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A mixed frequency BVAR for the euro area labour market

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Abstract

We introduce a Bayesian Mixed-Frequency VAR model for the aggregate euro area labour market that features a structural identification via sign restrictions. The purpose of this paper is twofold: we aim at (i) providing reliable and timely forecasts of key labour market variables and (ii) enhancing the economic interpretation of the main movements in the labour market. We find satisfactory results in terms of forecasting, especially when looking at quarterly variables, such as employment growth and the job finding rate. Furthermore, we look into the shocks that drove the labour market and macroeconomic dynamics from 2002 to early 2020, with a first insight also on the COVID-19 recession. While domestic and foreign demand shocks were the main drivers during the Global Financial Crisis, aggregate supply conditions and labour supply factors reflecting the degree of lockdown-related restrictions have been important drivers of key labour market variables during the pandemic.

Keywords: Labour market, Mixed Frequency Data, Bayesian VAR
JEL codes: J6, C53, C32, C11

Non-technical summary

Understanding labour market dynamics is of high importance for interpreting the macroeconomic developments in an economy. In particular, the need to cover the euro area labour market is relevant because its structure is quite different from the U.S. - which is typically analysed in the academic literature - in terms of regulations, composition of the labour force, as well as the dynamics of the ins and outs of unemployment (also known as the job market flows).

With this paper, we introduce an empirical model for the aggregate euro area labour market with the twofold purpose of (i) providing reliable and timely forecasts of key labour market variables and (ii) enhancing the economic interpretation of the main movements in the labour market. We develop a structural vector autoregression (SVAR) model, which includes mixed frequency data, and we identify shocks by means of sign restrictions. As a methodological contribution, we augment the methodology of Schorfheide and Song (2015), by including a step that draws impact matrices that fulfil the imposed restrictions, with the methodology of Rubio-Ramirez et al. (2010). Our sign-restricted SVAR also provides a powerful policy tool for practitioners, given that it allows the interpretation of the drivers of nowcasts in terms of the structural shocks.

We find satisfactory results in terms of forecasting, especially when looking at quarterly variables, such as employment growth and the job finding rate. Additionally, we also find that our model produces suitable industrial production growth forecasts. The unemployment rate is more difficult to predict, given that the information contained in its own lags is often already sufficient to provide a good forecast. As a further contribution of our paper, we consider the importance of the labour market flows. In contrast to the results in this literature, we find no evidence that the inclusion of job market flows in the model produces more accurate forecasts.

Further, we look into the shocks that drove the labour market and macroeconomic dynamics from 2002 to early 2020. We find noteworthy insights. First, demand shocks were the main drivers during the past Global Financial Crisis. Second, shocks originating in the labour market play an important role in explaining the period of low inflation and low wage growth from 2013 onward. We further dig into the corona virus (COVID-19) period, in order to find an early assessment of the shocks explaining this crisis. We find that, in contrast to the Global Financial Crisis, aggregate supply and labour supply are important drivers of key labour market variables. Furthermore, in the early estimates for the nowcast of the first quarter of 2021, we find signs of a recovery in the labour market mainly driven by aggregate supply, aggregate demand, and wage-bargaining shocks.

1 Introduction

Understanding labour market dynamics is of high importance for interpreting the macroeconomic developments in an economy. While there is a substantial literature to understand the dynamics of labour markets in the U.S. from a structural point of view (see, among many, Gertler et al. (2008), Mumtaz and Zanetti (2012), Christiano et al. (2016a)) and also for forecasting purposes (e.g. Montgomery et al. (1998), Askitas and Zimmermann (2009), and D'Amuri and Marcucci (2017)), not many studies are available to understand euro area labour market developments. The need to cover the euro area labour market is relevant because its structure is quite different from the US, in terms of regulations, composition of the labour force, as well as the dynamics of the ins and outs of unemployment (also known as the job market flows).

With this paper, we aim at filling this gap. We introduce an empirical model for the aggregate euro area labour market with the twofold purpose of providing reliable and timely forecasts of key labour market variables and enhancing the economic interpretation of the main movements in the labour market. An extensive literature is analysing the role of macroeconomic shocks, since the nature of the shocks is key to understand the economic consequences and to tailor monetary or fiscal policies in response. Standard demand and supply shocks can be accompanied in the analysis by shocks originated directly in the labour market. These types of shocks have been mostly analysed within general equilibrium models, and they are described as exogenous shifts in the dis-utility of supplying labour or as movements in wage mark-ups (see for example, see Galí (2011), Gali et al. (2007), Galí et al. (2012), and Phaneuf et al. (2018)). In this paper, we focus on the labour market from an empirical perspective, seeking to describe the role of different macroeconomic and labour market shocks in a semi-structural model. We develop a structural vector autoregression (SVAR) model, which includes mixed frequency data, and we identify shocks by means of sign restrictions. We are, in fact, interested in obtaining a "real-time" evaluation of the dynamics in the euro area labour market. Therefore, we face the fact that some variables are available at monthly frequency (such as the unemployment rate and survey measures), while other labour market indicators (such as employment and labour market flows) are available only at quarterly frequency and with different publication delays. While the literature on mixed frequency techniques is vast by now, in this paper we follow the approach of Schorfheide and Song (2015) and use a mixed frequency Bayesian VAR (MF-BVAR). The choice of this method is driven by the purpose of our study: first, we want to have a set of variables depicting the labour market and macroeconomic dynamics; second, we want to be able to provide a reliable forecast of the main variables; and third, we want to have a structural interpretation in light of economic shocks, which are likely to explain the history of the time series, as well as the projected forecasts. A Bayesian VAR set up is, therefore, very convenient for us, given that it allows to focus on a relatively small and timely set of variables and to obtain economically interpretable results. As a methodological contribution, we merge two strands of literature, the mixed-frequency and the sign restrictions fields, and we augment the methodology of Schorfheide and Song (2015), by including a step that draws impact matrices that fulfil the imposed restrictions, with the methodology of Rubio-Ramirez et al. (2010). While there are few examples of structural mixed frequency VARs (see Foroni and Marcellino (2014) and Ghysels (2016)), to the best of our knowledge, however, no previous papers use sign restrictions in mixed frequency VARs and, therefore, we aim at closing a methodological gap. Moreover, our sign-restricted SVAR also provides a powerful policy tool for practitioners, given that it allows the interpretation of the drivers of nowcasts in terms of the structural shocks.

We find satisfactory results in terms of forecasting, especially when looking at quarterly variables, such as employment growth and the job finding rate. These findings are aligned with most of the results available in the mixed frequency literature, which suggest that the content available in higher frequency data helps improve the forecasting accuracy of lower frequency variables. Additionally, we also find that our model produces suitable industrial production growth forecasts. The unemployment rate is more difficult to predict, given that the information contained in its own lags is often already sufficient to provide a good forecast.

Further, we look into the shocks that drove the labour market and macroeconomic dynamics from 2002 to early 2020. We find noteworthy insights. First, demand shocks were the main drivers during the past Global Financial Crisis. Second, shocks originating in the labour market play an important role in explaining the period of low inflation and low wage growth from 2013 onward. We further dig into the corona virus (COVID-19) period, in order to find an early assessment of the shocks explaining this crisis. We find that, in contrast to the Global Financial Crisis, aggregate supply and labour supply are important drivers of key labour market variables. Furthermore, in the early estimates for the nowcast of the first quarter of 2021, we find signs of a recovery in the labour market mainly driven by aggregate supply, aggregate demand, and wage-bargaining shocks.

As a further contribution of our paper, we consider the importance of the labour market flows. First, these variables are used to refine the shock identification and to enrich the set of shocks originating in the labour market. Second, we assess whether they play a role in the forecasting of labour market variables, as some papers in the literature show (see Barnichon and Nekarda (2012) and Barnichon and Garda (2016)). In contrast to the results in this literature, we find no evidence that the inclusion of job market flows in the model produces more accurate forecasts. The nature of this result lies in the significant publication delay of job market flows. Unlike the US, the availability of job market flows in the euro area has a delay of at least two quarters.

The remainder of paper is organised as follows. Section 2 describes the model we set up for interpreting the euro area labour market developments. Section 3 describes the methodology for the estimation of the SVAR model. Section 4 provides the main economic results, and a snapshot on the analysis of the economic developments during the COVID-19 crisis. Section 5 summarises the forecasting performance results. Section 6 concludes.

2 A model for the euro area labour market

We start our analysis by describing the empirical semi-structural approach to identify the main macroeconomic drivers behind the variables in our mixed-frequency BVAR model. Our model can exploit the information available at different frequencies to produce reliable forecasts and to derive a narrative based on the key structural shocks to them. Furthermore, we include labour market flows in the empirical model, first to enrich the shock identification strategy, and second to check whether these variables play a role in the forecasting of euro area labour market variables, as shown by Barnichon and Nekarda (2012) and Barnichon and Garda (2016) for the US economy.

Our VAR model includes eight variables: industrial production growth rate (Δip_t), an index to identify the relative strength of the manufacturing relative to the service sector (which for simplicity we will call as MS index (ms_t))[1], inflation (Δp_t), wage growth (Δw_t measured by compensation per employee), unemployment rate (u_t), employment growth (Δe_t), and job flows, specifically job finding (f_t) and job separation rates (s_t). A detailed description of the variables and of their transformations can be found in Appendix A. The number of lags included in the estimation is equal to 12.[2]

With this set of variables, we aim at identifying six shocks. Specifically, two demand shocks, domestic and foreign; a technology shock; and three shocks originating in the labour market, a labour supply shock, a wage bargaining shock, and a mismatch shock. The identification is obtained by means of sign restrictions, following theoretical results from a class of DSGE models featuring labour market variables (see, for example, Smets and Wouters

[1]The MS index is defined as the difference between the growth rates of the Purchasing Managers' Index (PMI) in manufacturing and services, respectively: $ms_t = \Delta \ln PMI_t^M - \Delta \ln PMI_t^S$. We consider surveys instead of the gross-value-added in both sectors, given that the former represent expectations and therefore react more promptly to the impact of shocks than the latter.

[2]As a robustness check, we estimate the model using 4, 6, and 12 lags. Results remained qualitatively robust. Given the mixture of monthly and quarterly variables, we choose a lag order of 12, in order to integrate the dynamics of a year for each variable.

(2007); Christiano et al. (2016b); Mumtaz and Zanetti (2015); Foroni et al. (2018a)).

We consider the following identification scheme, where all the restrictions are imposed on impact:

Table 1: Identification scheme via sign restrictions - baseline model

	Demand		Supply	Labour Market		
	Domestic	Foreign	Technology	Labour Supply	Mismatch	Wage Bargaining
u_t	-	-	-	-	-	-
Δip_t	+	+	+	-	+	+
Δp_t	+	+	-	+	///	-
Δw_t	///	///	+	+	+	-
Δe_t	///	///	///	///	///	///
ms_t	-	+	///	///	///	///
f_t	+	+	+	///	+	///
s_t	-	-	-	///	+	///

The sign + indicates a positive response of the variable on impact for that specified shock. The sign - indicates a negative response. The sign /// indicates no restrictions.

A demand shock represents a shift in the demand curve, which pushes up output (in our case industrial production growth) and inflation, while it lowers the unemployment rate. These dynamics are consistent with the effects induced by monetary policy, government spending, marginal efficiency of investment, discount factor, and most financial shocks. The MS index helps us distinguishing between domestic and foreign demand shocks. The manufacturing sector in the euro area is mostly affected by the global demand for tradable goods while the service sector is mostly non-tradable. As a consequence, a foreign demand shock is expected to generate a higher on-impact effect on the manufacturing sector compared to the service sector, and vice versa for a domestic demand shock. Therefore, a domestic demand shock moves the MS index positively ($ms_t > 0$), while a global demand shock negatively ($ms_t < 0$).

Following Mumtaz and Zanetti (2015), we further use the information of labour market flows for the identification of neutral technology shocks. A neutral technology shock represents an increase in productivity, which reduces the marginal costs for firms and, therefore, pushes inflation down. The production expansion creates incentives for increasing hiring and translates into a rise in the job finding rate. Moreover, the higher productivity makes firms more willing to keep their employees, therefore decreasing the job separation rate. Consequently, the unemployment rate also decreases. However, a positive technology shock also creates a positive shift in the labour demand curve, which increases output and wage growth.

Both labour supply and wage bargaining shocks generate an inverse co-movement between output and real wages (see Foroni et al. (2018a)). A positive labour supply shock is an exogenous increase in labour supply, or a reduction in the disutility of working, which increases the number of participants in the labour market. In the first case, an exogenous increase in labour supply leads to an increase in the number of job seekers and makes it easier for firms to fill vacancies and to reduce hiring costs. Thereby, leading to a decrease in wages and prices and to an increase in output.[3] A negative wage bargaining shock captures a reduction of the negotiation power of workers and at the same time firms can benefit more from a larger share of the bargaining surplus. That leads to a reduction in wages and to an increase in firms' vacancy posting conditions and hiring. That ultimately leads to a decrease in the unemployment rate.

Matching efficiency shocks refer to exogenous changes in the job-worker matching process and represent a wedge in the standard matching function. In line with search and matching models (see Pissarides (2000)), an exogenous increase in matching efficiency shifts the job creation curve outward, increasing both the labour market tightness and wages. The increase in efficiency makes it easier for workers to find a job and therefore the job finding rate increases. The higher efficiency of the matching process reduces the costs of firing workers. Firms get an incentive to search for a better candidate matching the job and react by increasing the job separation rate as well. Additionally, this shock shifts the Beveridge curve inward, which translates into lower unemployment (see Consolo and Da Silva (2019), for further details).

3 Methodology

Mixed-frequency vector autoregressions (MF-VARs) are well established models in the toolbox for macroeconomic analysis. While most of the studies with MF-VARs focus on forecasting (see Kuzin et al. (2011), Schorfheide and Song (2015), and Brave et al. (2019), among others), there are a few studies focusing on structural analysis with MF-VARs (see Foroni and Marcellino (2016) and Ghysels (2016)). In this chapter, we focus on a model that is jointly able to perform a structural analysis of the euro area labour market and, at the same time, has a good forecasting performance for main labour market variables. For this purpose, we take the Schorfheide and Song (2015) model as starting point and we extend their methodology to a structural VAR, where we identify key macroeconomic shocks by means of sign restrictions.

In this section, we summarise the main features of the Schorfheide and Song (2015)

[3] In table 1, we report the signs of a negative labour supply shock in order to have all the shocks standardized to a negative impact on the unemployment rate.

model, and the main ingredients of the Bayesian estimation (for a detailed description of the model and the estimation procedure, see Appendix B).

Let us define N_m monthly variables denoted by $x_{m,t}$ and N_q quarterly variables represented by $y_{q,t}$, for $i = 1, \ldots, T$ months. The block $y_{q,t}$ is a set of variables with missing observations and have available data only every third month. Further, we denote the latent monthly counterpart of the quarterly variables as $x_{q,t}$. We assume the economy evolves as a monthly frequency VAR with p lags:

$$x_t = c + A_1 x_{t-1} + \cdots + A_p x_{t-p} + u_t, \quad \text{where} \quad u_t \overset{iid}{\sim} N(0, \Sigma), \tag{1}$$

where $x_t = [x'_{m,t}, x'_{q,t}]$ of dimension $N = N_m + N_q$, which can be rewritten in a more compact way as

$$x_t = c + A_+ z_{t-1} + u_t. \tag{2}$$

where $A_+ = [A_1 \ldots A_p]$ and $z_{t-1} = [x_{t-1} \ldots x_{t-p}]'$.

Since the VAR contains latent variables, we need to write the model in a state-space representation in order to obtain estimates of both the parameters and the states. To do this, let us denote T as the sample size which is defined as the last month for which we have at least one observation in the monthly block; T_{bq} is the time period at which we have a quarterly balanced set and finally T_b is the data point for which we have a balanced panel in the monthly block. Notice that, not all monthly variables might be available between T_b and T and in a similar fashion we can face ragged edges within the quarterly set. We could then have three types of missing observations: (i) mixed frequencies from $t = 1, \cdots, T_b$, (ii) ragged edges in the quarterly set and (iii) ragged edges in the monthly variables. Until time $t = T_b$ the state vector only corresponds to the quarterly block. However, due to the presence of missing monthly observations between T_b and T, a subset of the monthly block becomes a state for $t > T_b$.

The bridge between observable and latent observations (both quarterly and also monthly, whenever ragged edges are present) is given by the measurement equation (13).

$$y_t = M_t x_t \quad \text{for} \quad t = 1, ..., T. \tag{3}$$

The key component is the selection matrix M_t which selects the variables that are observed at time t and are part of the information set (more details on the definition of those selection matrices are provided in Appendix B). This also means that the selection matrix M_t bridges the observable quarterly variables with their latent monthly counterpart through an accumulator, which in this case corresponds to the average. The state space representation is given by equations (1) and (3).

Schorfheide and Song (2015) develop a two-block Gibbs sampler in order to obtain draws from the conditional posterior distributions of the parameters and states of the model. We adapt the type of priors to our dataset, in particular we consider a Normal-inverse Wishart prior, i.e.

$$\text{vec}(\Phi)|\Sigma \sim \mathcal{N}\left(\text{vec}(\Phi_0), \Sigma \otimes V_0\right) \quad \text{and} \quad \Sigma \sim iW\left(S, d\right) \tag{4}$$

where $\Phi = [c, A_+]$ and Σ is the reduced-form covariance matrix. The moments of the normal distribution of the parameters follow a Minnesota structure (Litterman (1986), Sims and Zha (1998), Del Negro and Schorfheide (2011)). The scale covariance matrix $S = diag(s_1, \cdots, s_N)$ correspond to the standard deviation of each variable in the training sample. We set the degrees of freedom d to be $N+2$ which is the minimum number such that the mean of an inverse Wishart distribution exists, see Kadiyala and Karlsson (1997). The hyperparameter which drives the overall degree of shrinkage is chosen as the one which optimises the marginal data density (MDD) over a grid (for more details on the grid and hyperparameters, see Appendix B.2).

3.1 Shock identification with sign restrictions

In contrast to Schorfheide and Song (2015), we do not limit our study to forecasting but we use the MF-VAR for structural analysis. The literature on structural analysis with mixed frequency data is still scarce and relies on simple identification schemes, such as the Cholesky decomposition (see Foroni and Marcellino (2016)). With our paper, we further contribute to the literature by allowing identification of structural shocks through sigh restrictions in a mixed frequency Bayesian VAR framework.

We link the reduced-form VAR from equation (9) with the structural macroeconomic shocks, ε_t, as follows:

$$u_t = B\varepsilon_t, \;\; \varepsilon_t \overset{iid}{\sim} N(0, I_N) \tag{5}$$

where B is an $N \times N$ matrix of impact effects, such that $\Sigma = B'B$. The identification of the structural shocks driving the system hinges on identifying the columns of B. To do so, the sign restrictions approach relies on restricting the elements of the columns of B such that the impact of the shocks onto the variables in the VAR are backed by economic theory. To obtain draws of the posterior distribution of matrix B, we follow the methodology of

Rubio-Ramirez et al. (2010). This approach draws a candidate matrix B^*, defined as

$$B^* = PQ,$$

where Q is a rotation matrix, $P = \text{chol}(\Sigma)$, and chol denotes the lower-triangular Cholesky decomposition. To generate draws of the rotation matrix, Rubio-Ramirez et al. (2010) propose an algorithm based on a QR decomposition, which translates to an independent Haar-uniform prior of the rotation matrix. In our framework, once we obtain a draw of the states and the parameters of the VAR, we draw B^* for up to 100,000 candidate matrices until we find a draw that fulfils the sign restrictions. In this paper, we consider a partially-identified model, therefore, we corroborate that the non-identified shocks have a different set of signs than the restricted elements of the shocks of interest.

4 Drivers of the euro area labour market

Through the lenses of our BVAR model with sign restrictions, we aim at explaining the dynamics in the euro area labour market variable. We report the historical decomposition of the main labour market variables in Figures 1 to 5 (results for industrial production and inflation are also provided in Figures 6 and 7).

First, macroeconomic shocks such as aggregate demand and supply shocks are equally important to the shocks internally originated in the labour market to account for the business cycle fluctuations of employment, wages and job flows. Following the Global Financial Crisis, the model points at demand shocks (domestic and foreign) as the main factors explaining the dynamics of most of the variables, influencing both real and nominal variables. However, looking at the period starting in 2014 (typically identified as a period of low inflation), the role of shocks originated in the labour market (labour supply, bargaining power, and mismatch) become more important to explain low wage growth (and, consequently, low inflation). In particular, the wage bargaining shock plays an important role as a driver of the wage inflation, since it reflects the overall effects stemming from reforms in labour market institutions implemented in the euro area following the 2010 Sovereign Debt Crisis. This is consistent with the interpretation of the wage bargaining shock in our model, which captures both a change in the pure bargaining power of workers and a change in the workers' outside option. The latter indeed decreased both for the effect of the crisis and also for the increased flexibility of labour market institutions across some euro area countries (see Koenig et al. (2016)).

Furthermore, we also find that the mismatch shock complements the wage bargaining shock in explaining key labour market developments during the euro area Sovereign Debt Crisis, in line with a standard search and matching model à la Pissarides (2000). According to Consolo and Da Silva (2019), the degree of labour market mismatch has increased following the euro area Sovereign Debt Crisis, which is also visible in the outward shift of the euro area Beveridge curve as of 2011. In our model, this is reflected in the historical decomposition of the job finding rate, which suggests a negative contribution from job matching efficiency starting from 2011. Consistent with the search and matching framework, higher mismatch in the labour market has led to lower wage growth over the same period. As in Elsby et al. (2013), the dynamics of the unemployment rate during the Sovereign Debt Crisis are also driven by the mismatch shock, which features an important cyclical component.

As we detail in the next section, one advantage of our model is that it can provide an economic interpretation that goes beyond the historical data. In particular, we can derive a decomposition of the contribution of different shocks also for the nowcasting and forecasting period. Figures 1 – 7 show also the latest observation that is often not observable and derived using the outcome of the mixed-frequency BVAR estimation. Therefore, our contribution of augmenting the model of Schorfheide and Song (2015) to an SVAR provides a key tool for policy work.

Impulse responses are reported in Figures 9-14 in Appendix C. In particular, we report the cumulative responses for the variables in growth rates. The shaded areas show the 68% point-wise credibility bands, whereas the blue line depicts the point-wise median. We choose the median as the central tendency of the impulse responses, since it is the optimal solution of the sum of the absolute loss of the impulse responses (see Baumeister and Hamilton (2015, 2018)). We find that aggregate supply and aggregate foreign demand shocks have the most persistent effects on real variables. Moreover, we find that prices increase and remain significant for about a year after foreign demand and labour supply shocks. Although we remain agnostic about the prior sign of wages when a labour supply shock hits the economy, we find that a positive labour supply shock has a negative impact on wages, given that compensation per employee decreases for about six months after the shock.[4] From the side of the job flows, we find that most of the shocks lead to a negative correlation between job finding and job separation rates, but the mismatch shock. The job finding rate rises after aggregate supply, aggregate demand (both foreign and domestic), and mismatch shocks. However, the job separation rate only mildly decreases after an aggregate supply shock.

[4] See Figure 9 which shows a negative labour supply shock

4.1 An early assessment of the COVID-19 period

The euro area labour market has been severely hit by the coronavirus (COVID-19) pandemic and associated containment measures. Employment and total hours worked declined at the sharpest rates on record. Unemployment increased more slowly and to a lesser extent, reflecting the high take-up rate of job retention schemes and transitions into inactivity. The labour market adjustment occurred primarily via a strong decline in average hours worked and labour force participation (see Anderton et al. (2021), for an overview).

In this subsection, we re-estimate the MF-BVAR model considering a sample containing the COVID-19 crisis. Specifically, we now take a sample spanning from January 1998 to March 2021.[5] In this case, the ragged-edges structure of the data is summarised by table 2. As previously stated, the estimation of our benchmark model provides us with a nowcast for the missing months and quarters.

Table 2: Ragged-edges by March 2021

	Monthly variables				Quarterly variables			
	u_t	Δip_t	Δp_t	ms_t	Δw_t	Δe_t	f_t	s_t
Sep/Q3 20	x	x	x	x	x	x	x	x
Nov 20	x	x	x	x	NaN	NaN	NaN	NaN
Dec/Q4 20	x	x	x	x	NaN	x	NaN	NaN
Jan 21	x	NaN	x	x	NaN	NaN	NaN	NaN
Feb 21	NaN	NaN	x	x	NaN	NaN	NaN	NaN
Mar 21	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN

The table reports the data available as of 8 March 2021. ”x”represents one data point that it is available, while ”NaN” represents one missing data point.

In Figure 8, we report the historical decomposition of the variables for the months affected by the COVID-19 pandemic. The MF-BVAR model interprets the decline in the employment rate observed during the crisis as being induced primarily by a combination of supply-side and demand shocks. In particular, the negative impact of the labour supply shock captures workers who lost their jobs during the pandemic crisis and did not immediately search for new jobs. This reflects the impact of lockdown and containment measures introduced by national governments during the pandemic, which forced many shops and firms to temporarily close or reduce their operations. Demand shocks reflect constraints on the demand for services

[5] Data were downloaded on March 8, 2021

as a consequence of the lockdown measures, as well as other factors, such as an increase in uncertainty during the pandemic, which restrained consumption. Unlike in the COVID-19 pandemic, the dominant shock during the trough of the financial crisis in 2009 was related to demand, which accounted for a larger share of the decline in employment than the two supply-side shocks. A similar picture, although with a stronger role for demand shocks and a smaller role for labour supply shock, is found in the historical decomposition of industrial production. The small response of euro area unemployment to the decline in activity (which stayed well below the euro area average, as shown by the negative numbers in the figure) can be attributed to the job retention schemes that aimed to protect employment and limit unemployment, as well as to a large number of workers transitioning into inactivity, rather than into unemployment. Thus, the shock composition of the unemployment rate is therefore quite different than the one of employment. Shocks originated in the labour market play a big role also in explaining nominal variables (in particular, wages and, consequently, inflation), through the role of negative wage bargaining and labour supply shocks. This reflects the fact that even the measures to contain employment losses imply a reduction in wages per person.

5 Forecasting performance

We use the model described in Section 2 to estimate forecasts of the main euro area economic variables, with a particular focus on the labour market variables. Specifically, we evaluate forecasts up to a year ahead for compensation per employee growth, employment growth rate, as well as the in- and out-flows of unemployment. Moreover, we include the results for industrial production growth and inflation.[6]

Our sample spans from January 1998 to February 2021 and we carry out a pseudo real-time forecasting exercise for the period from January 2006 to the end of the sample. The data set we consider is mixed frequency and with ragged edges, given that the series have different publication delays. For this exercise, we assume that we update our data set on the tenth day of the month, such that we have the latest released figure of the unemployment rate. Table 3 shows an example of the ragged-edge pattern within the months of the quarter, where "x" means that the information is available, "x*" denotes a flash estimate provided by Eurostat, and "NaN" indicates a missing observation. We split the flow of data into three blocks: beginning, middle and end. This is because, for the case of the quarterly variables,

[6]We omit results from the MS variable, since it is not a variable that it is typically monitored. Although we initially introduce this variable for identification purposes, we keep it in the model since this specification gives more accurate forecasts in contrast to a model that excludes the variable.

we divide the forecast evaluation into these groups, based on the information set available in each month of the quarter when the forecast is computed. The first group corresponds to the first month of the quarter (January, April, July, and October; "beginning"); the second to the months of February, May, July, and November ("middle"); and the third, to March, June, September, and December ("end"). Forecasting horizons change according to the group. As a clarifying example, if we compute the forecasts in January, the forecast of the first quarter corresponds to a two-months ahead horizon, while if we are in February, the forecast of the first quarter corresponds to a one-month ahead forecast, and when we are in March, the nowcast corresponds to the forecast of the first quarter.

Table 3: Data flow within quarters

		Monthly variables				Quarterly variables			
		u_t	Δip_t	Δp_t	ms_t	Δw_t	Δe_t	f_t	s_t
Beginning	Sep/Q3	x	x	x	x	x*	x*	x	x
	Oct	x	x	x	x	NaN	NaN	NaN	NaN
	Nov	x	NaN	x	x	NaN	NaN	NaN	NaN
	Dec/Q4	NaN	NaN	x*	x	NaN	NaN	NaN	NaN
	Jan	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
Middle	Sep/Q3	x	x	x	x	x	x	x	x
	Oct	x	x	x	x	NaN	NaN	NaN	NaN
	Nov	x	x	x	x	NaN	NaN	NaN	NaN
	Dec/Q4	x	NaN	x	x	x*	x*	NaN	NaN
	Jan	NaN	NaN	x*	x	NaN	NaN	NaN	NaN
	Feb	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
End	Nov	x	x	x	x	NaN	NaN	NaN	NaN
	Dec/Q4	x	x	x	x	x	x	x*	x*
	Jan	x	NaN	x	x	NaN	NaN	NaN	NaN
	Feb	NaN	NaN	x*	x	NaN	NaN	NaN	NaN
	Mar	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN

The table reports a snapshot of the ragged-edges and the mixed frequency nature of our data set. "x" indicates an available data point, "x*" denotes a flash estimate, and "NaN" represents a missing observation.

We construct the forecasts based on an expansive window approach. For each date in the evaluation sample, we estimate the model based on 20000 draws using the initial 10000 as burn-in sample. We compute each h-step ahead prediction through an iterative forecasting equation, based on the reduced-form parameters of the VAR from equation (1).

To evaluate the forecasts, we first compute the root mean squared forecast error (RMSFE), defined as:

$$RMSFE(i,h) = \sqrt{\frac{\sum_{t=\tau_0}^{\tau_1-h}\left(\hat{y}_{i,t+h|t} - y^o_{t+h}\right)^2}{\tau_1 - h - \tau_0 + 1}}, \tag{6}$$

where y^o_{t+h} denotes the realised value of variable i and $\hat{y}_{i,t+h|t}$ is the h-step ahead forecast of variable i, τ_0 and τ_1 are the extremes of the evaluation sample. The computation of equation (6) changes depending the frequency of the variable. For monthly variables, the time index t and the horizon h denote months, where the forecasts are computed up to twelve-months ahead and the indices τ_0 and τ_1 correspond to the months of December 2005 and August 2019, respectively. For quarterly variables, the indices and the horizons are in terms of quarters.

As a benchmark, we compute forecasts based on univariate AR models, where for each variable and each month or quarter of the evaluation sample, we choose the optimal number of lags through the Bayesian Information Criterion (BIC). We estimate the AR model based on normal - inverse Gamma priors, where as in the case of the MF-BVAR, we estimate 20000 draws and keep the last 10000 for inference. For both the MF-BVAR and the AR, we consider the point forecasts based on the mean posterior distribution of the forecasts.

Results are reported in Table 4, where we show the RMSFE of our model relative to the RMSFE of the AR process. Therefore, whenever the number reported is smaller than one, it indicates a superior performance of the model relative to the AR. The table reports the forecasting performance both of quarterly (panel (a)) and monthly variables (panel (b)).

What we can see is that we obtain significant gains when predicting the quarterly labour market variables (table 4, panel (a)). This is consistent with most of the evidence concerning mixed frequency models, in which the higher frequency information typically helps improve the forecasting performance of low frequency variables. The results are particularly satisfactory for the employment growth and the job flows. The results on the good performance in forecasting the flows is novel in the literature and it is useful since it provides further insights to the labour markets, going beyond the classical variables of employment and unemployment. Further, the results confirm that the information flow during the quarter matters, since the performance tends to improve as more information is acquired over the quarter. Nevertheless, the forecast performance tends to improve also when the quarterly information on the previous period becomes available.

In Table 4, panel (b), we report the results for the monthly variables. Although less commonly applied, it is also possible to include quarterly information to predict monthly variables, if the content contained in the lower frequency information carries important information (see Foroni et al. (2018b)). In this case, results are more mixed. While in the case of industrial production the results are particularly encouraging, this is not the case for the other two monthly variables we have in the model, inflation and the unemployment rate. This is most likely based on the fact that the unemployment rate and inflation are relatively timely and quite persistent. Therefore, the information contained in the last month available is dominating over the information included in other series that are released with a bigger delay.

All in all, the results show that our model is well suited to forecast the euro area labour markets developments.

To further investigate this good performance, we also consider a sub-sample, the period from January 2008 to December 2012, corresponding to the Global Financial Crisis for the euro area.[7] The results are fully consistent with those on the full sample, and reported in Table 5.

Further, we investigate a simpler MF-BVAR specification by excluding the job market flows from the variables. The results show that the use of flows does not help in improving the forecasting performance of labour market variables. This actually goes against the findings of Barnichon and Nekarda (2012) and Barnichon and Garda (2016) for the unemployment rate. However, in our set up, we aim at forecasting the monthly unemployment rate and not the quarterly one, as in the literature. Thus, in our case, the job flows have the disadvantage of being significantly delayed in their release over the monthly information included in the unemployment itself. Nevertheless, these are important in the analysis of the labour market and, in the euro area, the lag in the availability of this type of information needs to be considered. However, given that the performance is not significantly deteriorated by them (and in some cases even improved), we include the job market flows in our main model, since we use them for explaining the economic intuition behind the analysis. Results without flows are reported in Table 6 and can be directly compared with the results in Table 4, given that the AR benchmark is the same.

Finally, we check whether the performance of our MF-BVAR is superior in terms of density forecasting. To do so, for each forecasting horizon, we evaluate density forecasts by computing the log-predictive likelihood (LPL, see Geweke and Amisano (2010)), for both the MF-BVAR and the AR. This criterion is defined as follows:

$$LPL(i,h) = \log p(y_{i,t+h} = y^o_{i,t+h}|Y), \tag{7}$$

where $p(y_{i,t+h} = y^o_{i,t+h}|Y)$ is the predictive density evaluated at the realised value of variable

[7] We did not consider the COVID sub-sample, because it would span less than one year of our sample and would not allow us to properly check the forecasting performance, especially at longer horizons.

i at time $t + h$ and Y denotes the data available until T. To compare between the two models, we follow Geweke and Amisano (2010) and compute the average of the differential between log-predictive likelihoods (ALPL) as follows:

$$ALPL(i, h) = \frac{1}{\tau_1 - h - \tau_0 + 1} \sum_{t=\tau_0}^{\tau_1 - h} LPL_{MF-VAR}(i, h) - LPL_{AR}(i, h). \tag{8}$$

As clarified by Korobilis and Pettenuzzo (2019), positive values of the differential means that, on average, the MF-BVAR model produces more accurate density forecasts than the AR. Similar as in the computation of the RMSFE, the time index and the forecast horizon change depending on the frequency of the variable.

Tables 7 to 9 show the results for density forecasts mirroring the previous tables for the RMSFE. Numbers in bold show the instances when the MF-BVAR produces more accurate density forecasts than the AR, in terms of the ALPL. Looking at the tables, our results are more mixed than for the point-forecast evaluation. The findings in terms of the RMSFE for the job finding rate and the industrial production growth are confirmed. However, we do not find a better performance of density forecasts from our model, for employment growth and the longer horizons of the job separation rate. In contrast, we find that our model improves the density forecasting of wage growth for longer horizons and the nowcasts of the current quarter for the job separation rate. Furthermore, we find that these gains are not so strongly present when assessing the Global Financial Crisis period (table 8). Similar to the results for the point-forecasts, we find no additional gain stemming from the inclusion of job flows in the model (table 9).

6 Conclusions

In this paper, we proposed a Bayesian mixed-frequency VAR model, with the twofold purpose of interpreting the main movements in the labour market variables through the lenses of structural shocks, and at the same time, being able to produce reliable and economically interpretable forecasts. Moreover, we shifted our focus to the euro area labour market and we did it from an empirical perspective, seeking to describe the role of different macroeconomic and labour market shocks in a signed-identified SVAR. We found satisfactory results in terms of forecasting, especially when looking at quarterly variables, such as employment growth and the job finding rate. Further, we looked into the shocks that drove the labour market and macroeconomic dynamics from 2002 to early 2020, with a first insight on the COVID-19 recession. First, demand shocks were the main drivers during the past Global Financial Crisis. In contrast, aggregate supply and labour supply shocks are important drivers of key labour market variables during the quarters affected by the COVID-pandemic. Finally, we re-considered the importance of the labour market flows and we found that, while these variables are useful to refine the shock identification, we found no evidence that the inclusion of job market flows in the model produces more accurate forecasts. The nature of this results lies in the significant publication delay of the euro area job market flows.

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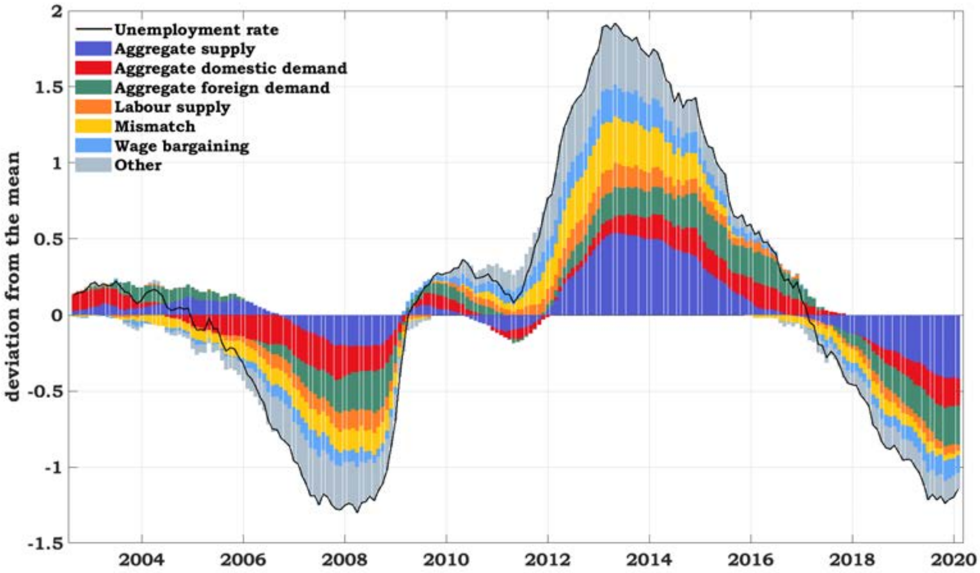
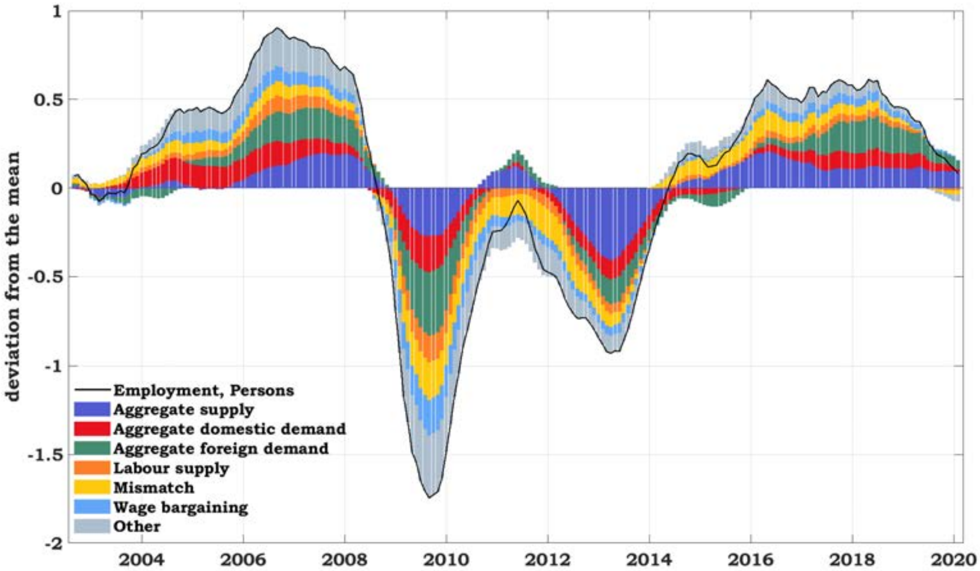
Figure 1: Historical decomposition of the unemployment rate



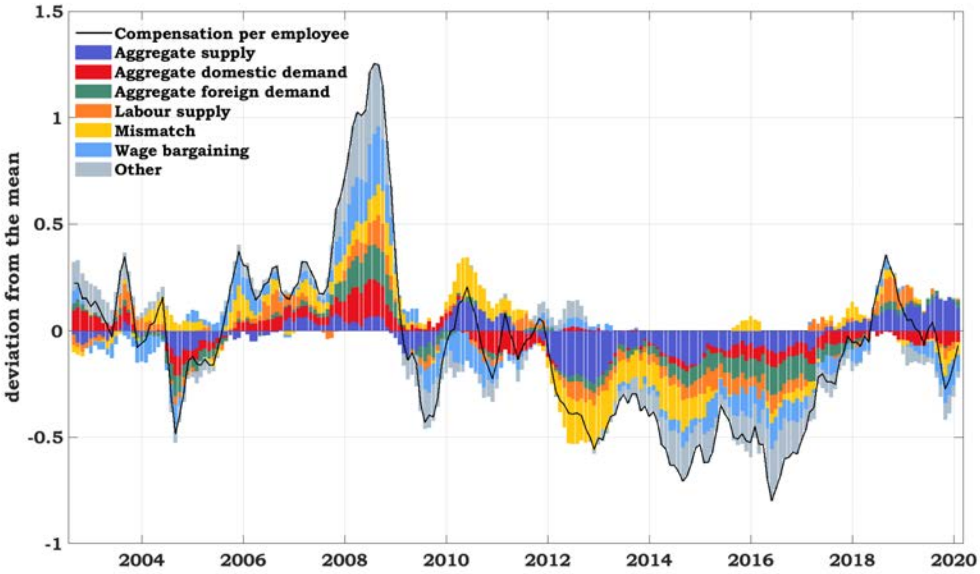
Note: The graph shows the median posterior distribution of the historical decomposition of the variables in deviation from their initial conditions.

Figure 2: Historical decomposition of the employment growth



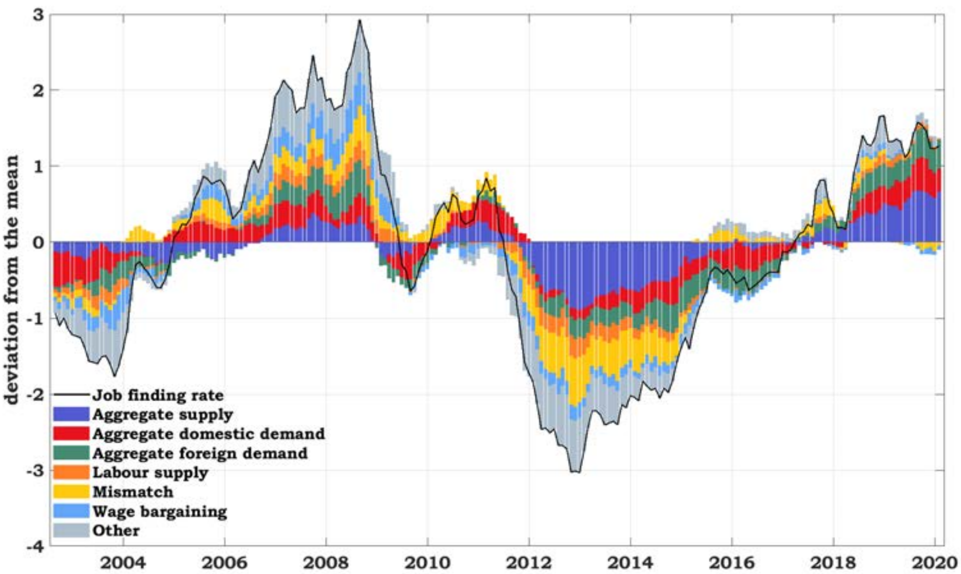
Note: See note in figure 1.

Figure 3: Historical decomposition of the wage growth, measured by compensation per employee



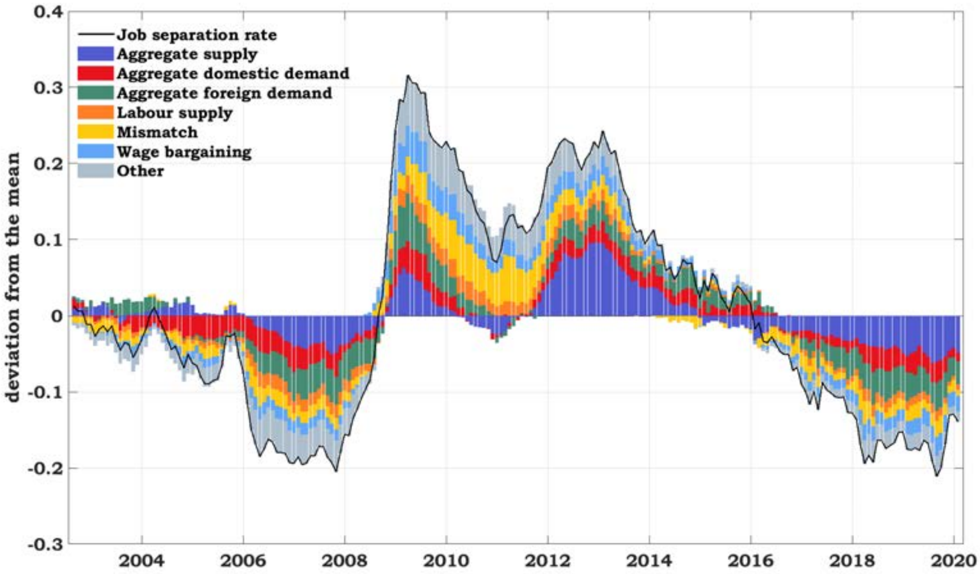
Note: See note in figure 1.

Figure 4: Historical decomposition of the job finding rate



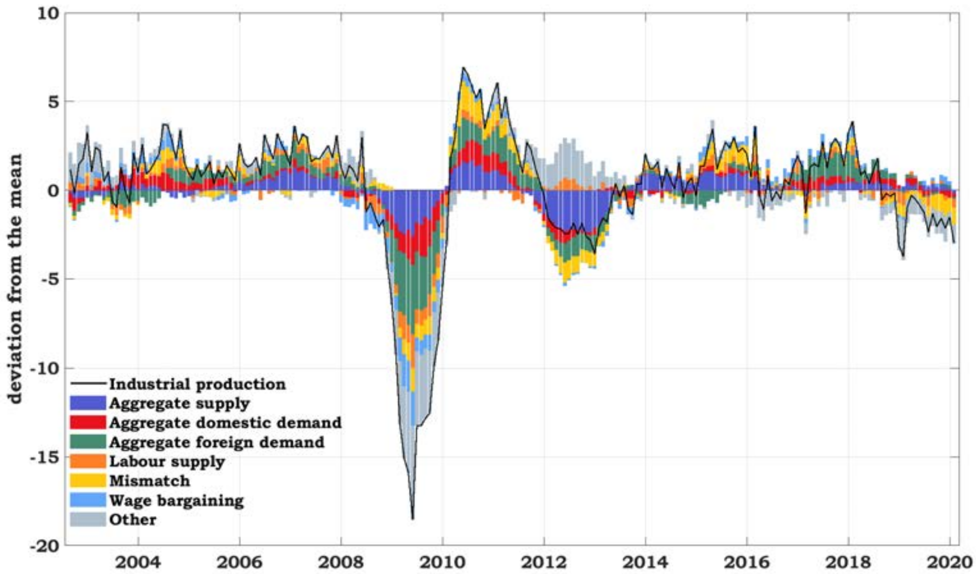
Note: See note in figure 1.

Figure 5: Historical decomposition of the job separation rate



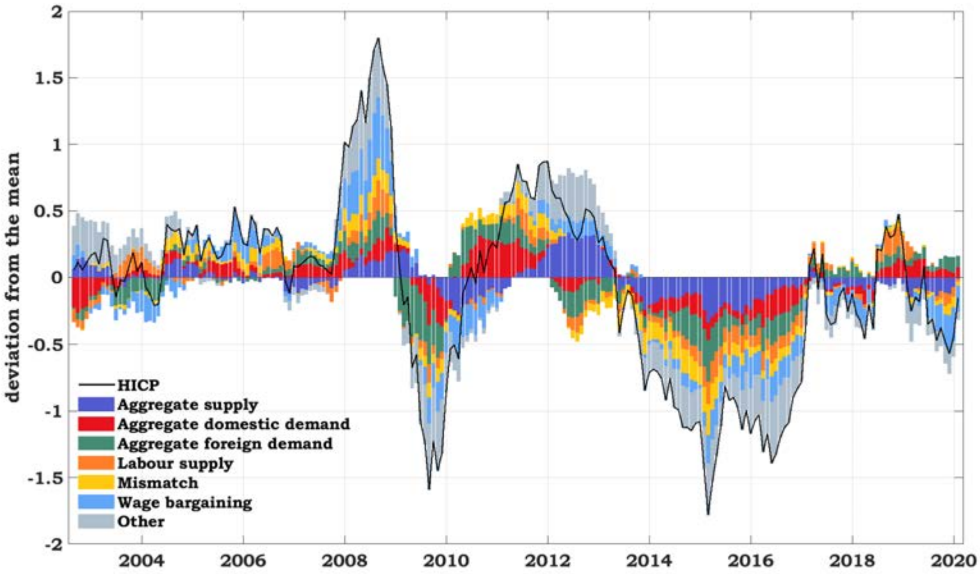
Note: See note in figure 1.

Figure 6: Historical decomposition of the industrial production growth



Note: See note in figure 1.

Figure 7: Historical decomposition of inflation



Note: See note in figure 1.

Figure 8: Historical decomposition of key labour market and macroeconomic variables during the COVID-19 sample

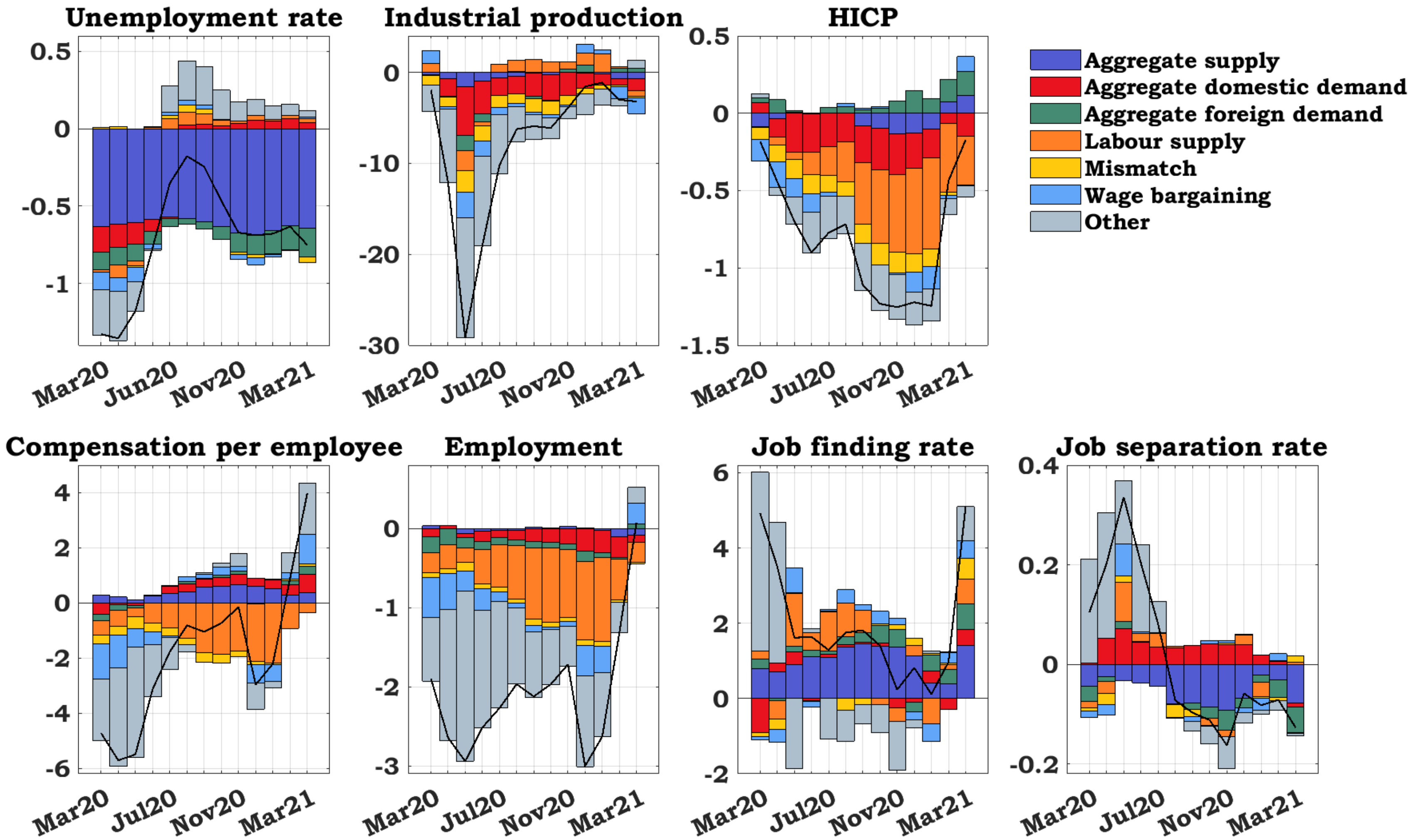


Table 4: Forecasting performance: RMSFE relative to an AR benchmark - Full sample

Panel (a): quarterly variables

	Employment growth			Wage growth			Job finding rate			Job separation rate		
	Beginning	Middle	End	Beginning	Middle	End	Beginning	Middle	End	Beginning	Middle	End
Q(-1)	1.63			1.10			1.27	**0.96**		1.61	3.07	
Q(0)	1.26	**0.94**	**0.90**	1.12	**0.96**	1.01	**0.92**	**0.92**	1.01	1.20	1.52	1.21
Q(+1)	1.06	**0.95**	**0.95**	1.07	1.09	1.12	**0.80**	**0.81**	**0.87**	**0.97**	1.03	1.00
Q(+2)	**0.95**	**0.92**	**0.96**	1.03	1.05	1.04	**0.77**	**0.75**	**0.85**	**0.89**	**0.87**	**0.89**
Q(+3)	**0.93**	**0.93**	**0.90**	1.02	1.03	1.08	**0.81**	**0.79**	**0.89**	**0.85**	**0.80**	**0.85**
Q(+4)	**0.90**	**0.92**	**0.91**	**0.99**	1.00	1.02	**0.84**	**0.83**	**0.93**	**0.82**	**0.76**	**0.82**

Panel (b): monthly variables

	Unemployment rate				Industrial production growth				Inflation			
	All	Beginning	Middle	End	All	Beginning	Middle	End	All	Beginning	Middle	End
M(-2)					**0.89**	**0.82**	**0.91**	**0.98**				
M(-1)	1.14	1.23	1.02	1.22	**0.96**	**0.87**	**0.98**	1.00				
M(0)	1.19	1.22	1.12	1.21	**0.96**	1.01	**0.92**	**0.96**	1.03	1.09	1.03	**0.97**
M(+1)	1.14	1.19	1.14	1.10	**0.95**	**0.90**	1.01	**0.98**	1.02	1.09	**0.97**	1.00
M(+6)	1.14	1.19	1.13	1.09	1.02	1.03	**0.99**	1.04	1.07	1.08	1.08	1.06

The table reports the RMSFE of the MF-BVAR relative to an AR benchmark with lag length selected according to the BIC criterion for each variable. Numbers less than one indicate a superior performance of the MF-BVAR. Panel (a) reports the forecasting performance for quarterly variables, panel (b) for monthly variables. The forecast horizon is expressed in months or quarters respectively, and with results organised depending on which month of the quarter the forecast is produced, since they are based on different information sets.

Table 5: Forecasting performance: RMSFE relative to an AR benchmark - Global Financial Crisis sample

panel (a): quarterly variables

	Employment growth			Wage growth			Job finding rate			Job separation rate		
	Beginning	Middle	End	Beginning	Middle	End	Beginning	Middle	End	Beginning	Middle	End
Q(-1)	1.70			1.08			1.45	1.01		1.88	3.78	
Q(0)	1.27	**0.92**	**0.89**	1.23	**0.94**	1.00	**0.99**	1.00	1.04	1.30	1.70	1.32
Q(+1)	1.03	**0.97**	**0.96**	1.19	1.16	1.22	**0.85**	**0.88**	**0.86**	1.01	1.09	1.05
Q(+2)	**0.94**	**0.92**	**0.97**	1.15	1.12	1.18	**0.81**	**0.82**	**0.86**	**0.90**	**0.89**	**0.90**
Q(+3)	**0.89**	**0.89**	**0.92**	**0.91**	1.23	1.29	**0.83**	**0.84**	**0.92**	**0.84**	**0.79**	**0.85**
Q(+4)	**0.82**	**0.87**	**0.88**	**0.94**	1.04	1.04	**0.85**	**0.88**	**0.97**	**0.80**	**0.73**	**0.81**

panel (b): monthly variables

	Unemployment rate				Industrial production growth				Inflation			
	All	Beginning	Middle	End	All	Beginning	Middle	End	All	Beginning	Middle	End
M(-2)					**0.79**	**0.62**	**0.84**	**0.97**				
M(-1)	1.33	1.66	1.12	1.34	**0.93**	**0.87**	1.00	**0.93**				
M(0)	1.32	1.36	1.20	1.40	**0.93**	1.00	**0.88**	**0.95**	1.06	1.30	1.09	**0.93**
M(+1)	1.23	1.24	1.22	1.22	**0.90**	**0.80**	**0.99**	**0.95**	1.04	1.18	**0.93**	1.00
M(+6)	1.15	1.19	1.16	1.11	**0.99**	**0.98**	**0.94**	1.06	1.14	1.16	1.20	1.10
M(+12)	1.08	1.11	1.10	1.03	**0.98**	**0.99**	**0.79**	1.15	1.18	1.23	1.22	1.12

Note: The table reports the RMSFE of the MF-BVAR model relative to an AR benchmark with lag length selected according to the BIC criterion for each variable. Numbers less than one indicate a superior performance of the SVAR. Panel (a) reports the forecasting performance for quarterly variables, panel (b) for monthly variables. The forecast horizon is expressed in months or quarters respectively, with results organised depending on which month of the quarter the forecast is produced, since they are based on different information sets. The sample considered in the forecast evaluation span January 2008 - December 2012.

Table 6: Forecasting performance: RMSFE relative to an AR benchmark - Benchmark model without flows

panel (a): quarterly variables

	Employment growth			Wage growth		
	Beginning	Middle	End	Beginning	Middle	End
Q(-1)	1.34			1.03		
Q(0)	1.09	**0.93**	**0.86**	1.08	**0.96**	**0.98**
Q(+1)	**0.96**	**0.90**	**0.80**	1.06	1.07	1.06
Q(+2)	**0.88**	**0.84**	**0.82**	1.03	1.06	1.03
Q(+3)	**0.90**	**0.91**	**0.81**	1.02	1.03	1.06
Q(+4)	**0.87**	**0.89**	**0.89**	**0.99**	1.00	1.02

panel (b): monthly variables

	Unemployment rate				Industrial production growth				Inflation			
	All	Beginning	Middle	End	All	Beginning	Middle	End	All	Beginning	Middle	End
M(-2)					**0.92**	**0.84**	**0.94**	1.01				
M(-1)	1.15	1.23	1.07	1.19	**0.97**	**0.90**	1.01	**0.98**				
M(0)	1.18	1.21	1.20	1.15	**0.96**	1.01	**0.91**	**0.96**	1.01	1.05	1.01	**0.99**
M(+1)	1.13	1.18	1.19	1.03	**0.94**	**0.89**	1.00	**0.98**	1.02	1.07	**0.99**	**0.98**
M(+6)	1.09	1.16	1.13	**0.98**	1.03	1.04	1.01	1.05	1.04	1.03	1.05	1.04
M(+12)	1.02	1.08	1.05	**0.94**	1.00	**0.99**	1.01	1.02	1.07	1.07	1.05	1.08

Note: The table reports the RMSFE of the MF-BVAR model without job market flows relative to an AR benchmark with lag length selected according to the BIC criterion for each variable. Numbers less than one indicate a superior performance of the MF-BVAR. Panel (a) reports the forecasting performance for quarterly variables, panel (b) for monthly variables. The forecast horizon is expressed in months or quarters respectively, with results organised depending on which month of the quarter the forecast is produced, since they are based on different information sets.

Table 7: Average log-predictive likelihood - AR vs benchmark model - Full sample

Panel (a): quarterly variables

	Employment growth			Wage growth			Jof finding rate			Job separation rate		
	Beginning	Middle	End	Beginning	Middle	End	Beginning	Middle	End	Beginning	Middle	End
Q(-1)	-0.08			-0.15			-0.01	**0.07**		**0.03**	-0.01	
Q(0)	-0.12	-0.34	**0.27**	-0.13	0.00	-0.09	**0.23**	**0.31**	**0.04**	**0.21**	**0.34**	**0.13**
Q(+1)	-0.04	-0.38	**0.09**	-0.22	-0.16	-0.16	**0.42**	**0.56**	**0.12**	-0.34	-0.66	-0.05
Q(+2)	**0.11**	-0.33	**0.06**	-0.20	-0.33	-0.25	**0.53**	**0.67**	**0.32**	-0.88	-1.20	-0.36
Q(+3)	-1.21	-4.84	**0.31**	**2.52**	-2.05	-0.23	**0.43**	**0.52**	**0.27**	-1.20	-1.72	-1.46
Q(+4)	-1.30	**0.66**	**0.79**	**6.32**	**1.55**	**3.56**	**0.39**	**0.43**	**0.03**	-1.00	-0.97	-2.91

Panel (b): monthly variables

	Unemployment rate				Industrial production growth				Inflation			
	All	Beginning	Middle	End	All	Beginning	Middle	End	All	Beginning	Middle	End
M(-2)					**0.09**	**0.17**	**0.06**	**0.03**				
M(-1)	-0.11	-0.20	**0.01**	-0.15	**0.04**	**0.06**	**0.02**	**0.03**				
M(0)	-1.04	-1.41	-0.67	-1.15	**0.10**	**0.04**	**0.07**	**0.21**	-0.03	-0.05	-0.10	**0.07**
M(+1)	-1.10	-1.28	-0.87	-1.34	**0.06**	**0.10**	**0.03**	**0.07**	-0.01	-0.06	**0.03**	-0.01
M(+6)	-1.97	-1.77	-4.18	-0.72	**0.00**	**0.00**	**0.04**	-0.02	-0.06	-0.07	-0.06	-0.07
M(+12)	-3.30	**0.10**	-4.56	-7.49	-0.16	-0.34	-0.14	-0.13	-0.09	-0.14	-0.07	-0.07

Note: The table reports the average of the differential between the log-predictive likelihood of the MF-BVAR and an AR model, where the number of lags is selected via the BIC criterion for each variable and each forecasting period. Panel (a) shows the results for quarterly variables and panel (b) for the monthly variables. The forecast horizon is expressed in months or quarters respectively, with results organised depending on which month of the quarter the forecast is produced, since they are based on different information sets.

Table 8: Forecasting performance: Average log-predictive likelihood - Benchmark model vs AR model - Global Financial Crisis sample

Panel (a): quarterly variables

	Employment growth			Wage growth			Jof finding rate			Job separation rate		
	Beginning	Middle	End	Beginning	Middle	End	Beginning	Middle	End	Beginning	Middle	End
Q(-1)	-0.24			0.00			-0.19	0.00		-0.24	-0.39	
Q(0)	-0.34	-1.40	**0.17**	-0.15	**0.14**	-0.02	-0.07	**0.31**	-0.22	-0.18	-0.04	-0.20
Q(+1)	-0.01	-1.93	-0.30	-0.41	-0.18	-0.28	**0.07**	**0.20**	-0.08	-2.21	-3.60	-0.82
Q(+2)	**0.58**	-1.42	-0.31	-0.61	-0.39	-0.51	**0.57**	**0.64**	**0.19**	-5.20	-6.81	-2.49
Q(+3)	**0.27**	-2.22	**0.97**	-0.61	-0.61	-0.87	**0.94**	**0.68**	-0.03	-8.92	-11.42	-8.90
Q(+4)	-1.77	**0.46**	-0.28	-1.33	-0.97	-0.96	**1.77**	**0.67**	-0.61	-13.51	-9.82	-23.72

Panel (b): monthly variables

	Unemployment rate				Industrial production growth				Inflation			
	All	Beginning	Middle	End	All	Beginning	Middle	End	All	Beginning	Middle	End
M(-2)					**0.23**	**0.51**	**0.14**	**0.07**				
M(-1)	-0.27	-0.51	-0.07	-0.30	**0.10**	**0.10**	**0.02**	**0.19**				
M(0)	-2.11	-2.50	-1.61	-2.97	**0.02**	-0.08	**0.06**	**0.07**	-0.10	-0.10	-0.34	**0.08**
M(+1)	-2.47	-2.49	-2.60	-3.55	**0.17**	**0.36**	**0.05**	**0.19**	-0.02	-0.20	**0.13**	0.00
M(+6)	-5.55	-6.11	-17.35	-2.29	**0.07**	**0.19**	**0.20**	-0.08	-0.15	-0.21	-0.30	-0.21
M(+12)	-10.50	**3.04**	-36.04	-61.52	**0.01**	**0.05**	**0.44**	-0.40	-0.19	-0.71	-0.51	-0.46

Note: The table reports the average of the differential between the log-predictive likelihood of the MF-BVAR and an AR model, where the number of lags is selected via the BIC criterion for each variable and each forecasting period. Panel (a) shows the results for quarterly variables and panel (b) for the monthly variables. The forecast horizon is expressed in months or quarters respectively, with results organised depending on which month of the quarter the forecast is produced, since they are based on different information sets. The sample considered in the forecast evaluation span January 2008 - December 2012.

Table 9: Forecasting performance: Average log-predictive likelihood - Benchmark model without flows

panel (a): quarterly variables

	Employment growth			Wage growth		
	Beginning	Middle	End	Beginning	Middle	End
Q(-1)	-0.02			-0.13		
Q(0)	-0.08	-0.31	**0.31**	-0.11	**0.00**	-0.09
Q(+1)	0.00	-0.31	**0.32**	-0.21	-0.14	-0.14
Q(+2)	**0.30**	-0.20	**0.33**	-0.20	-0.32	-0.23
Q(+3)	-0.71	-6.13	**0.23**	**2.14**	-1.66	-0.22
Q(+4)	-1.10	-0.16	-1.06	**6.21**	1.21	**3.15**

panel (b): monthly variables

	Unemployment rate				Industrial production growth				Inflation			
	All	Beginning	Middle	End	All	Beginning	Middle	End	All	Beginning	Middle	End
M(-2)					**0.07**	**0.17**	**0.04**	**0.00**				
M(-1)	-0.13	-0.21	-0.07	-0.14	**0.03**	**0.05**	**0.00**	**0.05**				
M(0)	-1.11	-1.35	-1.21	-0.91	**0.10**	**0.03**	**0.08**	**0.20**	-0.01	-0.01	-0.06	**0.04**
M(+1)	-1.25	-1.20	-1.78	-0.95	**0.07**	**0.11**	**0.03**	**0.07**	-0.01	-0.04	**0.01**	**0.01**
M(+6)	-2.57	-2.80	-4.42	-1.45	-0.01	-0.01	**0.03**	-0.04	-0.04	-0.04	-0.04	-0.05
M(+12)	-3.12	-2.36	-7.39	-2.10	**0.73**	**0.56**	-0.13	**0.17**	-0.06	-0.11	-0.04	-0.11

Note: The table reports the average of the differential between the log-predictive likelihood of the MF-BVAR without job market flows and an AR model, where the number of lags is selected via the BIC criterion for each variable and each forecasting period. Panel (a) shows the results for quarterly variables and panel (b) for the monthly variables. The forecast horizon is expressed in months or quarters respectively, with results organised depending on which month of the quarter the forecast is produced, since they are based on different information sets.

A Data description

Name	Description	Transformation	Frequency	Source
UR	Unemployment rate (as a % of labour force)	Levels	M	ECB-SDW
IP	Industrial production for the euro area	Δln(IP)	M	ECB-SDW
HICP	HICP - Overall index	Δln(HICP)	M	ECB-SDW
PMIM	Purchasing Managers' Index: Manufacturing	Δln(PMIm)	M	ECB-SDW
PMIS	Purchasing Managers' Index: Services	Δln(PMIs)	M	ECB-SDW
YIELD	Euro area 1-year Government Benchmark bond yield	Levels	M	ECB-SDW
SLOPE*	Slope of the yield curve, difference between Euro area ten-year and two-year yields, $r_{t,10Y} - r_{t,2Y}$	Levels	M	ECB-SDW
W	Compensation per employee	Δln(w)	Q	ECB-SDW
E	Employment (in thousands of persons)	Δln(e)	Q	ECB-SDW
F^+	Job finding rate	Levels	Q	ECB-SDW
S^+	Job separation rate	Levels	Q	ECB-SDW

*Own calculations; $^+$ computed by the Supply Side, Labour and Surveillance division at the ECB.

B The Schorfheide and Song (2015) model in detail

We consider a set of N variables divided in two different blocks, $x_t = [x'_{m,t}, x'_{q,t}]'$, for $t = 1, \dots, T$ months. The first block contains N_m monthly variables, which are originally available at this frequency; whereas the second block $x_{q,t}$ includes N_q variables that are the monthly counterpart of variables available at the quarterly frequency. Therefore, the variables in block $x_{q,t}$ are latent. Moreover, we define the vector $y_{q,t}$ as the observable quarterly variables, which have observations every third month and missing values otherwise. We assume that the vector x_t evolves as the following VAR :

$$x_t = c + A_1 x_{t-1} + \dots + A_p x_{t-p} + u_t, \quad u_t \overset{iid}{\sim} N(0, \Sigma), \tag{9}$$

where c is a vector of intercepts, A_i are matrices of reduced-form parameters, for $i = 1, \dots, p$, and u_t is a vector of reduced-form errors.

We re-write the VAR in equation (9) in terms of the two blocks as follows:

$$\begin{bmatrix} x_{m,t} \\ x_{q,t} \end{bmatrix} = \begin{bmatrix} c_m \\ c_q \end{bmatrix} + \begin{bmatrix} A_{mm} & A_{mq} \\ A_{qm} & A_{qq} \end{bmatrix} \begin{bmatrix} z_{m,t-1} \\ z_{q,t-1} \end{bmatrix} + \begin{bmatrix} u_{m,t} \\ u_{q,t} \end{bmatrix}, \tag{10}$$

where $z_{m,t-1} = [x'_{m,t-1}, x'_{m,t-2}, \cdots, x'_{m,t-p}]'$ is an $N_m p \times 1$ vector with the lags of the monthly variables and the $N_q p \times 1$ vector, $z_{q,t-1} = [x'_{q,t-1}, x'_{q,t-2}, \cdots, x'_{q,t-p}]'$, has the lags of the latent variables. The matrix of parameters has four sub-matrices: A_{mm}, an $N_m \times N_m p$ matrix of parameters governing the relationship among monthly variables; A_{mq}, an $N_m \times N_q p$ matrix containing the impact of the latent variables into the monthly block; A_{qm}, an $N_q \times N_m p$ matrix representing the impact of monthly variables into the equations of the latent variables; and A_{qq}, an $N_q \times N_q p$ matrix ruling the interactions among the latent variables. The blocks of constants, c_q and c_m, have the dimensions $N_q \times 1$ and $N_m \times 1$, respectively. The same dimensions apply for the blocks of error terms $u_{q,t}$ and $u_{m,t}$. In a similar way, we partition the covariance matrix into four blocks:

$$\Sigma = \begin{bmatrix} \Sigma_{mm} & \Sigma_{mq} \\ \Sigma_{qm} & \Sigma_{qq} \end{bmatrix}.$$

To obtain estimates of both the parameters and the latent variables, we write the state-space representation of the model. To do so, we denote T as the last month for which we have at least one observation in the monthly block; T_{bq} is the time period at which we have a quarterly balanced set; and T_b is the data point for which we have a balanced panel in the monthly block. Note that, not all monthly variables might be available between T_b and T. In a similar fashion, the quarterly set can also have ragged-edges. Summarising, we can have three types of missing observations: (i) mixed frequencies from $t = 1, \ldots, T_b$; (ii) ragged edges in the quarterly set; and (iii) ragged edges in the monthly variables. As an illustration of the different pattern of missing observations, in table 10, we consider an example of a data set with three monthly and three quarterly variables. The character "x" represents an available data point; whereas "NaN" denotes a missing observation.

Until time $t = T_b$, the state vector only corresponds to the latent quarterly block. However, due to missing observations in the monthly variables between T_b and T, a subset of the monthly variables becomes a state. For this reason, we split our problem into two state-space representations. The first state-space model approaches mixed frequencies, ragged-edges in the quarterly variables, and the fact that we are interested in obtaining an estimate of $x_{q,t}$, the monthly counterpart of the quarterly variables. The second state-space model deals with the publication delays in the monthly block.

Table 10: Type of missing observations

	$x_{m1,t}$	$x_{m2,t}$	$x_{m2,t}$	$y_{q1,t}$	$y_{q2,t}$	$y_{q3,t}$	
Jun 19	x	x	x	x	x	x	$t = T_{bq}$
Jul 19	x	x	x	NaN	NaN	NaN	
Aug 19	x	x	x	NaN	NaN	NaN	
Sep 19	x	x	x	x	x	NaN	
Oct 19	x	x	x	NaN	NaN	NaN	
Nov 19	x	x	x	NaN	NaN	NaN	
Dec 19	x	x	x	x	NaN	NaN	$t = T_b$
Jan 20	x	x	NaN	NaN	NaN	NaN	
Feb 20	x	NaN	NaN	NaN	NaN	NaN	$t = T$

Note: The table shows an example of the different types of missing observations. “x” represents one available data point and “NaN” represents a missing value.

B.0.1 A state-space for mixed-frequency variables

We define the state and measurement equations for $t = 1, \cdots, T_{bq}$. To do so, we partition the block-VAR of equation (10) into the observable part $x_{m,t}$ and the latent part $x_{q,t}$. We define the state vector, S_t, as follows:

$$S_t = \begin{bmatrix} x_{q,t} \\ x_{q,t-1} \\ \vdots \\ x_{q,t-p+1} \end{bmatrix}.$$

S_t stacks present and past values of the latent variables and it has a dimension of $N_s \times 1$, with $N_s = N_q(p+1)$. We assume S_t evolves as the following state equation:

$$S_t = \Gamma_c + \Gamma_z z_{m,t-1} + \Gamma_s S_{t-1} + \Gamma_u u_{q,t}, \quad (11)$$

where

$$\Gamma_c = \begin{bmatrix} c_q \\ 0_{(N_q \times p)\times 1} \end{bmatrix}, \quad \Gamma_z = \begin{bmatrix} A_{qm} \\ 0_{(N_q \times p)\times (N_m \times p)} \end{bmatrix}$$
$$\Gamma_s = \begin{bmatrix} A_{qq} & 0_{N_q} \\ I_{(N_q \times p)} & 0_{(N_q \times p)\times N_q} \end{bmatrix}, \quad \text{and} \quad \Gamma_u = \begin{bmatrix} I_{N_q} \\ 0_{(N_q \times p)\times N_q} \end{bmatrix}.$$

The dimensions of state matrices Γ_c, Γ_z, Γ_s, and Γ_u are $N_s \times 1$, $N_s \times N_m p$, $N_s \times N_s$, and $N_s \times N_q$, respectively.

To link the observable variables in $y_{q,t}$ with the latent states $x_{q,t}$, we introduce variable $\tilde{y}_{q,t}$, which is the monthly average of the latent states.[8] Thus,

$$\tilde{y}_{q,t} = \Lambda_s S_t, \tag{12}$$

where $\Lambda_s = \begin{bmatrix} \frac{1}{3} I_{N_q} & \frac{1}{3} I_{N_q} & \frac{1}{3} I_{N_q} & 0_{N_q \times (N_s - N_q)} \end{bmatrix}$, assuming the number of lags in the VAR is at least three. Therefore, the following relationship holds

$$y_{q,t} = M_{q,t} \tilde{y}_{q,t}, \tag{13}$$

where $M_{q,t}$ is an $N_q \times N_q$ selection matrix linking the average of the latent states with the observable quarterly variables every third month. This means that, in addition to the monthly block, we observe the quarterly average of the states every third month. Therefore, the measurement equation is:

$$y_t = \Lambda_c + \Lambda_z z_{m,t-1} + \Lambda_{ys} S_t + \Lambda_u u_{m,t}, \tag{14}$$

where $y_t = [x_t', y_{q,t}']'$ is a vector of observable variables, and

$$\Lambda_c = \begin{bmatrix} c_m \\ 0_{N_s \times 1} \end{bmatrix}, \quad \Lambda_z = \begin{bmatrix} A_{mm} \\ 0_{N_q \times N_m p} \end{bmatrix},$$

$$\Lambda_{ys} = \begin{bmatrix} 0_{N_m \times N_q} \quad A_{mq} \\ M_{q,t} \Lambda_s \end{bmatrix}, \quad \text{and} \quad \Lambda_u = \begin{bmatrix} I_{N_m} \\ 0_{N_q \times N_m} \end{bmatrix}.$$

To address potential ragged edges in the quarterly variables, which may occur for $t = T_{bq} + 1, \ldots, T_b$, we follow the approach in Durbin and Koopman (2012).

B.0.2 A state-space for monthly ragged-edges

The second state-space representation considers the case of ragged-edges in the monthly block. Although we treat ragged-edges in the quarterly set, this approach does not apply for the monthly variables. This is because, so far, the monthly variables are observable. However, when the monthly block contains ragged-edges, a subset of the monthly variables becomes

[8] We do not distinguish between stock and flow variables, given that assuming the average or the sum only affects the scale of the measurement equation. Note, however, that the assumption of the average (or sum) must be consistent when constructing the latent states and their potential transformations.

a state. We define the new state vector as $\tilde{z}_t = [x'_t, \ldots, x'_{t-p+1}]$, which has a dimension of $N_{\tilde{z}} \times 1$, with $N_{\tilde{z}} = Np + N$. This state vector is only defined for $t = T_b + 1, \cdots, T$, where only monthly variables are available. We assume the state vector $\tilde{z}_t$ evolves as the VAR:

$$\tilde{z}_t = \tilde{c} + \tilde{\Phi}\tilde{z}_{t-1} + \tilde{u}_t, \quad \tilde{u}_t \overset{iid}{\sim} N(0, \tilde{\Sigma}), \tag{15}$$

with covariance matrix

$$\Sigma_{\tilde{z}} = \begin{bmatrix} \Sigma & 0_{Np\times N} \\ 0_{N\times Np} & 0_{N\times N} \end{bmatrix}.$$

$\tilde{c}$, $\tilde{\Phi}$, and $\tilde{u}_t$ are defined as follows:

$$\tilde{c} = \begin{bmatrix} c \\ 0_{Np\times 1} \end{bmatrix}, \quad \tilde{\Phi} = \begin{bmatrix} A_+ & 0_{N\times N} \\ I_{Np} & 0_{Np\times N} \end{bmatrix}, \quad \tilde{u}_t = \begin{bmatrix} u_t \\ 0_{Np\times 1} \end{bmatrix},$$

and have dimensions $N_{\tilde{z}} \times 1$, $N_{\tilde{z}} \times N_{\tilde{z}}$, and $N_{\tilde{z}} \times 1$, respectively. The matrix $A_+ = [A_1, \ldots, A_p]$ stacks the matrices of reduced-form parameters from the VAR in equation (9).

The measurement equation exclusively depends on monthly variables that are observable from $t = T_b + 1, \ldots, T$. Thus, we define $N_{\tilde{m}}$ as the number of monthly variables available after T_b. The measurement equation is defined as:

$$\tilde{y}_t = M_{\tilde{z}}\tilde{z}_t, \tag{16}$$

where $M_{\tilde{z}}$ is an $N_{\tilde{m}} \times N_{\tilde{z}}$ selection matrix picking those variables with observations after T_b.

B.1 Bayesian estimation

Schorfheide and Song (2015) develop a two-block Gibbs sampler, in order to obtain draws from the conditional posterior distributions of the parameters and states of the model. Specifically, they sample the state vectors via the Carter-Kohn algorithm (see Carter and Kohn (1994)), which is the Bayesian counterpart of the Kalman filter and smoother. Given the states, the second block consists on sampling the parameters of the VAR in equation (9), i.e., $\Phi = [c, A_1, \ldots, A_p]$ and the reduced-form covariance matrix Σ.

To sample the parameters of the VAR, we consider a Normal - Inverse Wishart prior as follows:

$$\text{vec}(\Phi)|\Sigma \sim \mathcal{N}\left(\text{vec}(\underline{\Phi}), \Sigma \otimes \underline{V}\right) \quad \text{and} \quad \Sigma \sim iW\left(S, d\right), \tag{17}$$

where $\underline{\Phi} = [\underline{c}, \underline{A}_1, \ldots, \underline{A}_p]$ has the prior mean of the reduced-form parameters and $\underline{V}$ is a diagonal matrix with the prior variance of the reduced-form parameters. $\underline{V}_j$ denotes a block of $\underline{V}$ corresponding to equation j. Therefore, $\underline{v}_{j,k}$ denotes the k-th element in the diagonal of $\underline{V}_j$. Given our assumption of a Gaussian likelihood, the prior (17) is conjugate. Following the Minnesota prior version of Sims and Zha (1998) and Del Negro and Schorfheide (2011), the scale covariance matrix is $S = diag(s_1, \cdots, s_N)$, whose diagonal elements are the standard deviation of each variable during a training sample. We set the degrees of freedom to $d = N + 2$, the minimum number such that the mean of an inverse Wishart distribution exists, see Kadiyala and Karlsson (1997).

The moments of the normal prior distribution take the following form:

$$\underline{A}_{i,jk} = \begin{cases} \delta_i & \text{if} \quad j = k \quad \text{and} \quad i = 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\underline{v}_{j,k} = \begin{cases} \left(\frac{\lambda_1 s_j}{i}\right)^2 & \text{if} \quad j = k \\ \left(\frac{s_j \lambda_1}{s_k \; i^{\lambda_2}}\right)^2 & \text{if} \quad j \neq k \end{cases} \tag{18}$$

where $\underline{A}_{i,jk}$ is the (j,k)-th element of matrix $\underline{A}_i$, for $i = 1, \ldots, p$. We set a diffuse prior for the intercept term. When a variable is not stationary, e.g., the unemployment rate, we shrink the autoregressive parameters toward a random walk and therefore $\delta_i = 1$. In contrast, when a time series is stationary, e.g., the employment growth rate, we shrink it toward a white noise, thus $\delta_i = 0$. Therefore, the diagonal elements of parameter matrix A_1 equal one when the variables are I(1) and 0, otherwise. Moreover, the off-diagonal elements of A_1 and the elements of matrices A_i, for $i > 1$ are zero. For the prior covariance matrix $\underline{V}$, we assume it is diagonal, where we impose that the more distant lags and the coefficients from variable k have a smaller weight in the equation of $x_{j,t}$, for $j, k = 1, \cdots, N$. The overall shrinkage of the parameters is ruled by λ_1 and the hyperparameter λ_2 rules the shrinkage of higher-order lags. In general, the larger the hyperparameters, the stronger the shrinkage.

Following Schorfheide and Song (2015), we implement the prior through dummy observations (see Sims and Zha (1998)).[9] This approach augments the data with the following artificial observations:

$$x_d = \begin{bmatrix} \lambda_1 \text{diag}(s_1^2, \cdots, s_N^2) \\ 0_{N(p-1)\times N} \\ \text{diag}(s_1^2, \cdots, s_N^2) \end{bmatrix}, \quad z_d = \begin{bmatrix} J^{\lambda_2} \otimes \lambda_1 \text{diag}(s_1^2, \cdots, s_N^2) & 0_{Np\times 1} \\ 0_{N\times Np} & 0_{N\times 1} \end{bmatrix},$$

[9]For detailed explanation of how it is implemented, see also Del Negro and Schorfheide (2011).

where $J = \text{diag}(1, \cdots, p)$. We denote the augmented data by $X^* = [X', x_d']'$, $Z^* = [Z', z_d']'$, where X is the $T \times N$ matrix of data and Z is a $T \times (Np+1)$ matrix with the lagged values of the data and a constant. Therefore, we consider the following augmented VAR:

$$X^* = Z^* \Phi^* + U^*, \tag{19}$$

where $U^* = [U', u_d']'$, and the time dimension is $T^* = T + T_d$. The augmented model combines the prior and the likelihood of the data. Thus, the posterior distribution takes the following form:

$$\begin{aligned} \text{vec}(\Phi|X, \Sigma \sim \mathcal{N}\left(\hat{\Phi}, \Sigma \otimes \left(X^{*'} X^*\right)^{-1}\right) \\ \Sigma|X \sim iW(\hat{\Sigma}, T^* + 1 - Np), \end{aligned} \tag{20}$$

where $\hat{\Phi} = \left(Z^{*'} Z^*\right)^{-1} \left(Z^{*'} X^*\right)$ and $\hat{\Sigma} = (Y^* - X^* \Phi^*)' (Y^* - X^* \Phi^*)$.

The vector of hyperparameters, $\Lambda = [\lambda_1, \lambda_2]$, governs the prior, therefore an important issue to consider is its estimation. Due to the nature of latent states and observable time series in the VAR, the marginal data density (MDD) does not have a closed-form solution. Thus, the methodologies that obtain the optimal hyperparameters through the optimisation of the MDD are not feasible, e.g., Giannone et al. (2015), Chan et al. (2019). Schorfheide and Song (2015) approximate the MDD through the harmonic mean estimator of Geweke (1999). Once they obtain the approximation, the optimal parameters can be estimated over a grid. In this paper, we follow their approach for obtaining optimal values for the overall degree of shrinkage λ_1. We set the decay parameter $\lambda_2 = 2$, a standard value selected in the literature. We show the considered grid and the selected hyperparameter for the model in appendix B.2.

B.2 Results from MDD-hyperparameters optimisation

Following Schorfheide and Song (2015), we select the hyperparameters over a grid. We consider the following grids:

$$\Lambda_1 = [0.01\ 0.72\ 1.44\ 2.15\ 2.86\ 3.58\ 4.29\ 5.00\ 5.72\ 6.43\ 7.14\ 7.86\ 8.57\ 9.28\ 10]$$

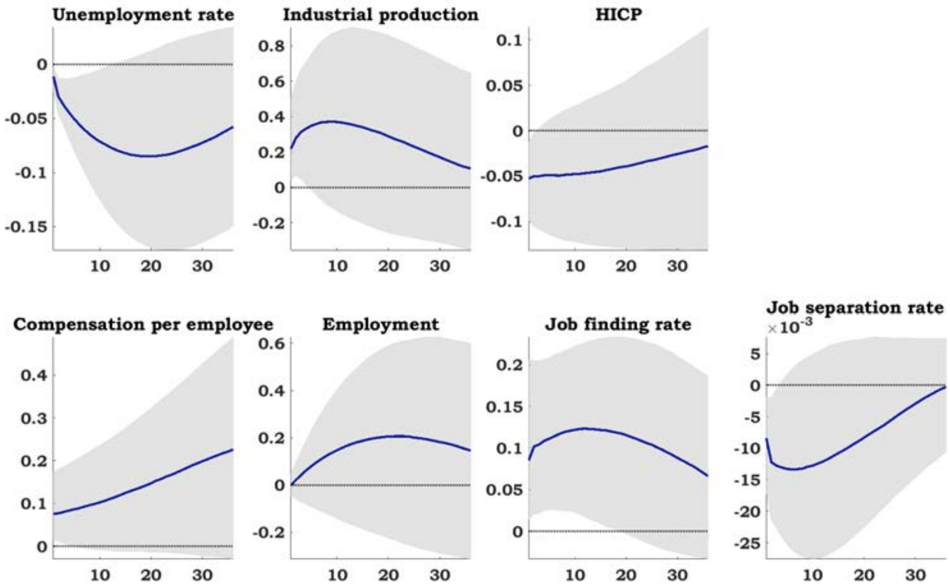
Table 11 shows the constellation of hyperparameters that yield the maximum MDD for each of the models considered.

Table 11: Optimal hyperparameters

Hyperparameters	Benchmark	No flows
λ_1	5.00	5.00

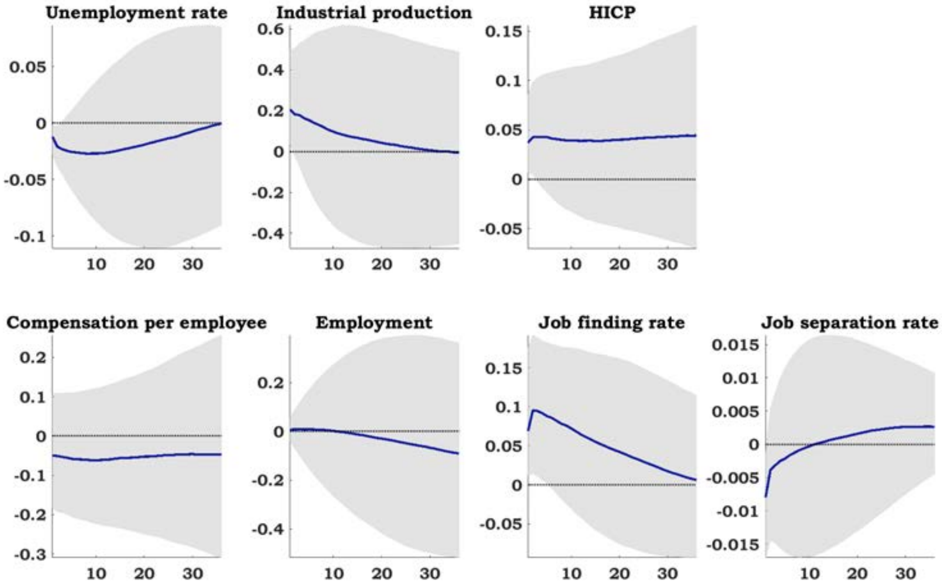
C Impulse responses of main macroeconomic and labour market shocks

Figure 9: Responses to an aggregate supply shock



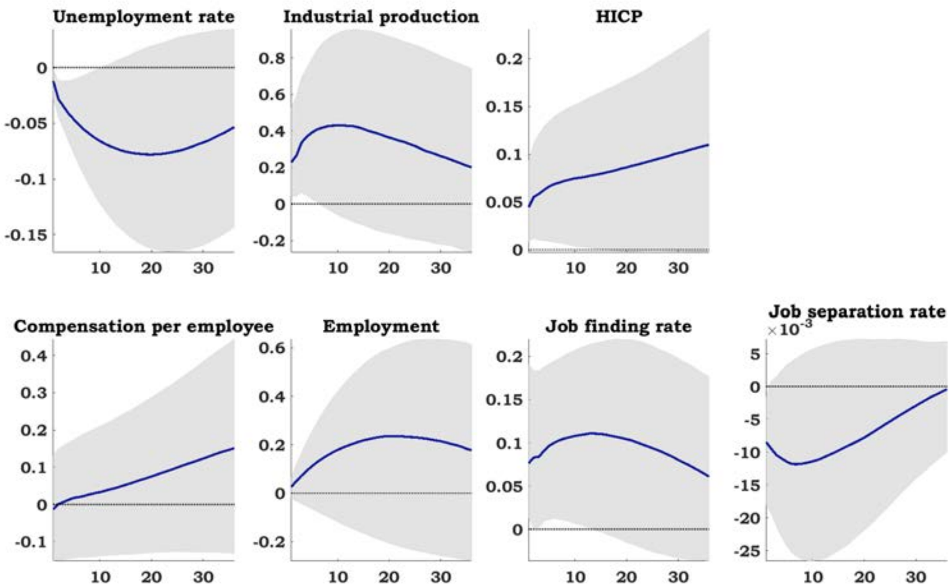
Note: Shaded areas show the 68% point-wise credibility bands, whereas the blue line shows the point-wise median.

Figure 10: Responses to an aggregate domestic demand shock



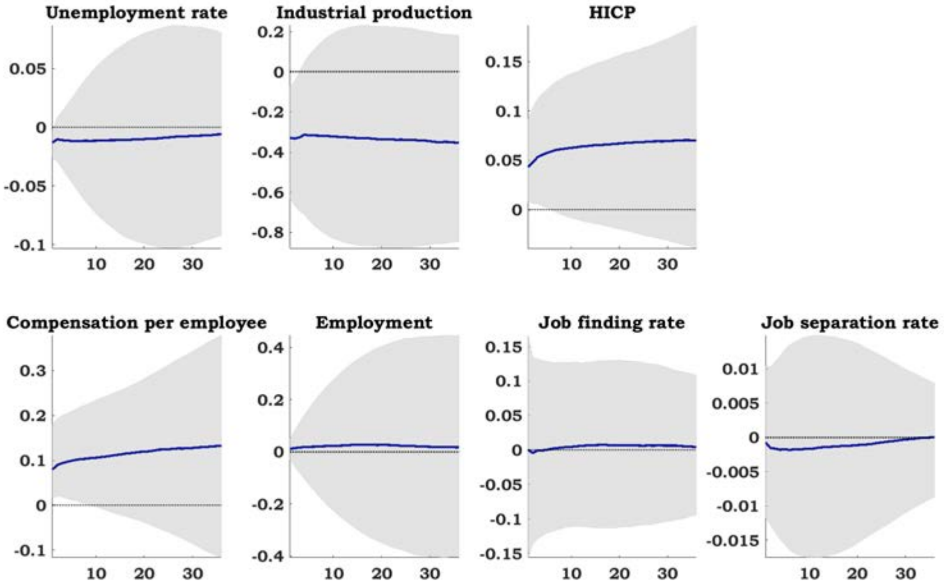
Note: See figure 9

Figure 11: Responses to an aggregate foreign demand shock



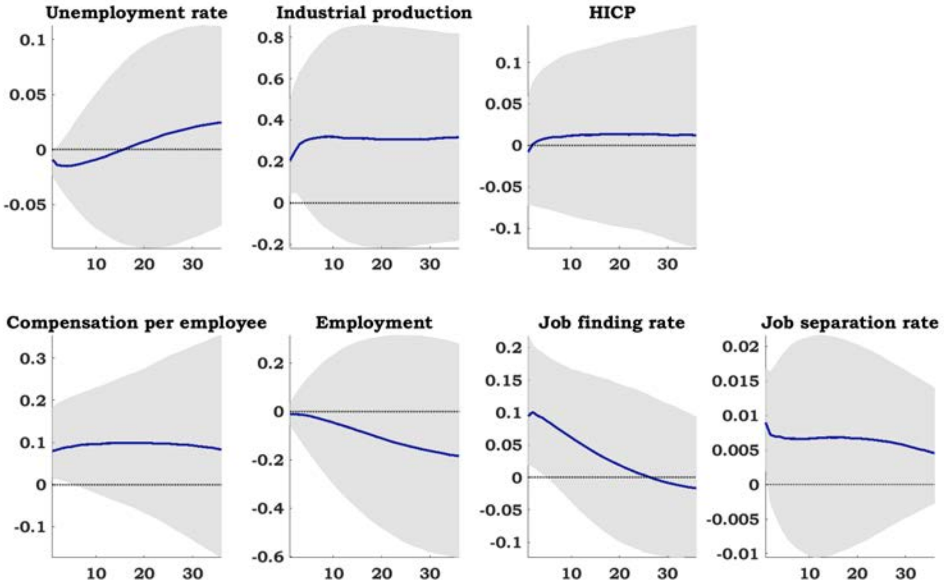
Note: See figure 9

Figure 12: Responses to a labour supply shock



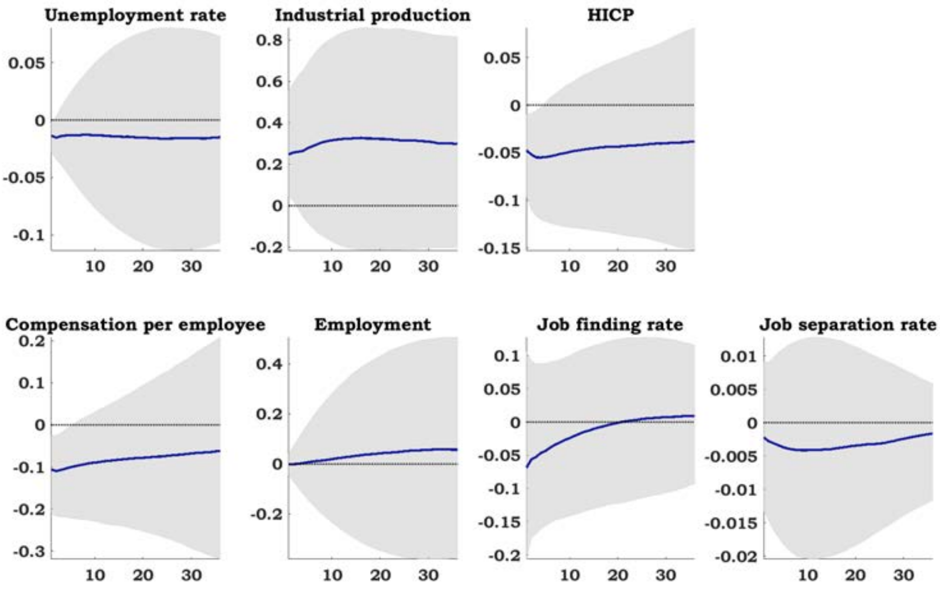
Note: See figure 9

Figure 13: Responses to a mismatch shock



Note: See figure 9

Figure 14: Responses to a wage bargaining shock



Note: See figure 9

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