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The financial accelerator mechanism:
does frequency matter?

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Abstract

We use mixed-frequency (quarterly-monthly) data to estimate a dynamic stochastic general equilibrium model embedded with the financial accelerator mechanism à la Bernanke et al. (1999). We find that the financial accelerator can work very differently at monthly frequency compared to the quarterly frequency, i.e. we document its inversion. That is because aggregating monthly data into quarterly leads to large biases in the estimated quarterly parameters and, as a consequence, to a deep change in the transmission of shocks.

Keywords: DSGE models, financial accelerator, Mixed-frequency data.

JEL codes: C52, E32, E52

1 Non-technical summary

General equilibrium models with financial frictions have gained a lot of attention during and after the Great Recession. Different distortions (financial frictions) in financial markets are shown to amplify (or de-amplify) the propagation of shocks through an economy. Common variables used to capture financial frictions are a measure of the spread between a risky return and the risk-free interest rate, or alternatively, measures of net worth, often proxied by stock market indexes. These variables typically react fast to economic conditions, and they are available at a higher frequency than the quarterly frequency common to most of the variables conventionally used to estimate DSGE models, such as GDP or investments.

Quarterly aggregation of high frequency data can wash out intra-quarterly effects and hide relevant information about the quarter. Foroni and Marcellino (2014) show that temporal aggregation can prevent the identification of structural DSGE models and lead to substantial bias in the identification of structural shocks and their transmission. They also show that the use of mixed frequency data can alleviate, and sometime eliminate, these problems.

Our main contribution is to estimate a state of the art DSGE model with financial frictions, as in Del Negro et al. (2015), using US mixed-frequency data for the period 1964m1-2008m9, and compare the quarterly with the monthly estimates. The main focus of our paper is to evaluate whether or not aggregating monthly data into quarterly leads to significant and relevant biases in the estimated quarterly parameters compared to the monthly ones. And if so, what those biases imply.

Our main result is that the aggregation process introduces large biases. We find that the transmission mechanism of shocks is greatly altered within the quarterly estimation, up to the point that the accelerator mechanism can be inverted. For example, a monetary policy shock leads to an acceleration in investments in the context of the quarterly frequency, in line with the literature, and to a deceleration in the mixed-frequency one. Vice-versa, investments decelerate in the quarterly realm following an investment-specific shock, as largely documented by previous papers, while they accelerate when our mixed-frequency specification is exploited.

The inversion of the accelerator critically depends on one key parameter, the one governing the investment adjustment cost function. We find a very low degree of investment adjustment costs in the estimated mixed-frequency model without financial frictions as compared to the same model estimated with quarterly data. That makes investment mov-

ing much more or much less than the quarterly model, depending on the shock. Moreover, the size of the inversion depends on the Calvo parameters (or the slope of the price and wages Phillips curves). The mixed-frequency financial frictions model estimates very steep Phillips curves which makes some shocks, like the monetary policy shock, propagate very little.

We also extend our analysis to other aspects of the financial frictions setup. In particular, we analyze how the relative importance of shocks changes when monthly data are used in the estimation. Our results suggest that in the quarterly model financial shocks play a relevant role to explain real variables fluctuations, in line with Christiano et al. (2014) and Jerman and Quadrini (2012), but they lose all their importance when monthly data are used, mainly in favor of price and wage mark-up shocks.

Finally, we corroborate our results with a Monte Carlo exercise. We show that also in a controlled environment the biases can be large once data are aggregated at quarterly frequency.

2 Introduction

General equilibrium models with financial frictions have gained a lot of attention during and after the Great Recession. Different financial frictions setups have been used to analyze many aspects of the crisis. Among others, the financial accelerator framework à la Bernanke et al. (1999) turned out to be a popular choice for academics and central bankers. That framework has been employed for instance to investigate the relevance of financial shocks, Christiano et al. (2014), the missing disinflation puzzle, Del Negro et al. (2015), the after crisis lost recovery, Cai et al. (2019), the crisis forecastability, Del Negro and Schorfheide (2013), and its implications for the conduct of optimal monetary policy, Furlanetto et al. (2021). The Federal Reserve Bank of New York regularly uses a model with a core based on that framework to forecasts the US economy and for policy analysis, see Del Negro et al. (2013).

A central feature of the accelerator framework is indeed the accelerator mechanism. It refers to the mechanism by which distortions (frictions) in financial markets amplify (or de-amplify) the propagation of shocks through an economy. Earlier pre-Great Recession papers, like Meier and Müller (2006), De Graeve (2008), Christensen and Dib (2008), and Gelain (2010), focused on the evaluation of its empirical relevance in the context of estimated New Keynesian Dynamic Stochastic General Equilibrium (DSGE) models. They found that data support the model with financial frictions, but the accelerator is small and not very statistically significant (if at all). It was common though for those models to not incorporate financial shocks and to be estimated without financial variables as observables.

On the contrary, the most recent post-Great Recession literature already mentioned ignores quantitative considerations about the accelerator, but it accounts for financial shocks and observables. A widespread variable used in the estimation is a measure of the spread between the relevant concept of the risky return and the risk free interest rate. But also measures of net worth often proxied by stock market indexes. Those variables typically react fast to economic conditions, and they are available at a higher frequency than the quarterly frequency common to most of the variables conventionally used to estimate DSGE models, such as GDP or investments.

Quarterly aggregation of high frequency data can wash out intra-quarterly effects and hide relevant information about the quarter. More generally, Foroni and Marcellino (2014) show that temporal aggregation can prevent the identification of structural DSGE models and lead to substantial bias in the identification of structural shocks and their transmission. They also show that the use of mixed frequency data can alleviate, and sometime eliminate,

these problems. We adopt a mixed-frequency approach. Our main contribution is to estimate a state of the art DSGE model with financial frictions, as in Del Negro et al. (2015), using US mixed-frequency data for the period 1964q1-2008q3 (1964m1-2008m9), and compare the quarterly with the monthly estimates. We stop the estimation in 2008Q3 to avoid modeling complications related to unconventional economic policies.

The main focus of our paper is to evaluate whether or not aggregating monthly data into quarterly leads to significant and relevant biases in the estimated quarterly parameters compared to the monthly ones. And if so, what those biases imply. Our main result is that the aggregation process introduces large biases. We find that the transmission mechanism of shocks is greatly altered within the quarterly estimation, up to the point that the accelerator mechanism can be inverted. For example, a monetary policy shock leads to an acceleration in investments in the context of the quarterly frequency, in line with the literature, and to a deceleration in the mixed-frequency one. Vice-versa, investments decelerate in the quarterly realm following an investment-specific shock, as largely documented by previous papers, while they accelerate when our mixed-frequency specification is exploited.¹

The inversion of the accelerator critically depends on one key parameter, i.e. the parameter governing the investment adjustment costs. This is not surprising because De Graeve (2008) already showed that in a quarterly model the assumption about investment adjustment costs is crucial to explain why some shocks lead to a deceleration rather than to an acceleration. We find a very low degree of investment adjustment costs in the estimated mixed-frequency model without financial frictions as compared to the same model estimated with quarterly data. That makes investment moving much more or much less than the quarterly model, depending on the shock. Moreover, the size of the inversion depends on the Calvo parameters (or the slope of the price and wages Phillips curves). If the quarterly model estimates quite flat Phillips curves, the mixed-frequency financial frictions model estimates very steep ones which makes some shocks, like the monetary policy shock, propagate very little.²

¹The inversion happens for all the shocks in our model, but for the technology shock, which always generates a deceleration.

²It is well documented, e.g. in Schorfheide (2008), that DSGE model-based estimates of the slope of the Phillips curve can vary widely across studies. The variation can be caused by a combination of *model specification*, dataset, and choice of the prior distribution. Del Negro and Schorfheide (2008) document that reasonable changes in the prior distribution can generate estimates of the Calvo parameter ranging between 0.54 and 0.84. In addition, Herbst and Schorfheide (2014) document that under a diffuse prior the Smets and Wouters model can generate a posterior distribution with two modes, one at 0.59 and another one at 0.70. Del Negro et al. (2015) document that estimating the Smets and Wouters model using spreads as an additional observable raises the estimate of the Calvo parameter from 0.65 to 0.81.

We also extend our analysis to other aspects of the financial frictions setup. In particular, we analyze how the relative importance of shocks changes when monthly data are used in the estimation. Our results suggest that in the quarterly model financial shocks play a relevant role to explain real variables fluctuations, in line with Christiano et al. (2014) and Jerman and Quadrini (2012), but they lose all their importance when monthly data are used, mainly in favor of price and wage mark-up shocks. This critically depends on the very flat estimated Phillips curves, as shown in Del Negro et al. (2015).

Finally, we corroborate our results with a Monte Carlo exercise. We show that also in a controlled environment the biases can be large once data are aggregated at quarterly frequency.

The paper is structured as follows. We review the related literature in the following subsection. In Section 3 we describe the model and we report all the equilibrium conditions. Section 4 provides the details of the estimation of the model. Section 5 describes our results in detail. Section 6 concludes.

2.1 Related literature

Our paper surely relates to the very large mixed-frequency literature in the context of time series models. It dates back to the 1980s. It is out of the scope of this paper to summarize it. We refer to Foroni et al. (2013) for a survey.

It is more important to highlight that the number of contributions about the estimation of DSGE models in the context of mixed-frequency data is still very limited. In particular only Giannone et al. (2016) use the same fully-blown-fully-fledged DSGE we use. They do not estimate the parameters of the model with monthly data, but they combine the quarterly model with a now-casting model designed to read timely monthly information as it becomes available.

There is a group of papers interested in methodological issues and/or performing mixed-frequency estimation with the aim of evaluating the impact on estimated parameters as compared with a different (often quarterly) frequency. Christensen et al. (2016) estimate a small general equilibrium (AK-Vasicek) model with both macro and financial data. Their focus is mostly methodological and related to the use of the martingale estimating functions.

Foroni and Marcellino (2014) are actually the first to highlight the problems of temporal aggregation and the use of mixed-frequency data in DSGE models. They show, first analytically with a three-equation New Keynesian model and then with the Smets and Wouters (2007) model, that the mismatch between the time scale of DSGE models and the data used

in their estimation translates into identification problems, estimation bias, and distortions in policy analysis. The authors find significant differences in the responses to structural shocks depending on whether the model is set at a quarterly frequency or at the monthly frequency. On top of having a more realistic model thanks to financial frictions, we improve upon them on the methodological side. In fact, we estimate our mixed-frequency model in a fully Bayesian context, as opposed to their Maximum Likelihood approach, consistently with what is now common practice in the DSGE literature.

Rondeau (2012) estimates a small open economy model featuring financial frictions (in the form of external debt) for twelve emerging countries. His mixed-frequency estimation with quarterly and annual data supports the view that transitory shocks are the main drivers of fluctuations in those countries as opposed to permanent shocks, while the use of only either quarterly or annual data assigns a larger role to permanent shocks. He also runs a Monte Carlo experiment to assess the relative merits of the mixed-frequency strategy. He finds that estimations based on quarterly data series exhibit large upward bias for the contribution of permanent technology shocks, leading to a wrong ranking of shocks importance. On the contrary, the mixed-frequency approach drastically reduces the bias.

Kim (2010) develops a new method for the estimation of DSGE models with mixed-frequency data based on a data augmentation technique within Bayesian estimation (missing observations are generated using Gibbs sampling). He shows, within a standard Real Business Cycle model, that the bias found in the total factor productivity persistence and standard deviation when the model is estimated with quarterly data is reduced at the monthly frequency. Kim (2012) first proposes a similar but different method to accommodate mixed-frequency data sets based on data augmentation method, by extending the MCMC algorithm with Rao-Blackwellization. He uses such a method to investigate whether or not frequency misspecification of a New Keynesian model results in temporal aggregation bias of the Calvo parameter. The answer is positive, and in bringing a medium-scale DSGE model to the data he finds that the average price duration implied by the monthly model is 6.76 months while that by the quarterly model is 17.03 months. For wage duration results suggest 6.36 versus 11.12 months.

There are some papers then which adopt the mixed-frequency approach because it is more convenient for the type of analysis they want to conduct. Justiniano and Michelacci (2012) estimate a business cycle model with search and matching frictions in the labor market combining quarterly and annual data for a set of countries. Yau (2015) estimates

a small open economy New Keynesian DSGE model with mixed-frequency Taiwanese data to get an estimates of the monthly GDP. Heung and Yau (2017) use a similar approach to nowcast the Taiwanese GDP growth. Both those papers show the superiority of the DSGE model against reduced form setups. Schorfheide et al. (2018) utilize mixed-frequency monthly-annual data in the context of a novel Bayesian state-space model. In such a way they identify a persistent conditional mean and a time-varying volatility component in consumption growth, using both simple AR(1) models and a structural long-run risks model.

Finally, our paper is also related to DSGE models with financial frictions à la Bernanke et al. (1999). We listed them in the introduction and we recall them in the text when necessary.

3 The model

The core of our model is based on Smets and Wouters (2007), a medium-scale model rich in nominal and real rigidities, that we extend with the inclusion of financial frictions as in Bernanke et al. (1999). We mainly follow Del Negro et al. (2015) both for the modellization and the estimation strategy. There are several agents in the model. We report here the log-linearized equilibrium conditions resulting from their maximization problems. While the Smets and Wouters model is well established and does not necessitate to be described in great detail, we spend a bit more time in flashing out the financial frictions part. All variables in the following equations are expressed in log-deviations from their nonstochastic steady state. Steady-state values are denoted by *-subscripts and steady-state formulas are provided in the technical Appendix of Del Negro and Schorfheide (2013).³ As in Del Negro et al. (2015) we first report the core equations and than we discuss the extension to financial frictions.

The model includes some non-stationary variables. We detrend them by $Z_t = e^{\gamma t + \frac{1}{1-\alpha} \tilde{z}_t}$, where γ is the steady-state growth rate of the economy, and \tilde{z}_t is the linearly detrended log-productivity process that follows the autoregressive law of motion⁴

$$\tilde{z}_t = \rho_z \tilde{z}_{t-1} + \sigma_z \epsilon_{z,t}, \quad \epsilon_{z,t} \sim i.i.d.N(0, 1)$$

³Also in the Furlanetto et al. (2021).

⁴All shocks innovations share the same statistical properties if not differently specified.

The growth rate of Z_t in deviation from γ , denoted by z_t , follows the process:

$$z_t = \ln(Z_t/Z_{t-1}) - \gamma = \frac{1}{1-\alpha}(\rho_z - 1)\tilde{z}_{t-1} + \frac{1}{1-\alpha}\sigma_z\epsilon_{z,t}$$

The relationship between households consumption c_t and labor l_t is established by the consumption Euler equation as follows

$$c_t = -\frac{(1 - he^{-\gamma})}{\sigma_c(1 + he^{-\gamma})}(R_t - E_t[\pi_{t+1}] + b_t) + \frac{he^{-\gamma}}{(1 + he^{-\gamma})}(c_{t-1} - z_t) \\ + \frac{1}{(1 + he^{-\gamma})}E_t[c_{t+1} + z_{t+1}] + \frac{(\sigma_c - 1)}{\sigma_c(1 + he^{-\gamma})}\frac{w_*l_*}{c_*}(l_t - E_t[l_{t+1}])$$

where R_t is the net risk-free nominal interest rate on households savings, π_t is the net inflation rate, b_t , also known as risk premium shock, is an exogenous AR(1) process with parameter ρ_b and σ_b which drives a wedge between the intertemporal ratio of the marginal utility of consumption and the riskless real return $R_t - E_t\pi_{t+1}$. The parameter σ_c is the degree of relative risk aversion, or the intertemporal elasticity of substitution, and h measures the degree of habit persistence in consumption.

The relationship between the value of capital in terms of consumption q_t^k and the level of investment i_t measured in terms of consumption goods is given by

$$q_t^k = S''e^{2\gamma}(1 + \bar{\beta})\left(i_t - \frac{1}{1 + \bar{\beta}}(i_{t-1} - z_t) - \frac{\bar{\beta}}{1 + \bar{\beta}}E_t[i_{t+1} + z_{t+1}] - \mu_t\right)$$

where S'' is the second derivative of the investment adjustment cost function, $\bar{\beta} = \beta e^{(1-\sigma_c)\gamma}$, β is the intertemporal discount rate in the utility function of the households, and μ_t is an exogenous AR(1) process called "the marginal efficiency of investment" with parameters ρ_μ and σ_μ .⁵

The capital stock \bar{k}_t evolves as

$$\bar{k}_t = \left(1 - \frac{i_*}{\bar{k}_*}\right)(\bar{k}_{t-1} - z_t) + \frac{i_*}{\bar{k}_*}i_t + \frac{i_*}{\bar{k}_*}S''e^{2\gamma}(1 + \bar{\beta})\mu_t$$

where i_*/\bar{k}_* is the steady-state ratio of investment to capital. The arbitrage condition

⁵Smets and Wouters (2007) label this shock as investment-specific technology shock.

between the return to capital and the riskless rate is

$$\frac{r_*^k}{r_*^k + (1 - \delta)} E_t[r_{t+1}^k] + \frac{1 - \delta}{r_*^k + 1 - \delta} E_t[q_{t+1}^k] - q_t^k = R_t + b_t - E_t[\pi_{t+1}] \quad (1)$$

where r_t^k is the rental rate of capital, r_*^k its steady-state value, and δ the capital depreciation rate. Given that capital is subject to variable capacity utilization u_t , the relationship between \bar{k}_t and the amount of capital effectively rented out to firms k_t is

$$k_t = u_t - z_t + \bar{k}_{t-1}$$

The optimality condition determining the rate of utilization is given by

$$\frac{1 - \psi}{\psi} r_t^k = u_t$$

where ψ captures the utilization costs in terms of forgone consumption. Real marginal costs for firms are given by

$$mc_t = w_t + \alpha l_t - \alpha k_t$$

where w_t is the real wage and α is the income share of capital (after paying markups and fixed costs) in the production function. From the optimality conditions of goods producers, it follows that all firms have the same capital-labor ratio:

$$k_t = w_t - r_t^k + l_t$$

The production function is

$$y_t = \Phi_p(\alpha k_t + (1 - \alpha)l_t) + (\Phi_p - 1)\frac{1}{1 - \alpha}\tilde{z}_t$$

where Φ_p is one plus the share of fixed costs in production, reflecting the presence of fixed costs in production.

The resource constraint is

$$y_t = g_t + \frac{c_*}{y_*}c_t + \frac{i_*}{y_*}i_t + \frac{r_*^k k_*}{y_*}u_t - \frac{1}{1 - \alpha}\tilde{z}_t$$

where the government spending g_t is assumed to follow the exogenous AR(1) process

$$g_t = \rho_g g_{t-1} + \sigma_g \epsilon_{g,t} + \eta_{gz} \sigma_z \epsilon_{z,t}$$

Price and wage setters are subject to Calvo (1983) nominal rigidities. Only a fraction of them can optimally set their price and wage, while the remaining fraction indexes them to previous period values. The result of the optimal price and wage setting are the price and wage Phillips curves

$$\pi_t = \frac{(1 - \zeta_p \bar{\beta})(1 - \zeta_p)}{(1 + \iota_p \bar{\beta}) \zeta_p ((\Phi_p - 1) \epsilon_p + 1)} m c_t + \frac{\iota_p}{1 + \iota_p \bar{\beta}} \pi_{t-1} + \frac{\bar{\beta}}{1 + \iota_p \bar{\beta}} E_t[\pi_{t+1}] + \lambda_{f,t}$$

and

$$w_t = \frac{(1 - \zeta_w \bar{\beta})(1 - \zeta_w)}{(1 + \iota_w \bar{\beta}) \zeta_w ((\lambda_w - 1) \epsilon_w + 1)} (w_t^h - w_t) - \frac{1 + \iota_w \bar{\beta}}{1 + \bar{\beta}} \pi_t + \frac{1}{1 + \bar{\beta}} (w_{t-1} - z_t - \iota_w \pi_{t-1}) + \frac{\bar{\beta}}{1 + \bar{\beta}} E_t[w_{t+1} + z_{t+1} + \pi_{t+1}] + \lambda_{w,t}$$

where the parameters ζ_p , ι_p , and ϵ_p are the Calvo parameters, the degree of indexation, and the curvature parameter in the Kimball aggregator for prices, and ζ_w , ι_w , and ϵ_w are the corresponding parameters for wages. w_t^h measures the households marginal rate of substitution between consumption and labor, and is given by

$$w_t^h = \frac{1}{1 - h e^{-\gamma}} (c_t - h e^{\gamma} c_{t-1} + h e^{\gamma} z_t) + \nu_l l_t$$

where ν_l characterizes the curvature of the disutility of labor (and would equal the inverse of the Frisch elasticity in the absence of wage rigidities). The mark-ups shocks $\lambda_{f,t}$ and $\lambda_{w,t}$ follow exogenous ARMA(1,1) processes

$$\lambda_{f,t} = \rho_{\lambda_f} \lambda_{f,t-1} + \sigma_{\lambda_f} \epsilon_{\lambda_{f,t}} - \eta_{\lambda_f} \sigma_{\lambda_f} \epsilon_{\lambda_{f,t-1}}$$

$$\lambda_{w,t} = \rho_{\lambda_w} \lambda_{w,t-1} + \sigma_{\lambda_w} \epsilon_{\lambda_{w,t}} - \eta_{\lambda_w} \sigma_{\lambda_w} \epsilon_{\lambda_{w,t-1}}$$

respectively. Finally, the monetary authority follows a generalized feedback rule

$$R_t = \rho_R R_{t-1} + (1 - \rho_R) (\psi_1 \pi_t + \psi_2 (y_t - y_t^f)) + \psi_3 ((y_t - y_t^f) - (y_{t-1} - y_{t-1}^f)) + r_t^m$$

where the flexible price/wage output y_t^f is obtained from solving the version of the model without nominal rigidities, and the residual r_t^m follows an AR(1) process with parameters ρ_{r^m} and σ_{r^m} .

Now to the version of the model with financial frictions, there is a type of agent, entrepreneurs, who use their own net worth n_t and borrow from a financial intermediary (that channels households' savings to entrepreneurs) to purchase raw capital \bar{k}_t from households.

After purchasing capital, entrepreneurs are subject to idiosyncratic productivity shock ω_t that transforms raw capital into effective capital. This shock is assumed to be independently drawn across time and across entrepreneurs and log-normally distributed with mean 1 and standard deviation $\tilde{\sigma}_{\omega,t}$. That standard deviation captures mean-preserving changes in the cross-sectional dispersion of ability across entrepreneurs as in Christiano et al. (2014), who label it risk shock. It follows an AR(1) process with parameters ρ_{σ_ω} and σ_{σ_ω} .

After observing the idiosyncratic shock, entrepreneurs choose the utilization rate u_t of its effective capital and rents an amount of capital services to intermediate goods-producing firms at the competitive real rental rate r_t^k . At the end of the period, entrepreneurs sell the left units of capital to households at price q_t^k . The gross nominal return on capital for entrepreneurs is \tilde{R}_t^k .

To cope with the asymmetric information about entrepreneurs idiosyncratic productivity, financial intermediaries enter into a financial contract with entrepreneurs. There is a cutoff value $\bar{\omega}_t$ such that entrepreneurs whose ω_t is lower than $\bar{\omega}_t$ declare bankruptcy and the intermediary must pay a monitoring cost μ^e proportional to the realized gross payoff to recover the remaining assets. Banks protect themselves against default risk by pooling all loans and charging a spread over the deposit rate. This spread may vary as a function of the entrepreneurs leverage and their riskiness.

The necessary adjustment to our core model to account for all that is to replacing equation 1 with the following two conditions

$$\tilde{R}_t^k - \pi_t = \frac{r_*^k}{r_*^k + (1 - \delta)} r_t^k + \frac{(1 - \delta)}{r_*^k + (1 - \delta)} q_t^k - q_{t-1}^k$$

and

$$E_t \left[\tilde{R}_{t+1}^k - R_t \right] = b_t + \zeta_{sp,b} (q_t^k + \bar{k}_t - n_t) + \tilde{\sigma}_{\omega,t} \quad (2)$$

The first condition defines the return on capital, while the second one determines the spread between the expected return on capital and the riskless rate and its relationship

with entrepreneurs leverage and riskiness. The following condition describes the evolution of entrepreneurial net worth

$$n_t = \zeta_{n, \tilde{R}_t^k} (\tilde{R}_t^k - \pi_t) - \zeta_{n, \tilde{R}_t^k} (R_{t-1} - \pi_t) + \zeta_{n, qK} (q_{t-1}^k + \bar{k}_{t-1}) + \zeta_{n, n} n_{t-1} - \frac{\zeta_{n, \sigma_\omega}}{\zeta_{sp, \sigma_\omega}} \tilde{\sigma}_{\omega, t-1}$$

All ζ_x coefficients are function of deep structural parameters and determined by steady state restrictions as in the technical Appendix of Del Negro and Schorfheide (2013).

4 Estimation

The estimation of all the versions of the model is conducted with Bayesian techniques. We use the following observed variables: real GDP growth, real consumption growth, real investments growth, real wage growth, hours worked, PCE inflation rate, nominal interest rate, spread between BAA corporate bonds yield and the 10-year government bond yield.⁶ Hours worked, PCE inflation rate, nominal interest rate, and the spread are all available also at monthly frequency and they will be used, together with the remaining quarterly variables, for the estimation of the mixed-frequency model. A detailed description of data and their transformation is in Appendix A.

The interpretation of time t across models' specifications is different. It can be a quarter or a month. In section 3 we presented the model for a generic time t . Now we need to distinguish between quarters and months for the sake of specifying the measurement equations. Hence we use the superscripts q , for quarterly, and m for monthly. The measurement equations for the quarterly specification are as follows

$$\begin{aligned} \text{Output}^q \text{ growth} &= \gamma^q + y_t^q - y_{t-1}^q + z_t^q \\ \text{Consumption}^q \text{ growth} &= \gamma^q + c_t^q - c_{t-1}^q + z_t^q \\ \text{Investment}^q \text{ growth} &= \gamma^q + i_t^q - i_{t-1}^q + z_t^q \\ \text{Real wage}^q \text{ growth} &= \gamma^q + w_t^q - w_{t-1}^q + z_t^q \\ \text{Hours}^q &= \bar{l} + l_t^q \\ \text{Inflation}^q &= \pi_* + \pi_t^q \\ \text{FFR}^q &= R_* + R_t^q \\ \text{Spread}^q &= SP_* + E_t \left[\tilde{R}_{t+1}^{k,q} - R_t^q \right] \end{aligned}$$

⁶Del Negro et al. (2015) use the GDP deflator to compute the inflation rate. This is not available at monthly frequency. We follow Forni and Marcellino (2014) and we use the PCE deflator. The quarterly estimation is not affected by that choice.

Turning to the monthly specification, for those variables observed at monthly frequency the measurement equations are

$$\begin{aligned}\text{Hours}^m &= \bar{l} + l_t^m \\ \text{Inflation}^m &= \pi_* + \pi_t^m \\ \text{FFR}^m &= R_* + R_t^m \\ \text{Spread}^m &= SP_* + E_t \left[\tilde{R}_{t+1}^{k,m} - R_t^m \right]\end{aligned}$$

where the constants are not marked differently from the quarterly case because their value is adjusted via the prior means. In particular, the monthly value of \bar{l} , π_* , and R_* is obtained by dividing by 3 the quarterly values. As in Del Negro et al. (2015), SP_* is set to 2 in the quarterly model, an annualized percent value. It is transformed into quarterly through the formula $(1 + SP_*/100)^{1/4}$; for the monthly case we simply set the prior mean at $SP_*/12$.

For the measurement equations of those variables observed only quarterly, we need to elaborate more and we report the technical details in Appendix B. We take output as an example, and the same holds for consumption, investments, and wages. The measurement equation for output growth is

$$\text{Output growth} = 3\gamma^m + y_t^q - y_{t-3}^q + z_t^m + z_{t-1}^m + z_{t-2}^m,$$

where $\gamma^m = \gamma^q/3$ is made consistent with the monthly frequency via the mean of its prior distribution and

$$\begin{aligned}y_t^q &= \frac{1}{1 + \frac{1}{e^\gamma} + \frac{1}{(e^\gamma)^2}} y_t^m + \frac{1}{e^\gamma \left[1 + \frac{1}{e^\gamma} + \frac{1}{(e^\gamma)^2} \right]} (y_{t-1}^m - z_t^m) + \\ &\quad \frac{1}{(e^\gamma)^2 \left[1 + \frac{1}{e^\gamma} + \frac{1}{(e^\gamma)^2} \right]} (y_{t-2}^m - z_t^m - z_{t-1}^m).\end{aligned}$$

It is extremely useful to define the variable y_t^q in the monthly model. Not only to correctly specify the measurement equations, but also because it facilitates the comparison between the output of the monthly model with the one from the quarterly model. In fact, for instance the monthly impulse response functions (IRFs) can be compared only if they are aggregated at a quarterly frequency first. One can follow different strategies, e.g. aggregate the IRFs of the monthly variables in the same way one would aggregate the data they refer to in the first place. But the easiest way to obtain the quarterly IRFs is to take

the IRFs of y_t^q and pick one value every three. Also in the data this variable is observed every three months.

As for the prior specification we follow Del Negro et al. (2015). For the quarterly model that is a natural choice for the sake of comparison. For the monthly model the choice is less obvious. In the baseline monthly estimation we assume the same quarterly moments for the prior distributions (with the exception of the constants in the measurement equations as explained above). We wanted to isolate and evaluate the contribution of the mixed-frequency data in driving our results, i.e., we wanted to make sure that differences in the posteriors come from the likelihood rather than from the priors. Nevertheless, according to the Bayesian philosophy, prior information, if available, should be used and incorporated into the prior distributions. For some parameters, e.g. Calvo parameters, the transformation from a quarterly to a monthly prior is straightforward. But for other parameters it is less so. That's another reason why the baseline monthly exercise uses the same quarterly priors, which are however in general rather uninformative, see Table 1 for details.

Some parameters are calibrated instead of estimated. Their quarterly values are 0.025 for the capital depreciation rate δ , 1.5 for the steady state wage mark-up shock λ_w , 0.18 for the government spending to output share g , 10 for both the curvature parameters in the Kimball aggregator for prices and wages ϵ_p and ϵ_w , 0.0076 for the steady state default probability of entrepreneurs $F(\bar{\omega})$ (3 percent yearly), and 0.99 for the survival rate of entrepreneurs γ^* . In the monthly specification, $\delta = 0.025/3$, $F(\bar{\omega}) = 1 - (1 - 0.03)^{1/12}$, and $\gamma^* = 1 - 0.01/3$.

The practical implementation of the mixed-frequency estimation is done by following the approach in Durbin and Koopman (2012). We treat the non-observed monthly values for (quarterly) output growth, consumption growth, investment growth, and real wages growth as missing values and we allow the Kalman filter to infer them.

The monthly model is particularly challenging to estimate, mainly because the sample size is relatively big, 534 observations referring to the period 1964m1-2008m9, and because there are many missing observations. To have reliable parameter estimates, we adopt the following iterative approach. In the first step, we run two Metropolis-Hastings chains of 1 000 000 draws each, with a 20 percent burn-in. In the second and subsequent steps, we repeat this procedure but initialize parameter estimates at the mode of the posterior distributions of the previous step. The stopping criterion is based on the marginal data density (MDD): when the differences in MDD in two estimation steps is smaller than 0.01,

we stop and keep those parameter estimates. This assures that parameters are the same as the previous round and that another estimation would not provide different estimated parameters. Moreover, this procedure guarantees appropriate convergence of the chains and well behaved posterior distributions. Posterior distributions, together with priors, and convergence diagnostics, only for our mixed-frequency baseline model with financial frictions, are in Figures 1, 2, and 3. The empirical results are commented in the next section.

5 Results

In this section we present our results. Our main focus is on estimated models with financial frictions, but for the sake of evaluating the accelerator mechanism we also need to estimate models without financial frictions. We then estimate four versions of the model: with and without financial frictions, at quarterly and mixed-frequency.

Before entering into the specific results, it is worth analyzing the estimated parameters. The median of the posterior distributions are in Table 2. To understand more the effects of the use of mixed-frequency data we start by describing their effect on those parameters that mostly are supposed to capture the nominal interest rate dynamics, namely the parameters of the monetary policy shock σ_{r^m} and ρ_{r^m} , and the interest rate smoothing ρ_R .⁷ The quarterly no financial frictions estimates are 0.2265, 0.1140, and 0.8460 respectively. Adding financial frictions leads to 0.2497, 0.0533, and 0.7765. The estimated standard deviations are substantially similar, but the persistence parameters are estimated at a lower value in the model with financial frictions. The reason is clear: that model is embedded with the accelerator mechanism, an endogenous mechanism that helps it to describe data properties better, allowing it to rely less on exogenous elements or on other persistence parameters.

Turning to the mixed-frequency estimation, we notice that the monthly no financial frictions model estimation gives 0.0619, 0.1895, and 0.9464.⁸ What does explain such different values compared with the quarterly estimation? The answer is in the moments of observed series. By construction, the monthly interest rate series has a one third the standard deviation of the quarterly series. In fact, the starting point to construct those series is the Fed funds rate observed either quarterly or monthly, but in annual terms.

⁷The other parameters of the Taylor rule are not that different.

⁸Belaygorod and Dueker (2009) estimate a three-equation New Keynesian model with monthly US data for the period 1959-2005. Their estimate of the monetary policy shock standard deviation is 0.0590. This corroborates very much our estimate in the mixed-frequency no financial frictions model.

Hence that series is divided by 4 to get the quarterly terms and by 12 to get the monthly terms. It is than clear why the standard deviation of the monetary policy shock is so lower: it has to fit a much less volatile series. As for the autocorrelation, monthly data has a slightly higher autocorrelation, 0.98 versus 0.95. Hence the persistence parameters are slightly higher too. Finally, the monthly financial frictions model estimates are 0.1488, 0.1288, and 0.4983. In this case, the estimation decides to accommodate the lower variance with a much lower smoothing parameter and compensate that with a bigger ρ_{r^m} to fit the autocorrelation.

A similar analysis can be done with the inflation rate. Also this series has a scale difference. The standard deviation of the quarterly series is 0.6249 and that of the monthly series is 0.2429, while the autocorrelations are 0.86 and 0.71 respectively. For inflation the relevant parameters are the Calvo parameter ζ_p , the indexation parameter ι_p and the autoregressive coefficient of the price mark-up shock ρ_{λ_f} . The quarterly model with financial frictions exploits the accelerator to reduce quite substantially ρ_{λ_f} , from 0.928 to 0.687, and compensates in part with higher ζ_p and ι_p . In the monthly case instead, the lower standard deviation and autocorrelation of monthly inflation is captured in the financial frictions model by a much lower ζ_p and ι_p . They drop to 0.2023 and 0.0605 from 0.7443 and 0.4226, respectively.⁹ The Calvo parameter is also interesting because it can be transformed into number of periods, making the comparison easy in this case. Firms can re-optimize their prices about every 8, 12, 19, and 1 months, if the model is estimated quarterly without financial frictions, quarterly with financial frictions, monthly without financial frictions, and monthly with financial frictions, respectively.¹⁰ The last value shows that the Phillips curve is estimated to be very very steep. This is one of the elements driving our results in terms of size of the inversion of the accelerator and relevance of financial shocks.

The analysis could go on for many other parameters. One has to bear in mind that also the spread and hours worked have different moments, along the lines of the variables already described.

All those examples, while informative, highlight the complications of trying to understand our results by looking at the estimated parameters values. The frequency of the data changes moments of observed variables, and it is not surprising that parameters adjust to account for those different moments. Hence, to compare quarterly and monthly estimates, it is preferable to rely on the aggregation of the monthly models output. We will do that in the next subsection.

⁹Liu et al. (2011) also estimate a low degree of price and wage rigidities.

¹⁰The duration for wages are 12, 9, 21, 1 months.

We conclude this section by first stressing that Figures 1 and 2 highlight that for all the discussed parameters the posterior distributions are quite different from the prior ones, which indicates that there are no identification issues in the mixed-frequency model.

Second, we elaborate on the habit parameter h . For the quarterly cases we estimate a value of 0.7094 for the model without financial frictions, in line with the macro literature, and a value of 0.2785 for the model with financial frictions, in keeping with Del Negro et al. (2015). For the monthly case instead, both values drop to about 0.10. This is very much supported by the evidence reported in Havranke et al. (2017) according to which estimates of habit in consumption obtained employing monthly frequency data tend to be substantially smaller than when quarterly or annual frequencies are used. They find that across all papers estimating the Euler equation with monthly data to get an estimate of the habit parameter, on average that estimate is 0.15.

5.1 The presence of the financial accelerator

In this section we investigate the impact of the mixed-frequency estimation on the financial accelerator mechanism. We first focus on two shocks, the monetary policy and the investment-specific shock, mainly because the former is representative of those shocks that are known from previous studies to generate an accelerator, while the latter a deceleration.¹¹

We report in Figure 4 the response of investments to a restrictive monetary policy shock in the quarterly models (upper-left panel) and in the mixed-frequency models (upper-right panel), while the lower panels report the response of the same variable to an investment-specific technology shock. In each there is the case of no financial frictions (blue-solid line) and the case of financial frictions (red-dashed line). Monthly impulse response functions are aggregated at the quarterly frequency for the sake of comparison, hence horizontal axes measure quarters after the shocks.

It is clear that we recover the standard result in the literature for the quarterly case.

¹¹It is well known that an investment-specific shock decelerates the dynamics of investments. The reason is that a positive investment-specific shock brings the economy into an expansionary period, with an increase in investments, consumption, and output. Nevertheless, this shock is a technology shock (it improves the technology of producing new capital goods) and as such it reduces the price of new capital goods. In our model this is the price of capital, i.e. the price at which the asset side of entrepreneurs' balance sheet is evaluated. The decreased asset value deteriorates entrepreneurs' financial position, so they have to pay a higher premium on their external finance (notice that the premium is pro-cyclical in this situation). So they cut borrowing and this has a negative second round effect on investments, which in turn increase less than otherwise. Or in other words investments experience a deceleration. See De Graeve (2008) and Gelain (2010) for a more elaborated explanation of the deceleration that might be present for other shocks.

When credit frictions are relevant, the response of investments is amplified after a monetary policy shock and de-amplified after an investment-specific shock. When we investigate the monthly case, we find the opposite: investment dynamics displays now a deceleration or an acceleration for the same shocks, respectively. The inversion takes place for all the other shocks with the exception of the technology shock.

There is one key parameter that explains the inversion of the accelerator. It is the second derivative of the investment adjustment cost function S'' . This parameter is not only the one that varies the most, but it also varies in opposite direction under the two cases of quarterly and mixed-frequency. In fact in the former case it decreases from 6.0404 when friction are off to 2.6698 when finance bites, while it increases from 0.8990 to 3.5410 in the latter case. This is already indicative of the fact that the accelerator might be working differently. A quarterly model with an accelerator always relies on that endogenous mechanism to explain investment dynamics, while relying less on investment adjustment costs. The mixed-frequency estimation seems instead to rely more on investment adjustment costs.

A very simple test to prove our conjecture of S'' being the main driver of our results, is to simulate the quarterly models under the assumption that S'' are those obtained with the mixed-frequency estimation, while keeping all other parameters at the quarterly estimates. Figure 5, left panel, shows that indeed a monetary policy shock generates a decelerator, while the investment specific shock an accelerator (right panel), the opposite of what a quarterly model normally predicts.¹²

Why is S'' so relevant for investment dynamics and the accelerator? This is related to the analysis in De Graeve (2008). He noted that the assumption on investment adjustment costs as opposed to the capital adjustment costs as in Bernanke et al. (1999) is responsible for affecting the sign of the accelerator. In particular, if the cost function is in terms of investment in deviation from the capital stock, $f(i_t/k_t)$, all shocks lead to an accelerator. On the contrary, if it is in terms of investment growth, $f(i_t/i_{t-1})$, some of the shocks lead to a deceleration.¹³ That's why the very low value of S'' that we estimate for the no financial frictions monthly model, 0.90, is so relevant to explain our results. When S'' approaches 0,

¹²We tried that with other parameters too, but none could invert the accelerator alone.

¹³There is in fact an interaction between financial frictions and the other frictions in the model. For instance, if the cost function is $f(i_t/i_{t-1})$, then once investments start rising due to a positive technology shock they will keep rising for a protracted period of time (the longer the higher the adjustment costs). This implies that the capital stock will soon outgrow net worth, thereby increasing borrowing needs over time (borrowing equals the capital bought by entrepreneurs $q_t^k \bar{k}_t$ minus its own funds n_t). The result is an increase in the external finance premium (i.e., a pro-cyclical premium). Because long lasting positive investment will be costly due to a high future premium for external finance and because agents are forward looking, investment will be lower in all periods, including current ones. Hence, they decelerate.

$f(i_t/i_{t-1})$ is approximately equal to $f(i_t/k_t)$, in the sense that investments have their pick response on impact and they go relatively quickly back to steady state (or in other words their response is not hump shaped). The agents' forward-lookiness and the time to adjust investments, very short if the adjustment cost function is $f(i_t/k_t)$, are crucial elements to explain the inversion. It is also worth mentioning that depending on the shock, the response of investment changes in response to different values of S'' . For instance in the model without financial frictions, the lower S'' the stronger the response of investment to a monetary policy shock. Vice-versa, the lower S'' the weaker the response of investment to an investment-specific shock. This is clear from the blue lines in Figure 4.

If the inversion itself depends on S'' , it is evident that its size depends on other parameters too. For instance, the response of investments in the monthly model with financial frictions after a monetary policy shock (red-dashed line in the upper-right panel in Figure 4) is so much muted compared to its counterpart in quarterly model. The change in S'' , from 2.67 to 3.54, is not big enough to fully account for the big change in the response of investments. The more muted response in the monthly model is then also due to the estimated Calvo parameter ζ_p at 0.20. This implies that monetary policy is almost exogenous and marginal costs react very little to changes in the interest rate, and so do the other real variables.¹⁴

In Figure 6 we evaluate the statistical significance of the accelerator by reporting the 5th and the 95th percentiles of the impulse response functions. If the impulse response functions do not overlap, then the accelerator is statistically significant.

The shocks that generate a statistically significant inversion are the monetary policy shock, the wage mark-up shock, and the price mark-up shock, the latter after few quarters. Does that imply that our results are not that relevant because the inversion takes place only for a small fraction of shocks? No, it does not. In fact, if one looks at the variance decomposition in Table 3, the wage mark-up shock, and the price mark-up shock alone explain more than 50 percent of the GDP variability. Therefore, those are key drivers of the business cycle fluctuations according to the monthly model, and hence properly identifying their implied dynamics is of paramount importance.

As already stressed, all impulse response functions from the monthly model are aggregated to be comparable with the quarterly ones. One might think that results could be driven by the aggregation. But there is another way of reading our results that does not rely on the aggregation. It consists on looking at the monthly impulse response functions.

¹⁴Note that the New Keynesian Philips curve becomes $mc_t + \kappa\lambda_f = 0$ when ζ_p converges to 0, implying that marginal costs are completely exogenous and equal to the inverse of the price mark-up shock.

We do not report them, but looking at them one would conclude that at a monthly frequency the accelerator is inverted exactly as we described above. Hence, using monthly data leads to very different evidence than using quarterly data.

5.2 Role of financial shocks

In this section we scrutinize what shocks drive the economy dynamics. The questions we want to answer are the following: what's the impact of the mixed-frequency estimation on the cocktail of shocks that turns out to be relevant to explain business cycle fluctuations? In particular, what is the effect of financial shocks?

Financial shocks have been at the center of the scene since the Great Recession. Many have investigated their role as a source of fluctuations originating in the financial sector and transmitting to the real economy. The analyses have been conducted both with structural and non-structural models. Generally, financial shocks are found to play a role in explaining the variance of real variables. But there is a wide variety of evidence classifying them from non-negligible, e.g. Fornari and Stracca (2012) find that financial shocks explain 12 percent of GDP, to major drivers, e.g. Furlanetto et al. (2019).

It is out of the scope of our paper to summarize the large number of papers on the topic. In our context it is more natural to compare our results with the DSGE literature, and restrict our attention to models similar to ours. The main reference is Christiano et al. (2014). They find that the risk shock alone explains 62 percent of GDP fluctuations in the US. Our model shares the core accelerator mechanism with them, but it differs greatly in many crucial respects. Our quarterly model cannot replicate their evidence about the risk shock for the following two main reasons: we do not use net worth and credit as observed variables and we do not have news on the risk shock.¹⁵ Also the different sample size (1985-2010 instead of 1964-2008) could matter to some extent. Another very relevant difference is that following Del Negro et al. (2015), who followed the seminal paper of Smets and Wouters (2007), we have the risk premium shock b_t in the model. This shock is absent in Christiano et al. (2014) because it is modelled as a discount factor (preference) shock. As shown in equation 2 both the risk premium and the risk shock enter that equation. Hence they concur in explaining the premium and as a consequence the rest of the endogenous

¹⁵Our analysis could be extended along those lines. It would be interesting especially to consider net worth as observable. This is proxied with a stock market index. So it is available at very high frequency. Nevertheless, the computational burden from including extra observables, even at a higher frequency than monthly, would be too high. We think that our results already indicate a pattern that in principle should hold also when financial shocks are much more prominent as in Christiano et al. (2014).

variables. In other words, they are both financial shocks, so we need to consider them both when we evaluate the relevance of the exogenous bits coming from the financial sector.¹⁶ This also means that we need to provide our own evidence about those shocks in the quarterly estimation to have a reference. In fact we are not aware of any other paper reporting evidence on those shocks.

To achieve our goal we rely on the variance decomposition. This is reported in Table 3. We only report the variance decomposition of output at different horizons. Also in this case the variance decomposition for the monthly model is computed for quarterly aggregated variables to make the comparison fair.

It highlights that when only quarterly data are used the most important drivers are the financial shocks. Together they account for about 28 percent of output variability in the very short run and about 44 percent in the very long run. It is worth noting that the risk shock counts very little, while the bulk of the explanation is loaded on the risk premium shock, for the reasons we explained earlier. This is not totally surprising. Absent Christiano et al. (2014) features one should expect a smaller role for the risk shock. They also acknowledge that in a sense. In their analysis the contribution of the unanticipated shock alone to the GDP variance is only 16 percent.

Our result is that when mixed-frequency data are used, the variance decomposition changes completely. Financial shocks lose entirely their power, while technology and both price and wage mark-up shocks regain a lot of importance. Mark-up shocks explain about 57 percent of output variability at all horizons. This has strong implications for monetary policy because those shocks are responsible for the trade-off between nominal and real stabilization. Big mark-up shocks big trade-offs. Small mark-up shocks small trade-offs, implying that monetary policy could more easily achieve good stabilization of the real and the nominal side of the economy at the same time, if not get closed to the divine coincidence. This greatly relates to the discussion in Justiniano et al. (2013) and Furlanetto et al. (2021).

Why do financial shocks lose importance and why mainly in favor of mark-up shocks? The explanation is in the relationship between the slope of the Phillips curve and the relevance of financial shocks versus the mark-up shocks.¹⁷ For a quarterly model this

¹⁶Despite the fact that Smets and Wouters (2007) describe their shock as having "similar effects as so-called net-worth shocks in Bernanke et al. (1999) and Christiano et al. (2003), which explicitly model the external finance premium (p. 589)", Fisher (2015) provides a different interpretation for the risk premium shock. He shows that it can be considered as shock to the demand of safe assets.

¹⁷For convenience we focus only on the price inflation Phillips curve, but the argument holds also for the wage curve.

argument is well developed in Del Negro et al. (2015). The starting point is the fact that in the data big drops of output are typically associated with small decreases in inflation.¹⁸ The quarterly model with financial frictions rationalizes that fact by estimating a flatter Phillips curve (higher Calvo parameter ζ_p) than the model without those frictions. A high degree of price rigidity allows movements in the demand curve alone (caused by demand shocks) to be consistent with large change in output and small change in inflation as in the data. On the contrary, a vertical (supply) Phillips curve would require a shift in the Phillips curve itself, via a price mark-up shock, to accommodate the empirical evidence, because a demand curve shift alone would generate large movement in both output and inflation. Hence, a vertical Phillips curve makes price mark-up shocks relevant, while a flatter curve makes them irrelevant. In our monthly model with financial frictions we estimate a very steep Phillips curve, $\zeta_p = 0.20$. This explains why financial shocks loose importance, mainly in favor of mark-up shocks.

5.3 Monte Carlo

We run a pseudo Monte Carlo experiment to evaluate whether or not the bias that we identify in the estimation materializes also in a controlled environment in which we know the data generating process. It is a pseudo analysis because we adopt a selection criterion to choose one set of simulated series for the endogenous variables of interest. This choice is dictated by the computational costs associated with the estimations.

The set we choose is supposed to be representative of an average set, out of the thousands normally simulated in a Monte Carlo set up. Or at least it should be a set of series that incorporates the features responsible to create the bias. Our procedure is as follows: we set the parameters values at the estimated posterior medians of the mixed-frequency models, both with and without financial frictions. For each model we simulate 1000 time series of 1000 months length. We aggregate them at quarterly frequency and we keep the last 178 values (the same length of our quarterly sample) of each series. Among those 1000 sets, we choose the one that contains the quarterly series of investment growth that best fits (in terms of minimizing the squared deviations) the observed quarterly investment growth series. We use that set to estimate the two models with quarterly data. And, using the corresponding monthly series which generated those "optimal" quarterly series, we also estimate the models with all monthly series and with mixed-frequency series.

¹⁸An example is the missing disinflation during the Great Recession. Moreover, this correlation is well documented in Christiano et al. (2010).

Results are in Table 4 and in Figure 7. In the Table we compare the estimated parameters with the "true" parameters and among themselves. Focusing on the parameter S'' , we note that the monthly model gets quite close to the true values. The results for the quarterly model, instead, clearly support our empirical evidence that a bias is introduced in this parameter once data are aggregated at quarterly frequency. Quarterly S'' values of about 7 and 5 are in line with the literature and highlight that at this frequency the accelerator seems to work in the right direction (the value for the no financial frictions model is higher than the model with financial frictions). As for the mixed-frequency estimation, this particular Monte Carlo simulation does not seem to perform particularly well in replicating the true parameters. However, it gets surely closer than the quarterly models. Moreover, like for the monthly models, the estimated value of S'' of the model without financial frictions, i.e., 3.3, is smaller than the value in the model with financial frictions, i.e., 5. The opposite of what happens in the quarterly case, suggesting that the accelerator might work the other way around, as we find in our previous section.

What do those numbers imply for the models dynamics? Do the impulse response functions reflect the evidence based on the estimated parameters? For the most part they do. In Figure 7 we report the response of investment to a set of selected shocks for the three frequency specifications. The monetary policy and the investment-specific shocks greatly confirm our results that the accelerator is inverted when higher frequency data are used compared to the quarterly frequency. Turning to the price mark-up shock, the quarterly model wrongly gives an accelerator, while the higher frequencies a deceleration, at least on impact. Finally, contrary to our results, the wage mark up shock does not reach the inversion. It correctly decelerates the response of investment at high frequencies, but it does not accelerate it at quarterly frequency. However, it pushes that latter dynamics a long way through the one from the quarterly estimation above.

Overall, this exercise stresses that the data aggregation at lower frequency introduces biases that deeply alter the model dynamics, up to the point of eventually inverting the accelerator mechanism.

6 Conclusions

In this paper we use monthly and quarterly data to estimate a canonical dynamic stochastic general equilibrium model with financial frictions à la Bernanke et al. (1999), a popular type of model after the Great Recession. We aim to understand the implications

of aggregating high frequency data into quarterly data, a common choice to estimate this type of models, and to assess whether the use of mixed frequency data can be useful in this context. Financial data are typically used to identify financial frictions and shocks. Those are available at high frequency, while real variables are not. Hence we rely on a mixed-frequency Bayesian approach.

Specifically, the literature has already highlighted that temporal aggregation can prevent the identification of structural DSGE models and lead to substantial bias in the identification of structural shocks and their transmission. We investigate, in the context of our model with financial frictions, the existence of such biases and eventually their implications for the embedded financial accelerator mechanism and for the relevance of financial shocks in explaining real variables variability.

We find that aggregating data to quarterly frequency has a big impact on the estimated parameters, indicating the existence of big biases, in particular with respect to those parameters related to investment adjustment costs and the slope of price and wage Phillips curves. This implies that the accelerator mechanism is different according to the frequency at which one estimates the model. In fact, our analysis shows that it can be inverted, namely it can happen that shocks that generate an accelerator in the quarterly model generate a deceleration in the monthly model and vice-versa.

We also find that financial shocks are important to explain real GDP fluctuations in the quarterly model, as in the related literature, while they are not at all in the monthly model.

Finally, an analysis based on simulated data supports our empirical conclusion that aggregating data at lower frequency can introduce substantial biases, and that the use of mixed frequency data can reduce these biases.

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Table 1: Priors for DSGE parameters

Parameter		Quarterly model		Mixed-frequency model		
		Density	Mean	St. Dev	Mean	St. Dev
Std technology	σ_z	InvG	0.178	0.30	0.178	0.30
Std risk premium	σ_b	InvG	0.178	0.30	0.178	0.30
Std price mark-up	σ_{λ_f}	InvG	0.178	0.30	0.178	0.30
Std wage mark-up	σ_{λ_w}	InvG	0.178	0.30	0.178	0.30
Std investment-specific	σ_μ	InvG	0.178	0.30	0.178	0.30
Std spending	σ_g	InvG	0.178	0.30	0.178	0.30
Std monetary policy	σ_{r^m}	InvG	0.178	0.30	0.178	0.30
Std risk	σ_{σ_w}	InvG	0.063	0.033	0.063	0.033
Auto. technology	ρ_z	Beta	0.50	0.20	0.50	0.20
Auto. risk premium	ρ_b	Beta	0.50	0.20	0.50	0.20
Auto. price mark-up	ρ_{λ_f}	Beta	0.50	0.20	0.50	0.20
Auto. wage mark-up	ρ_{λ_w}	Beta	0.50	0.20	0.50	0.20
Auto. investment-specific	ρ_{mu}	Beta	0.50	0.20	0.50	0.20
Auto. government spending	ρ_g	Beta	0.50	0.20	0.50	0.20
Auto. monetary policy	ρ_{r^m}	Beta	0.50	0.20	0.50	0.20
Auto. risk	ρ_{σ_w}	Beta	0.75	0.15	0.75	0.15
Arma price mark-up	η_{λ_f}	Beta	0.50	0.20	0.50	0.20
Arma wage mark-up	η_{λ_w}	Beta	0.50	0.20	0.50	0.20
Tech. in gov. spending	η_{gz}	Beta	0.50	0.20	0.50	0.20
Reaction inflation	ψ_1	Normal	1.50	0.25	1.50	0.25
Reaction output gap	ψ_2	Normal	0.12	0.05	0.12	0.05
Reaction output gap growth	ψ_3	Normal	0.12	0.05	0.12	0.05
Interest rate smoothing	ρ_R	Beta	0.75	0.10	0.75	0.10
Price stickiness	ζ_p	Beta	0.50	0.10	0.50	0.10
Wage stickiness	ζ_w	Beta	0.50	0.10	0.50	0.10
Capital share	α	Normal	0.30	0.05	0.30	0.05
Production fixed cost	Φ_p	Normal	1.25	0.12	1.25	0.12
Habit formation	h	Beta	0.70	0.10	0.70	0.10
Labour disutility	ν_l	Normal	1.25	0.12	2.00	0.75
Price indexation	ι_p	Beta	0.50	0.15	0.50	0.15
Discount factor	r^*	Gamma	0.25	0.10	0.25/3	0.10
SS inflation	π^*	Gamma	0.75	0.40	0.75/3	0.40
SS tech. Growth	γ	Normal	0.40	0.10	0.40/3	0.10
Invest. adj. costs	S''	Normal	4.00	1.50	4.00	1.50
Intertemporal elasticity	σ_c	Normal	1.50	0.37	1.50	0.37
Wage indexation	ι_w	Beta	0.50	0.15	0.50	0.15
Utilization costs	ψ	Beta	0.50	0.15	0.50	0.15
SS spread	SP^*	Gamma	2.00	0.10	2.00/12	0.10
Elasticity of EFP w.r.t. leverage	$\zeta_{sp,b}$	Beta	0.05	0.005	0.05	0.005
Mean hours worked	\bar{l}	Normal	-45	5.00	-45/3	5.00

Table 2: Posterior median for DSGE parameters

Parameter		Quarterly data		Mixed-frequency data	
		No financial friction	Financial friction	No financial friction	Financial friction
Std technology	σ_z	0.4625	0.4827	0.1995	0.2126
Std risk premium	σ_b	0.2154	0.0300	0.0387	0.0241
Std price mark-up	σ_{λ_f}	0.1500	0.1846	0.1283	0.1664
Std wage mark-up	σ_{λ_w}	0.2731	0.2817	0.2777	1.0840
Std investment specific	σ_{mu}	0.4097	0.5652	0.1520	0.1495
Std government spending	σ_g	2.9475	2.9381	1.7813	1.8691
Std monetary policy	$\sigma_{r,m}$	0.2265	0.2497	0.0619	0.1488
Std risk	σ_{σ_w}	–	0.0575	–	0.0144
Auto. technology	ρ_z	0.9608	0.9650	0.9994	0.9970
Auto. risk premium	ρ_b	0.3351	0.9839	0.9411	0.9923
Auto. price mark-up	ρ_{λ_f}	0.9280	0.6870	0.9761	0.9906
Auto. wage mark-up	ρ_{λ_w}	0.9713	0.9625	0.9849	0.9960
Auto. investment specific	ρ_{mu}	0.7481	0.7358	0.9562	0.9758
Auto. government spending	ρ_g	0.9800	0.9668	0.9998	0.9893
Auto. monetary policy	ρ_{rm}	0.1140	0.0533	0.1895	0.1288
Auto. risk	ρ_{σ_w}	–	0.9953	–	0.9976
Arma price mark-up	η_{λ_f}	0.7546	0.5961	0.9804	0.0973
Arma wage mark-up	η_{λ_w}	0.8982	0.9501	0.5616	0.1337
Tech. in gov. spending	η_{gz}	0.7830	0.8020	0.9217	0.8949
Reaction inflation	ψ_1	2.0019	1.1433	1.6987	1.3598
Reaction output gap	ψ_2	0.0889	0.0954	0.2629	-0.0278
Reaction output gap growth	ψ_3	0.2302	0.2350	0.4272	0.2927
Interest rate smoothing	ρ_R	0.8460	0.7765	0.9464	0.4983
Price stickiness	ζ_p	0.6358	0.7443	0.9510	0.2023
Wage stickiness	ζ_w	0.7504	0.8893	0.2817	0.0497
Capital share	α	0.1732	0.2083	0.0770	0.1414
Production fixed cost	Φ_p	1.7084	1.5764	2.0075	2.0059
Habit formation	h	0.7094	0.2785	0.1046	0.1066
Labour disutility	ν_l	2.5018	2.5602	6.7247	6.7427
Price indexation	ι_p	0.2609	0.4226	0.2709	0.0605
Discount factor	r^*	0.1669	0.1691	0.0108	0.1456
SS inflation	π^*	0.9229	0.8505	0.2918	0.2885
SS tech. Growth	γ	0.4069	0.3471	0.1539	0.1367
Invest. cdj. costs	S''	6.0404	2.6698	0.8990	3.5410
Intertemporal elasticity	σ_c	1.3408	1.4143	1.2762	0.3984
Wage indexation	ι_w	0.4378	0.3061	0.5084	0.3832
Utilization costs	ψ	0.6931	0.6176	0.1829	0.4468
SS spread	SP^*	–	1.8001	–	0.0805
Elasticity of EFP w.r.t. leverage	$\zeta_{sp,b}$	–	0.0530	–	0.0454
Mean hours worked	\bar{l}	-44.3261	-44.6400	-13.1391	-13.2586

Table 3: GDP variance decomposition

	Quarterly	Mixed-frequency
1-quarter ahead		
Technology, σ_z	9.83	23.24
Risk premium, σ_b	27.43	0.13
Price mark-up, σ_{λ_f}	1.75	42.60
Wage mark-up, σ_{λ_w}	1.67	14.31
Investment-specific, σ_{mu}	15.19	1.31
Government spending, σ_g	22.30	11.67
Monetary policy, σ_{r^m}	21.60	6.65
Risk, σ_{σ_ω}	0.24	0.11
4-quarter ahead		
Technology, σ_z	2.22	22.75
Risk premium, σ_b	31.62	0.03
Price mark-up, σ_{λ_f}	5.46	44.48
Wage mark-up, σ_{λ_w}	0.89	15.22
Investment-specific, σ_{mu}	27.57	5.85
Government spending, σ_g	10.86	8.70
Monetary policy, σ_{r^m}	21.25	1.51
Risk, σ_{σ_ω}	0.13	1.47
16-quarter ahead		
Technology, σ_z	2.67	25.46
Risk premium, σ_b	36.73	0.01
Price mark-up, σ_{λ_f}	5.41	43.17
Wage mark-up, σ_{λ_w}	2.38	16.56
Investment-specific, σ_{mu}	25.24	5.40
Government spending, σ_g	8.62	7.22
Monetary policy, σ_{r^m}	16.35	0.45
Risk, σ_{σ_ω}	2.59	1.72
Infinity		
Technology, σ_z	2.97	30.33
Risk premium, σ_b	39.36	0.02
Price mark-up, σ_{λ_f}	4.26	39.37
Wage mark-up, σ_{λ_w}	7.17	18.56
Investment-specific, σ_{mu}	19.74	4.08
Government spending, σ_g	9.49	6.06
Monetary policy, σ_{r^m}	12.74	0.26
Risk, σ_{σ_ω}	4.26	1.31

Table 4: Monte Carlo estimations

	"True" parameter	No financial frictions			"True" parameter	Financial frictions		
	value, no ff	M	MF	Q	value, ff	M	MF	Q
σ_z	0.200	0.230	0.127	1.424	0.213	0.261	0.197	2.023
σ_b	0.039	0.054	0.049	0.176	0.024	0.029	0.033	0.035
σ_{λ_f}	0.128	0.125	0.127	0.230	0.166	0.177	0.163	0.243
σ_{λ_w}	0.278	0.272	0.178	0.287	1.084	0.924	0.581	0.580
σ_{mu}	0.152	0.197	0.123	0.571	0.150	0.119	0.101	0.442
σ_g	1.781	1.698	1.479	2.357	1.869	1.846	1.762	2.776
σ_{r^m}	0.062	0.062	0.067	0.237	0.149	0.127	0.124	0.202
σ_{σ_ω}	–	–	–	–	0.014	0.016	0.016	0.082
ρ_z	0.999	0.999	0.985	0.982	0.997	0.996	0.985	0.917
ρ_b	0.941	0.929	0.922	0.445	0.992	0.981	0.983	0.979
ρ_{λ_f}	0.976	0.960	0.958	0.931	0.991	0.961	0.977	0.623
ρ_{λ_w}	0.985	0.972	0.966	0.730	0.996	0.993	0.994	0.987
ρ_{mu}	0.956	0.965	0.841	0.620	0.976	0.969	0.949	0.752
ρ_g	1.000	0.998	0.997	0.987	0.989	0.988	0.976	0.941
ρ_{r^m}	0.190	0.190	0.313	0.256	0.129	0.108	0.085	0.121
ρ_{σ_ω}	–	–	–	–	0.998	0.997	0.991	0.994
η_{λ_f}	0.980	0.968	0.967	0.961	0.097	0.086	0.154	0.563
η_{λ_w}	0.562	0.554	0.458	0.244	0.134	0.124	0.075	0.550
η_{gz}	0.922	0.793	0.702	0.138	0.895	0.742	0.532	0.159
ψ_1	1.699	1.268	1.534	0.978	1.360	1.326	1.320	1.084
ψ_2	0.263	0.141	0.201	0.066	-0.028	-0.028	-0.016	0.006
ψ_3	0.427	0.404	0.393	0.202	0.293	0.261	0.251	0.095
ρ_R	0.946	0.914	0.916	0.782	0.498	0.568	0.621	0.736
ζ_p	0.951	0.946	0.948	0.945	0.202	0.236	0.230	0.910
ζ_w	0.282	0.306	0.428	0.673	0.050	0.055	0.119	0.207
α	0.077	0.106	0.101	0.147	0.141	0.147	0.152	0.143
Φ_p	2.008	1.909	1.938	1.312	2.006	1.593	1.927	1.297
h	0.105	0.125	0.158	0.694	0.107	0.126	0.223	0.281
ν_l	6.725	6.603	5.569	0.456	6.743	6.515	6.459	0.411
ι_p	0.271	0.316	0.304	0.403	0.061	0.137	0.141	0.405
r^*	0.011	0.157	0.096	0.292	0.146	0.145	0.157	0.294
π^*	0.292	0.268	0.233	1.066	0.289	0.410	0.417	0.598
γ	0.154	0.153	0.151	0.466	0.137	0.134	0.138	0.421
S''	0.899	0.573	3.297	6.996	3.541	4.315	5.039	5.192
σ_c	1.276	0.774	0.996	1.522	0.398	0.343	0.247	0.776
ι_w	0.508	0.531	0.625	0.352	0.383	0.405	0.156	0.176
ψ	0.183	0.287	0.402	0.748	0.447	0.430	0.336	0.483
SP_*	–	–	–	–	0.081	0.073	0.088	1.737
$\zeta_{sp,b}$	–	–	–	–	0.045	0.042	0.043	0.057
\bar{l}	-13.139	-12.722	-13.515	-53.381	-13.259	-12.493	-12.390	-41.622

Figure 1: Prior (light gray) and posterior (black) distributions for the mixed-frequency baseline model with financial frictions

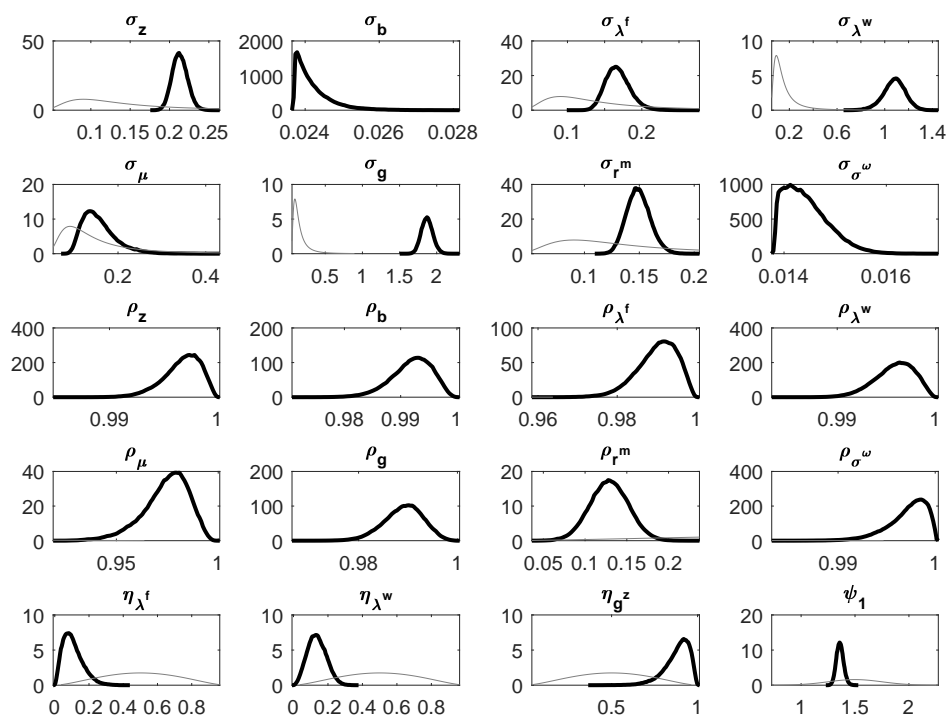


Figure 2: Prior (light gray) and posterior (black) distributions for the mixed-frequency baseline model with financial frictions

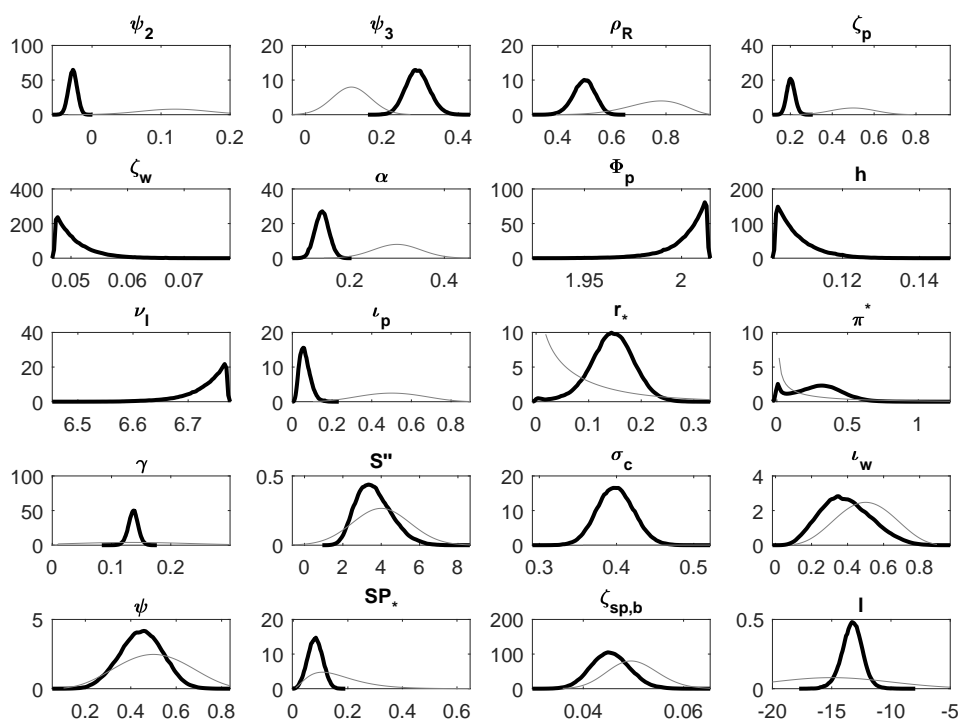


Figure 3: Brooks and Gelman (1998) convergence diagnostics for the mixed-frequency baseline model with financial frictions.

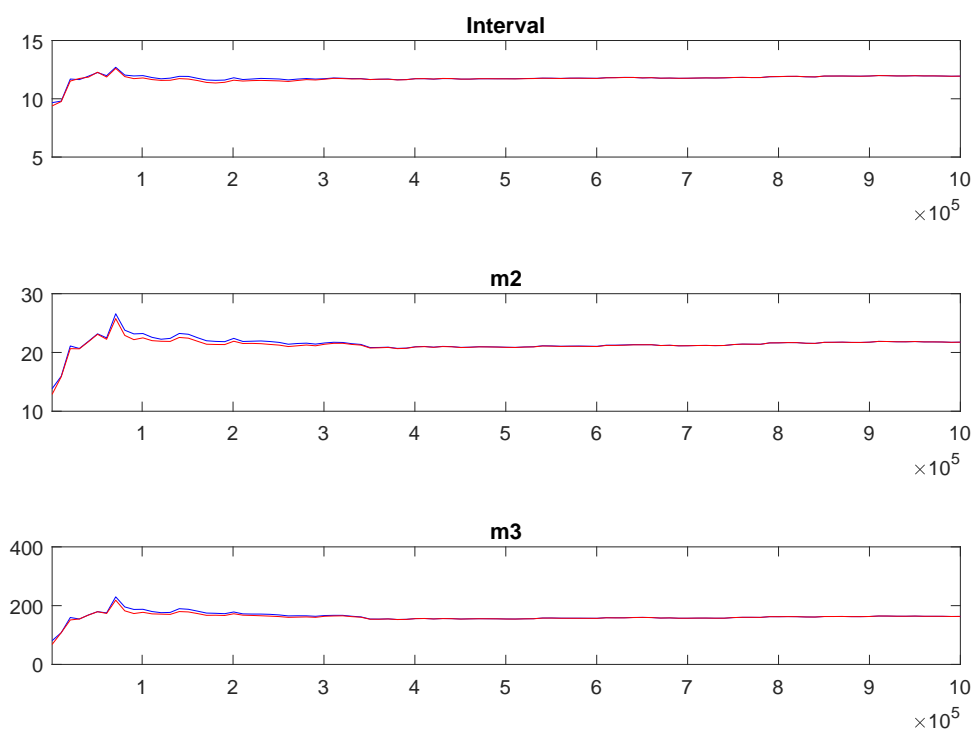


Figure 4: Monetary policy and investment-specific shocks: the financial accelerator.

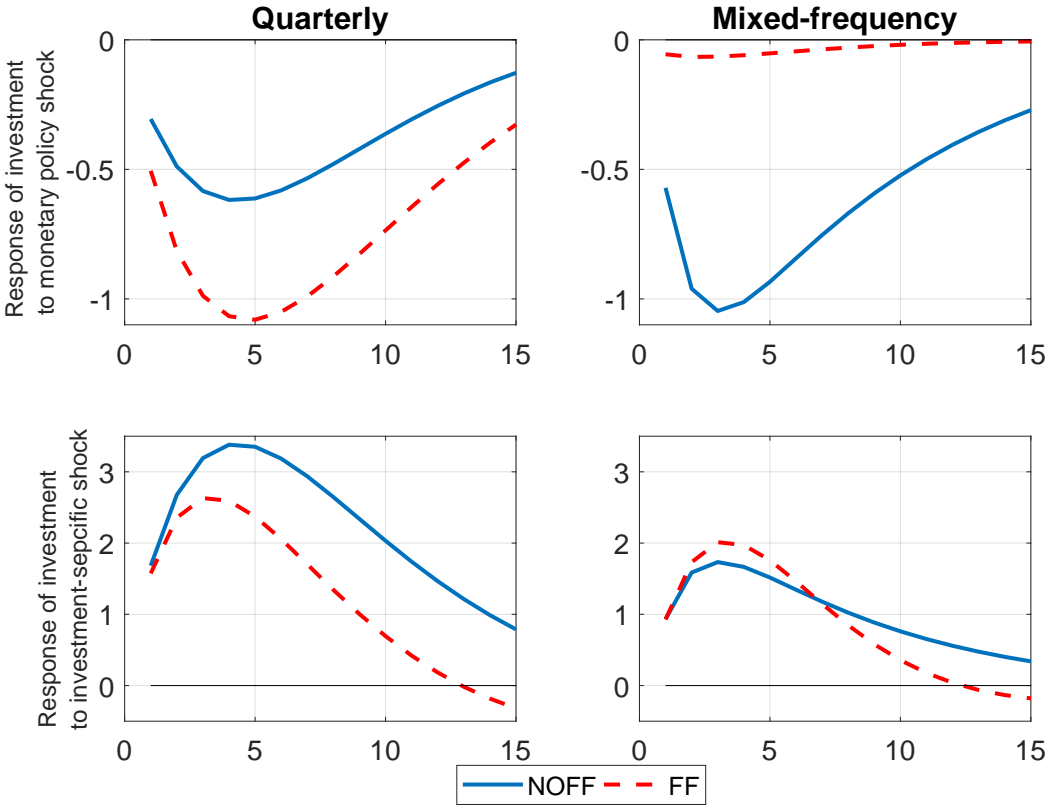


Figure 5: Monetary policy and investment-specific shocks: counterfactual accelerator for the quarterly model.

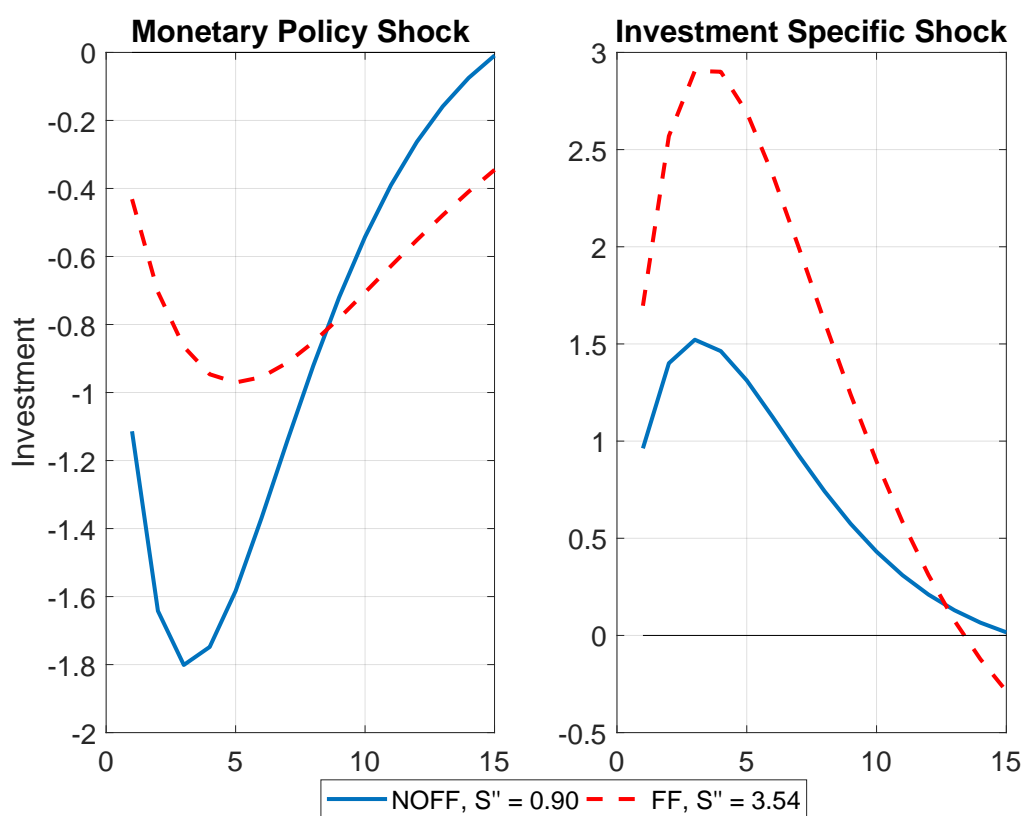


Figure 6: All shocks Bayesian IRFs

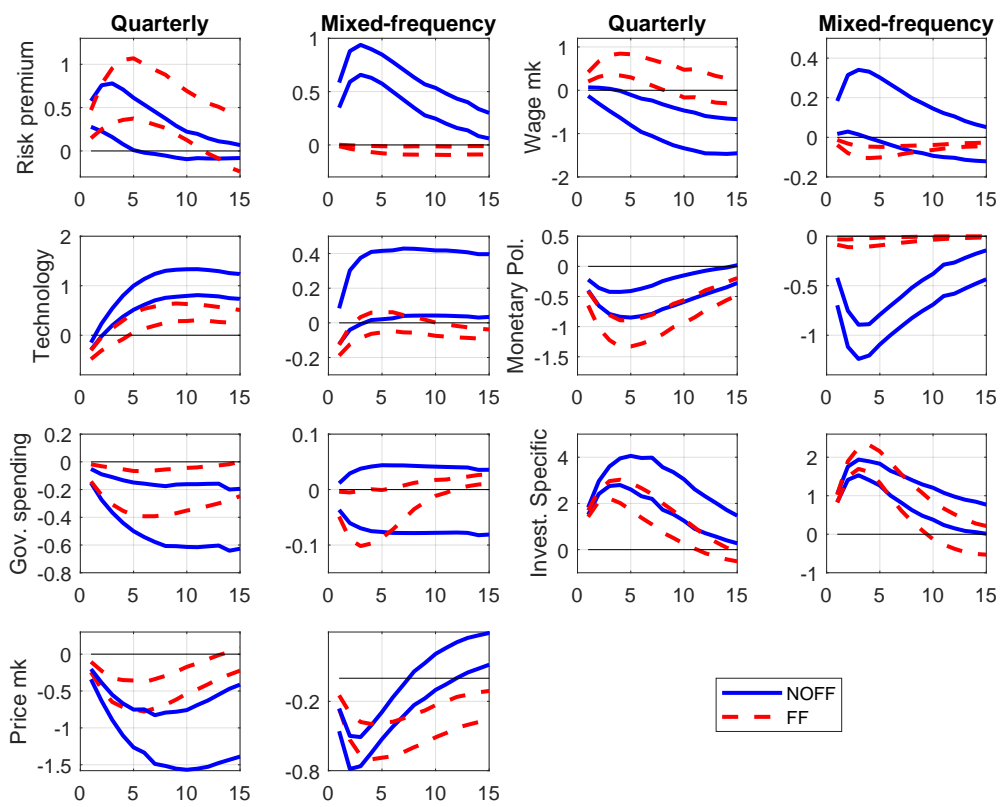
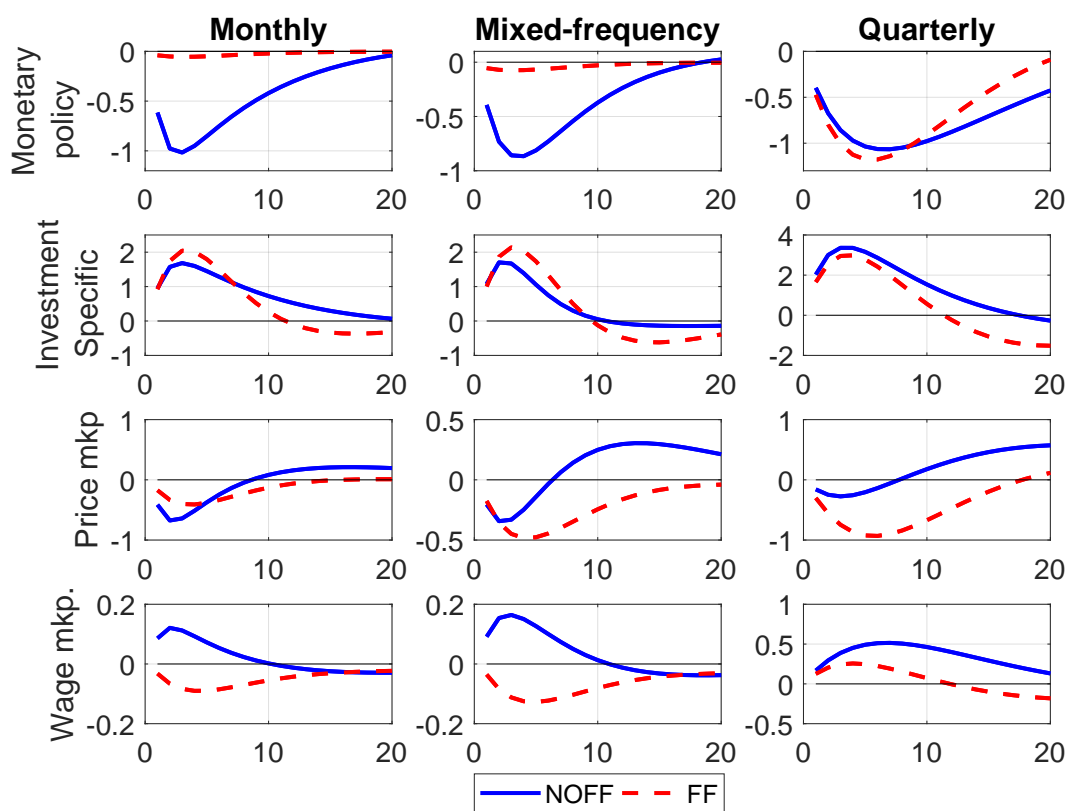


Figure 7: Monte Carlo impulse response functions. Response of investments to a selected set of shocks.



A Data

Most of the data are taken from Del Negro et al. (2015). For the quarterly model we use the exact same data as downloadable from the *American Economic Journal: Macroeconomics* Data Set. The only exception is the series for inflation. They use the GDP deflator (U.S. Bureau of Economic Analysis (BEA), Gross Domestic Product: Implicit Price Deflator [GDPDEF], retrieved from FRED, Federal Reserve Bank of St. Louis; <https://fred.stlouisfed.org/series/GDPDEF>) to compute it. Given our monthly estimation, we rely on the Personal Consumption Expenditures: Chain-type Price Index (U.S. BEA, Personal Consumption Expenditures: Chain-type Price Index [PCEPI], <https://fred.stlouisfed.org/series/PCEPI>). The remainder of the series: real GDP (U.S. BEA, Real Gross Domestic Product [GDPC1], <https://fred.stlouisfed.org/series/GDPC1>), nominal personal consumption expenditures (U.S. BEA, Personal Consumption Expenditures [PCEC], <https://fred.stlouisfed.org/series/PCEC>), nominal fixed private investment (U.S. BEA, Fixed Private Investment [FPI], <https://fred.stlouisfed.org/series/FPI>), average weekly hours of production and nonsupervisory employees for total private industries (U.S. Bureau of Labor Statistics (BLS), Average Weekly Hours of Production and Nonsupervisory Employees, Total Private [AWHNONAG], <https://fred.stlouisfed.org/series/AWHNONAG>), civilian employment (U.S. BLS, Employment Level [CE16OV], <https://fred.stlouisfed.org/series/CE16OV>), the civilian non-institutional population (U.S. BLS, Population Level [CNP16OV], <https://fred.stlouisfed.org/series/CNP16OV>) transformed in LNSINDEX, compensation per hour for the non-farm business sector (U.S. BLS, Nonfarm Business Sector: Compensation Per Hour [COMP-NFB], <https://fred.stlouisfed.org/series/COMP-NFB>), the annualized federal funds rate (Board of Governors of the Federal Reserve System (US), Effective Federal Funds Rate [FEDFUNDS], <https://fred.stlouisfed.org/series/FEDFUNDS>), and the annualized Moody's Seasoned Baa Corporate Bond Yield spread over the 10-Year Treasury Note Yield at Constant Maturity (Federal Reserve Bank of St. Louis, Moody's Seasoned Baa Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity [BAA10Y], <https://fred.stlouisfed.org/series/BAA10Y>). All data are transformed following Smets and Wouters (2007). Let Δ denote the temporal difference operator. Then:

$$\begin{aligned} \text{Output growth} &= 100\Delta\text{LN}(GDPC1/LNSINDEX) \\ \text{Consumption growth} &= 100\Delta\text{LN}((PCEC/GDPDEF)/LNSINDEX) \\ \text{Investment growth} &= 100\Delta\text{LN}((FPI/GDPDEF)/LNSINDEX) \\ \text{Real wage growth} &= 100\Delta\text{LN}(COMP-NFB/GDPDEF) \\ \text{Hours worked} &= 100\text{LN}((AWHNONAG * CE16OV/100)/LNSINDEX) \\ \text{Inflation} &= 100\Delta\text{LN}(PCEPI) \\ \text{FFR} &= (1/4) * \text{FEDERAL FUNDS RATE} \\ \text{Spread} &= (1/4) * (\text{BaaCorporate} - 10\text{yearTreasury}) \end{aligned}$$

For the monthly model

$$\begin{aligned} \text{FFR} &= (1/12) * \text{FEDERAL FUNDS RATE} \\ \text{Spread} &= (1/12) * (\text{BaaCorporate} - 10\text{yearTreasury}) \end{aligned}$$

B Monthly model measurement equations

In this Appendix we derive the measurement equations of the the variables observed only quarterly in the monthly model. We take output as an example, and the same holds for consumption, investments, and wages. What we observe is the quarterly value in levels, and this can be considered as the sum of an unobserved monthly output over the three months of the quarter:

$$Y_t^q = Y_t^m + Y_{t-1}^m + Y_{t-2}^m. \quad (3)$$

However, what we are finally interested in are the variables as they enter in the measurement equations. Therefore, we need to construct the measure for the log-linearized system and consider the growth trend explicitly. As already described in the paper, in our model all non-stationary variables are detrended by Z_t . For our purposes, then, let us define what we observe in the data in terms of growth rate of quarterly output

$$\text{Output growth} = \log(Y_t^{obs,q}) - \log(Y_{t-3}^{obs,q}),$$

which is observed every third month.

We can define output growth in the model. We recall that the variables in the model need to be defined as detrended, that is we define $\hat{Y}_t^q = \frac{Y_t^q}{Z_t^m}$. With this definition in mind, we can write:

$$\begin{aligned} \Delta \log(Y_t^q) &= \log(Y_t^q) - \log(Y_{t-3}^q) \\ &= \log(\hat{Y}_t^q Z_t^m) - \log(\hat{Y}_{t-3}^q Z_{t-3}^m) \\ &= y_t^q - y_{t-3}^q + \log\left(\frac{Z_t^m}{Z_{t-3}^m}\right) \\ &= y_t^q - y_{t-3}^q + \log\left(\frac{Z_t^m}{Z_{t-1}^m} \frac{Z_{t-1}^m}{Z_{t-2}^m} \frac{Z_{t-2}^m}{Z_{t-3}^m}\right) \\ &= y_t^q - y_{t-3}^q + z_t^m + z_{t-1}^m + z_{t-2}^m \end{aligned}$$

We need still one more step and define y_t^q . To do that, we start from equation (3), and combine it with the definition of the stationary variables $\hat{Y}_t^q = \frac{Y_t^q}{Z_t^m}$. We obtain that:

$$\hat{Y}_t^q Z_t^m = \hat{Y}_t^m Z_t^m + \hat{Y}_{t-1}^m Z_{t-1}^m + \hat{Y}_{t-2}^m Z_{t-2}^m,$$

and from here

$$\widehat{Y}_t^q = \widehat{Y}_t^m + \widehat{Y}_{t-1}^m \frac{Z_{t-1}^m}{Z_t^m} + \widehat{Y}_{t-2}^m \frac{Z_{t-2}^m}{Z_{t-1}^m} \frac{Z_{t-1}^m}{Z_t^m}$$

Linearizing around the steady state $\widehat{Y}^q = \widehat{Y}^m \left[1 + \frac{1}{e^\gamma} + \frac{1}{(e^\gamma)^2}\right]$, we obtain:

$$\widehat{Y}^q y_t^q = \widehat{Y}^m y_t^m + \frac{\widehat{Y}^m}{e^\gamma} (y_{t-1}^m - z_t^m) + \frac{\widehat{Y}^m}{(e^\gamma)^2} (y_{t-2}^m - z_t^m - z_{t-1}^m),$$

which can be rewritten as

$$\begin{aligned} y_t^q &= \frac{\widehat{Y}^m}{\widehat{Y}^q} y_t^m + \frac{\widehat{Y}^m}{\widehat{Y}^q e^\gamma} (y_{t-1}^m - z_t^m) + \frac{\widehat{Y}^m}{\widehat{Y}^q (e^\gamma)^2} (y_{t-2}^m - z_t^m - z_{t-1}^m), \\ &= \frac{1}{1 + \frac{1}{e^\gamma} + \frac{1}{(e^\gamma)^2}} y_t^m + \frac{1}{e^\gamma \left[1 + \frac{1}{e^\gamma} + \frac{1}{(e^\gamma)^2}\right]} (y_{t-1}^m - z_t^m) + \\ &\quad \frac{1}{(e^\gamma)^2 \left[1 + \frac{1}{e^\gamma} + \frac{1}{(e^\gamma)^2}\right]} (y_{t-2}^m - z_t^m - z_{t-1}^m). \end{aligned}$$

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The views stated herein are those of the authors and are not necessarily those of the Federal Reserve Bank of Cleveland or the Board of Governors of the Federal Reserve System, the European Central Bank and the Eurosystem.

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