $x(j)$ and have the same information content as the IR-based approach presented here.

The standard spatial direct and indirect effects in eq. (28) are computed as partial derivatives with respect to a change in the explanatory variable and are based solely on estimated coefficients. Therefore, they do not consider responses to shocks (as is standard in the GVAR literature). However, one can also derive the direct and indirect (spillover) *point-in-time*, as well as *time cumulative* effects (see also Debarsy et al. (2012)) of a transitory shock via one or more error terms. The point-in-time direct and indirect (spillover) effects at time t when the shock occurs and at any other point $t+h$ can be computed as follows:

$$\left[\frac{\partial Y_t}{\partial \epsilon_{y,t,S}}\right] = (I - \text{diag}(\delta_y)W_1)^{-1}\,\epsilon_{y,t,S} \tag{30}$$

$$\left[\frac{\partial Y_{t+h}}{\partial \epsilon_{y,t,S}}\right] = (I - \text{diag}(\tau_y)I - \text{diag}(\delta_y)W_1)^{-1}\left[\frac{\partial Y_{t+h-1}}{\partial \epsilon_{y,t,S}}\right] \tag{31}$$

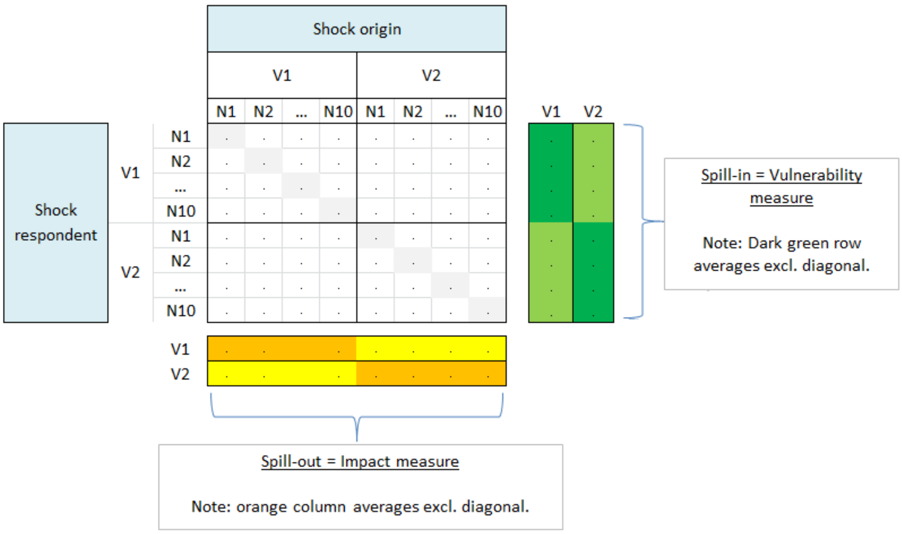
where $\epsilon_{y,t,S}$ is an $N \times 1$ vector of zeros except for the unit(s) where the shock takes place. The size of this shock is generally set equal to one standard deviation of σ^2.

The direct and indirect effects become more complicated in a system of equations, since mutual dependencies between the dependent variables, in the example of C_t on Y_t in eq. (25) and of Y_t on C_t in eq. (26), also need to be accounted for.[34] The point-in-time direct and indirect (spillover) effects for a system take the following forms:

[33] Taken and adjusted from Gross and Kok (2013) (Figure 9) who presented a conceptually equivalent schematic impact/vulnerability matrix based on generalized impulse responses; rows and columns are interchanged.

[34] In the spatial econometrics literature, Vega and Elhorst (2014) are among the first to determine direct and indirect effects within a system of equations.

Figure 3: Spill-out (impact) and spill-in (vulnerability) measures



Note: The chart illustrates the derivation of average spill-out (impact) measures and spill-in (vulnerability) measures which can be derived based on row and column averages of the elements of a matrix that contain the direct/indirect effects estimates or some feature of impulse responses (point-in-time or time-cumulative). See text for details.

$$\begin{bmatrix} \frac{\partial Y_t}{\partial \epsilon_{y,t,S}} \\ \frac{\partial C_t}{\partial \epsilon_{c,t,S}} \end{bmatrix} = G_0^{-1} \begin{bmatrix} \epsilon_{y,t,S} \\ \epsilon_{c,t,S} \end{bmatrix}, \text{ where } G_0 = \begin{bmatrix} I - \text{diag}(\delta_y)W_1 & -\text{diag}(\theta_y)W_2 \\ -\text{diag}(\theta_c)W_4 & I - \text{diag}(\delta_c)W_3 \end{bmatrix} \tag{32}$$

$$\begin{bmatrix} \frac{\partial Y_{t+h}}{\partial \epsilon_{y,t,S}} \\ \frac{\partial C_{t+h}}{\partial \epsilon_{c,t,S}} \end{bmatrix} = G_0^{-1} G_1 \begin{bmatrix} \frac{\partial Y_{t+h-1}}{\partial \epsilon_{y,t,S}} \\ \frac{\partial C_{t+h-1}}{\partial \epsilon_{c,t,S}} \end{bmatrix}, \text{ where } G_1 = \begin{bmatrix} \text{diag}(\tau_y)I & 0 \\ 0 & \text{diag}(\tau_c)I \end{bmatrix} \tag{33}$$

We use the symbols G_0 and G_1 here to denote the correspondence with the model solution of the GVAR model presented in Section 3.5 (the G matrices above are mathematically equivalent to those in eq. (24)).[35] The block-structure of the matrices G_0 and G_1 in eqs. (32) and (33) make clear that it no longer makes sense to consider single indirect effects calculated over all off-diagonal elements in a row or column, but that these effects on different dependent variables need to be separated along this structure.[36] In line with the blue axes in Figure 2, the upper right and bottom left blocks in Figure 3 should be used to determine the spill-in and spill-out effects from one variable to another. In contrast to the other two blocks, the diagonal elements do not have to be excluded here.

The *time-cumulative* effects are the sum of the point-in-time effects, usually computed over a chosen horizon $\tilde{h}$. If this horizon is chosen sufficiently long such that the corresponding point-in-time responses have converged to zero, the cumulative responses will converge to a constant, which can be calculated by:

$$\sum_{\tilde{t}=t}^{t+\tilde{h}} \begin{bmatrix} \frac{\partial Y_{\tilde{t}}}{\partial \epsilon_{y,t,S}} \\ \frac{\partial C_{\tilde{t}}}{\partial \epsilon_{c,t,S}} \end{bmatrix} \tag{34}$$

[35] The G_1 matrix in eq. (31) is relatively simple, however if eq. (25) would also contain the variable $W_1 Y_{t-1}$ and eq. (26) the variable $W_3 C_{t-1}$, see Section 6, the off-diagonal blocks of G_1 matrix will become non-zero as well.

[36] The statistical significance of direct and indirect effects is generally determined by drawing the parameters M times from the coefficient variance-covariance matrix, which is obtained after estimating the model by OLS or ML, by calculating the direct and indirect effects for each draw using the above formulas, and by approximating their standard errors by the standard deviation of the mean value over these draws.

We recall that the formulas in eqs. (32) and (33) are in fact the non-factorized impulse response functions (NIRs) used in the GVAR literature. Their outcomes are therefore reflected in Figure 3. NIRs ignore the contemporaneous cross-section correlation, should there be any, across the residuals of the models. Alternatively, generalized impulse responses can be employed which do account for possible remaining cross-section correlation (see Koop et al. (1996)). The discussion about structural identification and economic interpretation of IRs (e.g. via sign restrictions) in either spatial systems or GVAR representations is beyond the scope of this paper.

5 A practical guide to model selection and spillover measurement

A practical guide to measuring spillovers in any model covering potentially cross-sectional dependent data can be conceived as a sequence of steps which involve statistical tests on the raw data, a joint selection of model and estimation method and, finally, the actual computation of spillovers. Our guidance partly builds on but also extends BHP (2016) using the main findings of this paper (see also Table 2) as follows:

1. **Assess the degree of strong cross-sectional dependence in the raw data.** Apply the CD-test of Pesaran (2004) and Pesaran (2015a) and compute the corresponding estimator of α- exponent of BHP (2016) to each variable. An agnostic (generic) assumption is that the raw data is characterized by a mix of weak and strong cross-sectional dependence. A significant CD-test statistic and a value of α not significantly different from 1 suggests the presence of at least global common factors (strong cross-sectional dependence). This could be addressed by "de-factoring" the raw data by a standard common factor model (observed or unobserved but without involving weights).[37] By contrast, a non-significant CD-test statistic indicates that the data are possibly still weakly dependent. A stronger (judgmental) supposition is to assume that, given a value of α not significantly different from 1, the raw data is characterized only by strong cross-sectional dependence. If so, "de-factoring" the data by a model with a dense (e.g. trade) connectivity matrix W would be an appropriate choice of model from the start. An α greater than 3/4 points to weak exogeneity of the spatial lag WY_t (the "foreign" variable Y_t^*) in which case the OLS estimator may be used (although a small-sample bias is possible if N is (too) small).

2. **Assess the degree of weak cross-sectional dependence in the residuals from Step 1.** Apply the CD test on the "de-factored" observations from Step 1. Failure to reject the null indicates remaining weak cross-section dependence.[38] The appropriate method would then be a GVAR (or equivalently spatial heterogeneous system) with a sparse connectivity matrix W estimated by means of ML/IV/GMM.[39] Note that if the judgmental assumption of only strong cross section dependence in Step 1 was correct, the system residuals should not display any remaining cross-sectional dependence at this point.[40]

3. **Select between a homogenous or heterogeneous version of the model.** Once the model and estimation method are chosen, conduct an LR test to select between an unrestricted (heterogeneous) and a restricted (homogenous) version of the model. This is for the sake of enhancing *efficiency* of

[37] Alternatively, one might adjust the covariance matrix of the OLS coefficient estimates for the presence of common factors, as suggested by Andrews (2005).

[38] This should be associated with an estimate of $\alpha < 3/4$, though note that α can only be consistently estimated if it is also greater than 1/2. If $\alpha < 1/2$ then its estimate will hover around the lower bound of 1/2 (see section 3.2). Further note that α may be somewhat upward biased when N is small (BKP, 2016).

[39] The reason to recommend using a standard common factor model (without weights) if a mix of cross-sectional dependence is suspected in the data is that it is difficult to justify that two different sets of weights (one dense and one sparse—see Step 2) characterize the economic linkages among variables in the same cross-section at the same time, which would be the implicit assumption if a single cross-section GVAR with dense weights is used as an approximation to a global factor model in Step 1.

[40] This relates to the meta approach as suggested by BHP (2016) and employed in Vega and Elhorst (2016). To address both types of cross-section dependence simultaneously, the approach could only combine a sparse weight connectivity matrix (to address CWD) with non-connectivity based common factors (to address CSD) for the reasons set out in the preceding footnote. An alternative approach, modelling weak cross-sectional dependence and adjusting the covariance matrix for the presence of common factors, following Andrews (2005), is set out in Kuersteiner and Prucha (2015).

the model estimates in case that a homogenous coefficient set up is not rejected by the test. Should the homogenous coefficient structure be suggested, test the residuals again for any remaining cross-sectional dependence. A traditional spatial econometric application in which N is large and T is small will probably not consider the heterogeneous model. However, this does not mean that the first two steps of the practical guide are also irrelevant. The CD test and the α estimator can still be used to test whether weak and/or strong cross-sectional dependence need to be accounted for, and related to that, whether the spatial weight matrix should be sparse or dense.

4. **Conduct ex post analysis with the model.** Assuming a choice of model structure in the previous steps, compute the direct and spillover, point-in-time and time cumulative effects using the estimated model coefficients and a sequence of shocks (see Section 4), or develop unconditional or conditional forecasts from the model.

6 Empirical application

To anchor the methodological discussion from the first part of the paper and to illustrate the guidance developed in the previous section, we present and discuss the results of a small empirical application relating GDP and bank credit growth rates, respectively denoted by Y and C, for a sample of 17 EU countries and banking systems over the period 2001Q1-2015Q4 ($T = 60$). The credit stock variable is a consolidated bank credit measure, which means that business of a banking system belonging to one country across borders to other countries is included.[41] Empirical applications involving measures of credit, economic activity, and others, are relevant for monetary policy- and macroprudential policy-related analyses, for which dynamic panel systems are often employed. The empirical application with the two variables that we pursue here is a simple one and would need to be augmented by numerous other macro-financial variables. It is meant to serve as an example for illustrating the practical guidance presented in the previous section.

We first apply the CD test and estimate α set out in Section 3.1. For GDP growth, the CD test statistic equals 41.4 with an average pairwise correlation coefficient of 0.46 and an α-estimate of 0.864 with standard error 0.018. The significant value of the CD test statistic and the finding that α appears to be significantly greater than 0.75 indicate that strong cross-sectional dependence with respect to GDP growth needs to be accounted for. The fact that the latter coefficient is also significantly smaller than 1 indicates that controlling for common factors based on a dense weight matrix is an appropriate description of the cross-sectional dependence of GDP growth across countries. Furthermore, just because the elements of such a matrix will converge to zero slower than the $\sqrt{N}$-rate according to Lee (2002)'s condition C and Pesaran et al. (2004)'s smallness condition D, estimation of the coefficients of the common factors by OLS shall be consistent. A similar outcome is obtained for bank credit growth. Its CD test statistic takes a value of 27.4 with an average pairwise correlation coefficient of 0.30 and its α-estimate a value 0.851 with standard error 0.013. In view of these results, we estimate a GVAR first-order system with heterogeneous coefficients, as set out in eq. (2), for these two variables by OLS. This model specification extends the structure in eqs. (24) and (25) with additional time autoregressive spatial lags (the set of variables $Y^*_{i,t-1}$ in eq. (2)), so as to obtain a full first-order system in both space and time.

Four weight matrices are used to link the two cross-sections, whose structure is summarized in Table 3. Since GDP and credit growth may also affect each other mutually within one country at time t, these terms have been considered separately, cf. $X_t\beta$ in eq. (1).[42] An ML procedure has been used to modify the OLS estimation technique for the Jacobian term to reflect this mutual relationship so as to avoid a potential

[41] See Gross et al. (2016) and Gross et al. (2017) for applications that involve consolidated banking system measures in a Mixed-Cross-Section (MCS)-GVAR model which are similar in structure compared to the one that we use for the empirical application presented in this paper.

[42] Note that even though the dimension in each cross-section is the same, these are effectively two cross-sections: one of EU economies, the other of EU banking systems. On the GVAR model side, such a structure has been referred to as mixed-cross-section (MCS-)GVAR (Gross and Kok (2013); Gross et al. (2016)), a straightforward extension to the standard GVAR structure. Note that in this respect the choice of a consolidated as opposed to a locational credit measure has implications for the choice of the weights in the different equations.

simultaneity bias. It is a two-way mutual, local dominance structure of the kind discussed in Section 3.3 that indeed characterizes the relationship between consolidated credit and GDP, reflecting the fact that all consolidated banking systems in the sample (despite some of them being quite active across borders) provide the largest share of the credit they generate to their host economies.

Table 3: Connectivity (weight) matrices W that link GDP and credit growth in exemplary spatial equation system

		RHS	
		GDP	**Credit**
LHS	**GDP**	W11 = Trade	W12 = Transpose of loan exposure based weights
	Credit	W21 = Loan exposures (non-financial private sector)	W22 = Financial institutions' loan exposures

Tables 4 and 5 report the estimation results respectively for the GDP and credit growth equations. The system is dynamically stable under both the homogeneous and heterogeneous slope assumption, with the maximum of the modulus of the eigenvalues of the matrix $G_0^{-1}G_1$ in eq. (23) and (33) for our example specification being less than one.

First, we test whether the heterogeneous slopes of the variables (including the intercept) may be replaced by homogeneous coefficients. For this reason we also estimate the two homogeneous equation sets (results are not reported) and carry out a likelihood ratio (LR) test, based on the log-likelihood function values of the heterogeneous and homogenous models. The resulting LR test statistic provides strong evidence against the hypothesis of homogeneous coefficients. For the GDP equation, LR=-2*(2838.3-3384.5)=1092.4, with $p < 0.01$ given eight (coefficients)*16 (restrictions)=128 degrees of freedom (df). Similarly, the LR-statistic for the credit equation amounts to 2*(2179.5-2571.3)=783.6, with $p < 0.01$ and 128 df.

Second, we test whether the heterogeneous model is free of any remaining cross-sectional dependence by carrying out the CD-test on its residuals. For the credit equation we obtain a CD test statistic of -0.751 which is insignificant, but for the GDP equation the CD test statistic amounts to 3.102 which is still significant. As $N = 17$, this might be due to a small sample bias.

Table 4 shows that the time lagged variable Y_{t-1} and especially the spatially lagged variable $W_{11}Y_t$ are the two most relevant determinants of GDP. For the majority of countries these variables are weakly (10% significance level) to strongly significant (1% significance level). The remaining variables turn out to be significant for approximately one-third (5 to 6) of the countries being considered. A similar picture emerges in Table 5, though the number of coefficients that appears to be significant decreases slightly. The coefficient of the spatially and time lagged variable $W_{12}C_{t-1}$ is significant for only one country in Table 5, and therefore could have been removed from the model equally well.

Table 6 reports the short-term or point-in-time direct and spillover effects at $h = 0$ of a one standard deviation shock in each country's GDP growth, based on the estimate $\hat{\sigma}$ for each country. The direct or own country effect of this shock for QoQ GDP growth varies between about 0.09pp for France to 1.88pp for Luxembourg and is significant in all cases. The spillover effect of such a shock on GDP growth in other countries is reported in the column "GDP spill-in". Each country may be affected by a shock in one of the other sixteen countries. To reduce the amount of output, the reported spillover effect is calculated as the average over these sixteen outcomes. As expected, the spillover effect is smaller than the corresponding direct effect; the ratio between these numbers amounts to 0.11 with a minimum of 0.01 and a maximum of 0.22. Table 6 shows that the spill-out and spill-in effects are not the same, although they are on average of the same order of magnitude since they are based on the same concept. A country such as France is hardly sensitive to GDP growth spill-in effects and also hardly produces any GDP growth spill-out effects. By contrast, Latvia appears to be much more sensitive to these kinds of spill-in effects, while Denmark produces relatively strong GDP spill-out effects.

Table 4: Estimation results GDP growth equations

Country	Intercept	Y_{t-1}	$W_{11}Y_t$	$W_{11}Y_{t-1}$	Own C_t	$W_{12}C_t$	Own C_{t-1}	$W_{12}C_{t-1}$
AT	0.2351*	-0.2360*	0.6118***	0.1878	0.0031	0.006	0.0473	0.0208
BE	0.4101***	-0.1908	0.7870***	0.0308	0.0740**	0.1071	0.0148	-0.1865**
CY	-0.1386	-0.3240***	0.5661***	0.5881***	0.1679**	-0.2180**	0.2665***	-0.0806
DE	0.2022**	0.1617	1.1998***	-0.2996	0.0229	-0.0738	0.1087***	-0.1366**
EE	0.6869*	0.2653*	0.8450***	0.2452	0.0063	-0.0023	-0.0065	-0.058
ES	-0.0066	0.8304***	0.3573***	-0.13	-0.0032	0.0144	-0.0004	-0.0134
FI	-0.7880***	-0.3558***	1.4201***	0.195	0.0851*	0.0269	-0.0015	-0.0081
FR	0.5202***	-0.2091	1.0577***	0.3515	-1.1373***	0.1288	0.0318	0.2966**
GR	-0.4974	0.3791***	0.7698*	-0.3679	0.0028	-0.0134	0.1089	0.045
IE	-1.3586**	-0.1104	0.7076	2.3643***	-0.8827***	-0.4789	0.4581***	0.9417**
IT	-0.3778***	-0.3215***	0.8236***	0.3369**	0.0179	0.1077*	-0.0490**	-0.0306
LU	0.4219	-0.2568**	1.8195***	0.1452	-0.1041	0.0209	0.2473	-0.1065
LV	-1.3703***	0.0088	1.5332***	1.4719***	-0.0793	0.0692	0.0880*	-0.0649
NL	-0.1706**	-0.1176	0.5120***	0.5197***	-0.0259	0.1324**	-0.1141***	0.0991**
PT	-0.0942	0.1702	1.2875***	-0.5569**	-0.0803	0.0549	0.0713	-0.0332
SI	-0.2039	0.2842**	1.0126***	-0.0186	0.0298	0.1590***	0.0198	-0.1123**
SK	0.0463	0.0612	1.2637***	0.3027	0.0353	0.4128***	0.0574	0.006

Note: *** significant at 1%, ** significant at 5%, * significant at 10%.

Table 5: Estimation results credit growth equations

Country	Intercept	C_{t-1}	$W_{22}C_t$	$W_{22}C_{t-1}$	Own Y_t	$W_{21}Y_t$	Own Y_{t-1}	$W_{21}Y_{t-1}$
AT	-0.2523	0.2775**	0.5071**	0.0751	-0.1703	0.4504**	0.5838*	-0.0085
BE	0.3553	0.1157	0.0234	0.4745	-2.3469***	1.8095**	-0.6673	1.4718**
CY	0.0606	0.4076***	0.2672	0.3229	0.0331	0.144	0.2041	0.8392**
DE	-0.1506	0.1941	0.1118	0.4174	-0.3402	0.1467	0.4237	-0.4745
EE	0.5787	0.7041***	-0.3661	0.1432	0.0077	0.1218	0.3165	-0.1907
ES	0.2317	-0.0509	1.5261***	-0.0954	0.8096	0.0506	-0.172	-0.7023
FI	1.1310*	0.6863***	-0.0903	-0.1236	-0.5268	0.1031	0.2661	0.4884
FR	-0.3759	0.7843**	0.2339	-0.3024	0.6384***	-0.4157***	0.5787	-1.6066**
GR	0.5837	0.0936	0.1529	0.7175	0.0393	-0.1355	0.3997*	0.2321
IE	-1.6538*	0.4660***	0.8595*	0.2760*	0.6805***	-0.6271	0.3254**	-0.26
IT	-0.332	-0.001	0.2002	0.8598	-0.2641	0.2633	0.0074	0.18
LU	-0.0723	0.8009***	0.1059	0.0958	-0.0506	-0.0628	-0.0271	0.4876***
LV	0.4691	0.3578***	1.4834**	-0.3462	0.8428**	-0.3878	-0.1934	-0.2667
NL	-0.4166	0.1442	0.3554	0.0708	0.9300*	0.0404	0.3301	-0.3848
PT	-0.0418	0.5688***	0.2245**	-0.0254	0.5973***	-0.7164***	0.0807	0.4477*
SI	-0.1353	0.6619***	0.502	0.0961	-0.1647	0.4325	0.0531	0.1084
SK	0.9376*	0.6647***	0.1121	0.1077	-0.1346	0.6311	0.0136	0.19

Note: *** significant at 1%, ** significant at 5%, * significant at 10%.

Table 6: Short-term effects at $h = 0$ of a one standard deviation GDP growth shock in every country

Country	Direct effect	GDP spill-in	Credit spill-in	GDP spill-out	Credit spill-out
AT	0.6925***	0.0712***	-0.0083	0.1199***	0.0004
BE	0.5447***	0.0554***	-0.0034	0.1650***	0.0422
CY	1.2252***	0.0694***	0.0165	0.0134***	0.0207**
DE	0.7198***	0.0873***	-0.0135	0.3374***	-0.0008
EE	1.4079***	0.1474***	-0.0167	0.0628***	0.0003
ES	0.4179***	0.0304**	-0.0068	0.0859***	-0.0153
FI	0.7193***	0.1531***	-0.012	0.0512***	0.0194
FR	0.0916***	0.0133***	-0.0195***	0.0345**	-0.1332***
GR	1.5536***	0.0760*	-0.0083	0.0597***	0.0031
IE	1.8440***	0.0258	-0.1565***	0.0822***	-0.1878***
IT	0.5756***	0.0709***	-0.009	0.1500***	-0.0406
LU	1.8846***	0.1644***	-0.0247	0.0399***	-0.0059
LV	1.8232***	0.2141***	-0.0530*	0.0416***	-0.0318
NL	0.4841***	0.0471***	-0.0105	0.1843***	-0.0391
PT	0.6354***	0.0902***	-0.0215*	0.0234***	-0.0099
SI	0.6571***	0.1158***	-0.0062	0.0181***	0.0054
SK	1.8347***	0.1318***	-0.0106	0.0944***	0.0085

Note: *** significant at 1%, ** significant at 5%, * significant at 10%.

Table 7: Short-term effects at $h = 0$ of a one standard deviation credit growth shock in every country

Country	Direct effect	Credit spill-in	GDP spill-in	Credit spill-out	GDP spill-out
AT	1.5906***	-0.0201	0.0588**	-0.0102	0.0467**
BE	1.9984***	-0.1964***	0.036	-0.0942**	0.1011***
CY	1.7641***	0.0019	0.0331	0.0038	0.0051
DE	1.6263***	-0.0389	0.0157	-0.0065	0.1634***
EE	4.9817***	0.0129	-0.061	0.0213	0.0012
ES	1.8563***	0.0756	0.1271***	0.0419*	0.0351
FI	1.9806***	-0.1035*	-0.0133	-0.0115	-0.0015
FR	0.1067***	0.1177***	-0.0004	0.0888**	0.0286
GR	2.6628***	0.0055	0.0124	0.0063	0.0184
IE	1.9684***	0.1132***	0.0566	0.0797***	0.018
IT	2.4862***	-0.0154	0.0188	-0.0044	0.1287***
LU	0.5019***	-0.0138	0.0122	-0.0099	0.0114
LV	4.2638***	0.1953	0.1626*	0.0791**	-0.0206
NL	1.6116***	0.0706	0.0307	0.0216	0.0362
PT	0.9524***	0.0850***	0.0234	0.0329***	0.0196**
SI	2.1325***	-0.0363	0.0685	-0.0064	0.0064**
SK	2.1471***	-0.0331	0.0202	-0.0121	0.0035

Note: *** significant at 1%, ** significant at 5%, * significant at 10%.

The spillover effect of a GDP shock on credit in the own or in another country is reported in the column "Credit spill-in". Since they reflect the sensitivity of another variable (credit) than the variable that has initially been shocked (GDP), these spillovers are smaller, approximately 0.23 (in absolute values), than the GDP spill-in effects, which already appeared to be approximately one-tenth of the direct effects. Only 3 or 4 of the credit spill-in or spill-out effects in Table 6 are significant. It shows that it is much harder to find empirical evidence in favor of significant spillover effects than in favor of significant direct effects. This is because spillover effects depend on many more parameters. At $h = 0$, the direct effects reported in Table 6 only depend on the parameters of WY_t reported in Table 4 which almost all turned out to be significant. By contrast, the spillover effects also depend on the parameters of WC_t in Table 4 and WY_t in Table 5 of which respectively only 5 and only 4 are significant. The more parameters being insignificant, the greater the probability that the spillover effects derived from them will also be insignificant. It validates the proposition that a shock to one variable affects another one or the same variable in another unit is a strong one.

Table 7 reports the point-in-time direct and spillover effects at $h = 0$ of a one standard deviation shock in each country's QoQ credit growth. A notable finding is that we do find more significant spill-out effects across variables than in Table 6, i.e., of credit growth shocks in Austria, Belgium, Denmark, Italy, Portugal and Slovakia on GDP growth in other countries. Since the corresponding spill-in effects of these countries, except for Austria, appear to be insignificant, these results also illustrate that the distinction between spill-in and spill-out effects is a useful one.

Since the system was stable, i.e. the largest eigenvalue of the matrix of the system of equations smaller than unity, the long-term point-in-time responses to the shocks converge to zero and the cumulative effects to a constant. These cumulative effects are reported in Tables 8 and 9 and are calculated over a horizon of 5 years (20 quarters) which is long enough for the cumulative effects to converge since the half-life of the original impulse responses calculated (averaged over the whole matrix of dimensions 34 by 34) amounts to about 2.7 quarters.

Table 8: Cumulative effects of a one standard deviation GDP growth shock in every country

Country	Direct effect	GDP spill-in	Credit spill-in	GDP spill-out	Credit spill-out
AT	0.5980***	0.0764**	0.0062	0.1776***	0.0835*
BE	0.5378	0.0816	-0.0149	0.1723	0.1167*
CY	1.0043***	0.1543**	0.0106	0.0220**	0.0709
DE	0.7626*	0.0572	-0.0852***	0.4970**	0.1533*
EE	1.9366***	0.2658***	-0.0414	0.1182***	0.0146
ES	1.8730***	0.0868	-0.0338	0.1035	0.0021
FI	0.5403***	0.1331***	-0.004	0.0915***	-0.0194
FR	0.2607	-0.0001	0.0915	0.0917*	-0.1619*
GR	2.7929***	0.0652	0.081	0.1001***	0.0146
IE	3.7900**	0.1568	0.1286	0.1289***	-0.102
IT	0.4946**	0.0837	0.0016	0.185	0.012
LU	1.3923***	0.1426	-0.0016	0.0157	0.0478
LV	2.2784***	0.4862***	-0.0364	0.0614***	-0.0072
NL	0.3826	0.0524	0.0277	0.2648**	-0.0338
PT	0.8145***	0.027	0.0023	0.0562**	0.0345
SI	0.9160***	0.1707***	0.0229	0.0258**	0.0086
SK	2.2350***	0.2107	0.1264**	0.1385***	0.047

Note: *** significant at 1%, ** significant at 5%, * significant at 10%.

Table 9: Cumulative effects of a one standard deviation credit growth shock in every country

Country	Direct effect	Credit spill-in	GDP spill-in	Credit spill-out	GDP spill-out
AT	2.5729***	0.1453	0.1079	-0.08	0.197
BE	2.4812***	-0.0392	0.2449	-0.6071	0.072
CY	3.5511***	0.2296	0.2079*	0.0417	0.121
DE	2.1364***	-0.2311	0.0902	-0.4437	0.3782
EE	15.4275***	0.3107	-0.1127	0.1865	-0.1129
ES	1.7467***	0.0508	0.1122	-0.2952	0.1202
FI	5.8496***	0.1675	-0.3073**	-0.0338	0.0601
FR	2.5723***	-1717	0.1199	0.4067	0.1033
GR	3.4749***	0.0584	0.2494*	0.1589	0.1117
IE	3.8118***	0.0269	0.2707	0.3077	0.1926
IT	2.7590***	-0.6037	0.206	-0.4615	0.3729
LU	2.1619***	-0.0509	0.1270**	-0.3475	0.1339
LV	6.7674***	0.0759	0.1712	0.1046	-0.088
NL	1.8690***	0.0237	0.0847	-0.1846	0.2132
PT	2.1644***	0.1347	0.0348	0.0494	0.0817
SI	6.0440***	0.028	0.223	-0.0331	0.1028
SK	6.5325***	0.113	0.1223	-0.0467	-0.1073

Note: *** significant at 1%, ** significant at 5%, * significant at 10%.

7 Conclusions

The purpose of our paper was to provide a comprehensive review of two fundamentally related econometric model frameworks that can be used to model cross-sectional dependence and to measure spillovers. In addition to a detailed discussion of the analytical underpinnings and the appropriate estimation methods for these models, we show the convergence between these two frameworks, building on the specific extensions that have been adopted in some of the recent literature. We further show how both spatial systems and GVARs can be used to measure spillovers defined in a broad sense, while at the same time remaining cognizant of their limitations. We derive specific analytical formulas for spillovers, based on estimated coefficients (conditional on the choice of some connectivity (weight) matrix W) and a sequence of shocks, and show that these measurements are equivalent across spatial systems and GVAR representations. Finally, we develop a practical, step-by step guidance to applied researchers on model selection and measuring spillovers in these classes of models and illustrate the guidance with an empirical example involving GDP and credit growth for a sample of European countries and banking systems.

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