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by Bartosz Maćkowiak[2]
and Mirko Wiederholt[3]

In 2009 all ECB publications feature a motif taken from the €200 banknote.

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Abstract

This paper presents a model in which price setting firms decide what to pay attention to, subject to a constraint on information flow. When idiosyncratic conditions are more variable or more important than aggregate conditions, firms pay more attention to idiosyncratic conditions than to aggregate conditions. When we calibrate the model to match the large average absolute size of price changes observed in micro data, prices react fast and by large amounts to idiosyncratic shocks, but prices react only slowly and by small amounts to nominal shocks. Nominal shocks have strong and persistent real effects. We use the model to investigate how the optimal allocation of attention and the dynamics of prices depend on the firms' environment.

Keywords: rational inattention, sticky prices, real effects of nominal shocks.

JEL Classification: E3, E5, D8.

Non-Technical Summary

Phelps (1970) proposed the idea that real effects of monetary policy are due to imperfect information. Lucas (1972) formalized this idea by assuming that agents observe the state of monetary policy with a delay. The Lucas model has been criticized on the grounds that information concerning monetary policy is published with little delay. However, Sims (2003) points out that, if agents cannot attend perfectly to all available information, there is a difference between publicly available information and the information actually reflected in agents' decisions. We think that a convincing model of real effects of monetary policy due to imperfect information must have two features. First, information concerning the current state of monetary policy must be publicly available. Second, it must be optimal for agents to pay little attention to this information. This paper develops a model with both features. The model helps explain micro and macro evidence on prices.

In the model, price setting firms decide what to pay attention to, subject to a constraint on information flow. When idiosyncratic conditions are more variable or more important than aggregate conditions, firms pay more attention to idiosyncratic conditions than to aggregate conditions. When we calibrate the model to match the large average absolute size of price changes observed in micro data, prices react fast and by large amounts to idiosyncratic shocks, but prices react only slowly and by small amounts to nominal shocks. Nominal shocks have strong and persistent real effects. We use the model to investigate how the optimal allocation of attention and the dynamics of prices depend on the firms' environment.

"An optimizing trader will process those prices of most importance to his decision problem most frequently and carefully, those of less importance less so, and most prices not at all. Of the many sources of risk of importance to him, the business cycle and aggregate behavior generally is, for most agents, of no special importance, and there is no reason for traders to specialize their own information systems for diagnosing general movements correctly." (Lucas, 1977, p. 21)

1 Introduction

Phelps (1970) proposed the idea that real effects of monetary policy are due to imperfect information. Lucas (1972) formalized this idea by assuming that agents observe the state of monetary policy with a delay. The Lucas model has been criticized on the grounds that information concerning monetary policy is published with little delay. However, Sims (2003) points out that, if agents cannot attend perfectly to all available information, there is a difference between publicly available information and the information actually reflected in agents' decisions. We think that a convincing model of real effects of monetary policy due to imperfect information must have two features. First, information concerning the current state of monetary policy must be publicly available. Second, it must be optimal for agents to pay little attention to this information. This paper develops a model with both features. The model helps explain micro and macro evidence on prices. The model can be used to study how the optimal allocation of attention and the dynamics of prices depend on the economic environment.

In the model price setting firms decide what to pay attention to. Firms' inability to attend perfectly to all available information is modeled as a constraint on information flow, as in Sims (2003). Firms can change prices every period at no cost. The profit-maximizing price depends on the aggregate price level, real aggregate demand and an idiosyncratic state variable reflecting firm-specific demand or cost conditions. Firms face a trade-off between paying attention to aggregate conditions and paying attention to idiosyncratic conditions. We close the model by specifying exogenous stochastic processes for nominal aggregate demand and the idiosyncratic state variables reflecting firm-specific conditions.

The model makes the following predictions. Firms adjust prices every period and yet impulse responses of prices to shocks are sticky – dampened and delayed relative to the impulse responses under perfect information. The extent of dampening and delay in a particular impulse response depends on the amount of attention allocated to that type of shock. When idiosyncratic conditions are more variable or more important than aggregate conditions, firms pay more attention to idiosyncratic conditions than to aggregate conditions. In this case, price responses to idiosyncratic shocks are strong and quick whereas price responses to aggregate shocks are dampened and delayed. In addition, there are feedback effects, because firms track endogenous aggregate variables (the price level and real aggregate demand). When firms pay limited attention to aggregate conditions, the price level responds less to a nominal shock than under perfect information. If prices are strategic complements, this implies that the profit-maximizing price responds less to a nominal shock. Firms find it optimal to pay even less attention to aggregate conditions. The price level responds even less and so on.

We calibrate the stochastic process for nominal aggregate demand using U.S. macro data. We calibrate the stochastic process for the idiosyncratic state variables so as to match the average absolute size of price changes in U.S. micro data. Bils and Klenow (2004) and Klenow and Kryvtsov (2005) study U.S. micro data that the Bureau of Labor Statistics collects to compute the consumer price index. Bils and Klenow (2004) find that half of all non-housing consumer prices last less than 4.3 months. Klenow and Kryvtsov (2005) find that, conditional on the occurrence of a price change, the average absolute size of the price change is over 13%. To match the large average absolute size of price changes in the data, idiosyncratic volatility in the model has to be one order of magnitude larger than aggregate volatility. This implies that firms allocate almost all attention to idiosyncratic conditions. Therefore prices respond strongly and quickly to idiosyncratic shocks, but prices respond only weakly and slowly to nominal shocks. Nominal shocks have strong and persistent real effects. The model can explain the combination of observations that individual prices move around a lot and, at the same time, the price level responds slowly to monetary policy shocks.[1] In fact, it is precisely the observation that individual prices move around a lot that generates the slow response of the price level.

We use the model to study how the optimal allocation of attention and the dynamics of prices depend on the firms' environment. When the variance of nominal aggregate demand increases, firms shift attention toward aggregate conditions and away from idiosyncratic conditions. Since firms allocate more attention to aggregate conditions, a given nominal shock has smaller real effects. However, the reallocation of attention is not large enough to compensate fully for the fact that the size of nominal shocks has increased. On average firms make larger absolute mistakes in tracking aggregate conditions and the variance of real aggregate demand increases. In addition, since firms allocate less attention to idiosyncratic conditions, firms also make larger mistakes in tracking idiosyncratic conditions. The prediction that real volatility increases when nominal shocks become larger differs markedly from the Lucas model.[2] At the same time, our model is consistent with the empirical finding of Lucas (1973) that the Phillips curve becomes steeper as the variance of nominal aggregate demand increases.

The model has some shortcomings. First, the model cannot explain why prices remain fixed for some time. In the model prices change every period. One could add a menu cost. It may be that reality is a combination of a menu cost model and the model presented here. Adding a menu cost is likely to increase the real effects of nominal shocks even further.[3] Second, in some models of price setting the optimal decision is so simple that it may be unclear why firms make mistakes at all. We think that in reality setting the profit-maximizing price is complicated. In this paper we start from the premise that setting the profit-maximizing price is complicated and we study the implications. We focus on the ten-

[1] A variety of different schemes for identifying monetary policy shocks yield the result that the price level responds slowly to monetary policy shocks. See, for example, Christiano, Eichenbaum and Evans (1999), Leeper, Sims and Zha (1996) and Uhlig (2005). Uhlig (2005) finds that only about 25% of the long-run response of the U.S. GDP deflator to a monetary policy shock occurs within the first year after the shock.

[2] In the Lucas model an increase in the variance of nominal aggregate demand implies that prices become more precise signals of nominal aggregate demand. Real volatility decreases.

[3] For menu cost models calibrated to micro data on prices see, for example, Gertler and Leahy (2006), Golosov and Lucas (2006), Midrigan (2006) and Nakamura and Steinsson (2007a).

sion between attending to aggregate conditions and attending to idiosyncratic conditions.[4] Third, it is difficult to calibrate the parameter that bounds the information flow. We do not provide independent evidence on the right value for this parameter. We choose a value for the parameter such that firms set prices that are close to the profit-maximizing prices. We think this is realistic.

This paper builds on Sims (1998, 2003). Sims argues that agents cannot attend perfectly to all available information and proposes to model this inability as a constraint on information flow. The firms' problem in our model is, after a log-quadratic approximation to the profit function, similar to the quadratic tracking problem with an information flow constraint studied in Section 4 of Sims (2003). One difference is that firms in our model face a trade-off between tracking aggregate conditions and tracking idiosyncratic conditions. Another difference is that firms in our model track endogenous variables. This introduces the feedback effects.[5]

This paper is also related to the recent literature on real effects of monetary policy due to imperfect information. Woodford (2002) studies a model in which firms observe nominal aggregate demand with exogenous idiosyncratic noise. Woodford assumes that firms pay little attention to aggregate conditions. In contrast, we identify the circumstances under which firms find it optimal to pay little attention to aggregate conditions and we study how the optimal allocation of attention and the dynamics of prices depend on the firms' environment.[6] Mankiw and Reis (2002) develop a different model in which information disseminates slowly. Mankiw and Reis assume that every period an exogenous fraction of firms obtains perfect information concerning all current and past disturbances, while all other firms continue to set prices based on old information. Reis (2006) shows that a model

[4] Zbaracki et al. (2004) provide some evidence consistent with the view that setting the profit-maximizing price is complicated. Zbaracki et al. study price adjustment practices of a large manufacturing firm. They find that price adjustment costs comprise 1.2% of the firm's revenues and 20.3% of the firm's net margin. Furthermore, they find that managerial costs of price adjustment ("thinking costs") are much larger than physical costs of price adjustment ("menu costs").

[5] Other papers that build on Sims (2003) include Luo (2006), Mondria (2006), Moscarini (2004) and Van Nieuwerburgh and Veldkamp (2006).

[6] Woodford's (2002) model has been extended in a number of directions. Adam (2005) studies optimal monetary policy. Hellwig (2002) studies the role of public information.

with a fixed cost of obtaining perfect information can provide a microfoundation for this kind of slow diffusion of information. Note that in Mankiw and Reis (2002) and Reis (2006) prices react with equal speed to all disturbances. In contrast, in our model prices react quickly to some shocks and slowly to other shocks. Therefore the model can explain the combination of observations that individual prices move around a lot and, at the same time, the price level responds slowly to monetary policy shocks.

The rest of the paper is organized as follows. Section 2 introduces the tools that we use to quantify information flow. Section 3 presents the model. Section 4 derives the firms' price setting behavior for a given allocation of attention. In Section 5 we solve a special case of the model analytically. Afterwards we return to the model in its general form. In Section 6 we study the firms' attention problem. In Section 7 we compute the rational expectations equilibrium. Section 8 contains extensions and discusses shortcomings. Section 9 concludes.

2 Quantifying information flow

In this section we present the tools from information theory that we use to quantify information flow.[7]

The basic idea of information theory is to quantify information as reduction in uncertainty. In information theory uncertainty is measured by entropy.[8] The entropy of a random variable X that has a normal distribution with variance σ^2 is

$$H(X) = \frac{1}{2}\log_2\left(2\pi e\sigma^2\right).$$

The entropy of a random vector $X^T = (X_1, ..., X_T)$ that has a multivariate normal distribution with covariance matrix Ω is

$$H\left(X^T\right) = \frac{1}{2}\log_2[(2\pi e)^T \det\Omega]. \tag{1}$$

In the univariate normal case, entropy is a function of the variance. In the multivariate normal case, entropy is a function of the number of random variables and their covariance

[7] See Cover and Thomas (1991) for a detailed exposition of information theory.

[8] Entropy as a measure of uncertainty can be derived from axioms. See Ash (1990). Moreover, entropy turns out to be the answer to a number of questions in communication theory and statistics. See Cover and Thomas (1991).

matrix. Entropy as a measure of uncertainty has appealing properties. For example, the entropy of a random vector with a given number of random variables and given variances is largest when the random variables are independent. Furthermore, when the random variables are independent, the entropy of the random vector equals the sum of the entropies of the individual random variables.[9]

In information theory conditional uncertainty is measured by conditional entropy. When $X^T = (X_1, ..., X_T)$ and $Y^T = (Y_1, ..., Y_T)$ have a multivariate normal distribution, then the conditional entropy of X^T given Y^T is

$$H\left(X^T|Y^T\right) = \frac{1}{2}\log_2\left[(2\pi e)^T \det \Omega_{X|Y}\right], \tag{2}$$

where $\Omega_{X|Y}$ is the conditional covariance matrix of X^T given Y^T.

Equipped with measures of uncertainty and conditional uncertainty one can quantify the amount of information that one random vector contains about another random vector as the difference between unconditional uncertainty and conditional uncertainty. For example, the amount of information that $Y^T = (Y_1, ..., Y_T)$ contains about $X^T = (X_1, ..., X_T)$ is

$$I\left(X^T; Y^T\right) = H\left(X^T\right) - H\left(X^T|Y^T\right). \tag{3}$$

This measure of information is called mutual information. The name derives from the fact that mutual information is symmetric: $I\left(X^T; Y^T\right) = I\left(Y^T; X^T\right)$.

One can also quantify the information flow between two stochastic processes as the average per period amount of information that one process contains about the other process. Let $X_1, ..., X_T$ and $Y_1, ..., Y_T$ denote the first T elements of the processes $\{X_t\}$ and $\{Y_t\}$, respectively. The information flow between the processes $\{X_t\}$ and $\{Y_t\}$ can be defined as

$$\mathcal{I}\left(\{X_t\};\{Y_t\}\right) = \lim_{T\to\infty}\frac{1}{T} I\left(X_1, ..., X_T; Y_1, ..., Y_T\right). \tag{4}$$

The processes $\{X_t\}$ and $\{Y_t\}$ can be vector processes. Two examples may help build intuition. First, if $\{X_t, Y_t\}$ is a bivariate Gaussian white noise process

$$\mathcal{I}\left(\{X_t\};\{Y_t\}\right) = \frac{1}{2}\log_2\left(\frac{1}{1-\rho^2_{X,Y}}\right), \tag{5}$$

[9] An alternative measure of uncertainty of a random vector is the determinant of the covariance matrix. This measure of uncertainty satisfies the first property but fails to satisfy the second property. For the second property to hold one needs to take the log.

where $\rho_{X,Y}$ is the correlation coefficient. See Appendix A. This example illustrates that information flow is invariant to scaling of the variables and is bounded below by zero. Second, if $\{X_t, Y_t\}$ is a bivariate stationary Gaussian process one can show that

$$\mathcal{I}\left(\{X_t\};\{Y_t\}\right) = \frac{1}{4\pi}\int_{-\pi}^{\pi} \log_2\left(\frac{1}{1-\mathcal{C}_{X,Y}(\omega)}\right) d\omega, \tag{6}$$

where $\mathcal{C}_{X,Y}(\omega)$ is the coherence between the processes $\{X_t\}$ and $\{Y_t\}$ at frequency ω.[10] This example illustrates that information flow takes into account comovement at all frequencies.

We will use the definition of information flow (4) to bound the amount of information that a decisionmaker can absorb per period.

3 The model

3.1 Description of the economy

Consider an economy with a continuum of firms indexed by $i \in [0,1]$. Time is discrete and indexed by t.

Firm i sells good i. Every period $t = 1, 2, \ldots$, the firm sets the price of the good, P_{it}, so as to maximize the expected discounted sum of profits

$$E_{it}\left[\sum_{\tau=t}^{\infty} \beta^{\tau-t}\pi\left(P_{i\tau}, P_\tau, Y_\tau, Z_{i\tau}\right)\right], \tag{7}$$

where E_{it} is the expectation operator conditioned upon information of firm i in period t, $\beta \in (0,1)$ is a discount factor, and $\pi\left(P_{it}, P_t, Y_t, Z_{it}\right)$ are real profits of the firm in period t. The real profits depend on the price set by the firm, P_{it}, the price level, P_t, real aggregate demand, Y_t, and an idiosyncratic state variable reflecting firm-specific demand or cost conditions, Z_{it}. We assume that the function π is twice continuously differentiable and homogenous of degree zero in its first two arguments, that is, real profits only depend on the relative price P_{it}/P_t. We also assume that π is a single-peaked function of P_{it} for given P_t, Y_t and Z_{it}.[11]

[10] See Cover and Thomas (1991), pp. 273-274, or Sims (2003).

[11] For example, in a standard model of monopolistic competition with Dixit-Stiglitz preferences

$$\pi\left(P_{it}, P_t, Y_t, Z_{it}\right) = Y_t\left(\frac{P_{it}}{P_t}\right)^{1-\theta} - C\left(Y_t\left(\frac{P_{it}}{P_t}\right)^{-\theta}, Y_t, Z_{it}\right),$$

Prices are physically fully flexible, that is, firms can change prices every period at no cost. Firms take as given the stochastic processes for the price level, $\{P_t\}$, real aggregate demand, $\{Y_t\}$, and the idiosyncratic state variables, $\{Z_{it}\}$. These assumptions imply that the price setting problem is a purely static problem:

$$\max_{P_{it}} E_{it}[\pi\left(P_{it}, P_t, Y_t, Z_{it}\right)]. \tag{8}$$

We specify the aggregate environment of firms by postulating an exogenous stochastic process for nominal aggregate demand.[12] Let

$$Q_t \equiv P_t Y_t \tag{9}$$

denote nominal aggregate demand and let $q_t \equiv \ln Q_t - \ln \bar{Q}$ denote the log-deviation of nominal aggregate demand from its deterministic trend. We assume that q_t follows a stationary Gaussian process with mean zero and absolutely summable autocovariances.

The price level is defined by

$$\ln P_t = \int_0^1 \ln P_{it} di. \tag{10}$$

One obtains the same equation in a standard model of monopolistic competition after a log-linearization.[13]

We specify the idiosyncratic environment of firms by postulating an exogenous stochastic process for the idiosyncratic state variables. Let $z_{it} \equiv \ln Z_{it} - \ln \bar{Z}$ denote the log-deviation of the idiosyncratic state variable i from its deterministic trend. We assume that the z_{it}, $i \in [0,1]$, follow a common stationary Gaussian process with mean zero and absolutely summable autocovariances. Furthermore, we assume that the processes $\{z_{it}\}$, $i \in [0,1]$, are

where Y_t is the consumption aggregator, P_t is the corresponding price index and $Y_t\left(P_{it}/P_t\right)^{-\theta}$ with $\theta > 1$ is the demand for good i. Costs of production C depend on the firm's output and may also depend on aggregate output through factor prices. In this example Z_{it} denotes firm-specific productivity. If $C_{11} \geq 0$ then π is a single-peaked function of P_{it} for given P_t, Y_t and Z_{it}.

[12] This approach is common in the literature. For example, Lucas (1972), Woodford (2002), Mankiw and Reis (2002) and Reis (2006) also postulate an exogenous stochastic process for nominal aggregate demand.

[13] In a standard model of monopolistic competition with Dixit-Stiglitz preferences the price level is defined by $P_t = \left(\int_0^1 P_{it}^{1-\theta} di\right)^{\frac{1}{1-\theta}}$. Log-linearizing this equation around any point with the property that all the P_{it} are equal yields equation (10).

pairwise independent and independent of $\{q_t\}$. Thus[14]

$$\int_0^1 z_{it} di = 0. \tag{11}$$

Next we formalize the idea that agents cannot attend perfectly to all available information. Following Sims (2003), we model attention as an information flow and we model the inability to attend perfectly to all available information as a constraint on information flow. Let s_{it} denote the signal that the decisionmaker in firm i receives in period t. This is the new information that the decisionmaker uses in period t. Let $s_i^t = \{s_i^1, s_{i2}, ..., s_{it}\}$ denote the sequence of all signals that the decisionmaker has received up to period t. The signal s_{it} can be vector valued. We place a bound on the flow of information:

$$\mathcal{I}\left(\{P_t, Z_{it}\};\{s_{it}\}\right) \leq \kappa. \tag{12}$$

The operator $\mathcal{I}$ measures the information flow between aggregate and idiosyncratic conditions (summarized by P_t and Z_{it}) and the signal s_{it}. The information flow constraint states that the average per period amount of information that the signal contains about economic conditions cannot exceed the parameter κ. Thus the decisionmaker can only absorb a limited amount of information every period.

We model the idea that decisionmakers can only observe and process a limited amount of information every period due to limited cognitive ability. We formalize this idea as a constraint on the information flow between aggregate and idiosyncratic conditions (summarized by P_t and Z_{it}) and the signal s_{it}. There are several alternative formulations of the information flow constraint that yield the same equilibrium. First, instead of including the price level in the information flow constraint we could have included any other macro variable (or any set of macro variables) in the information flow constraint. This yields the same equilibrium. We prove this result below. The reason is that all aggregate variables are driven by the same innovations – the innovations to nominal aggregate demand. Second, instead of restricting the information content of the signal we could have also restricted directly the information content of the price setting behavior. This also yields the same equilibrium. We show this below. The only reason why we decided to think of price setting

[14] See Uhlig (1996), Theorem 2.

behavior as based on signals is that this facilitates comparison of our model to the large literature on models with an exogenous information structure.[15]

We think that information flow is a good reduced form description of the mental resources required to take good decisions. When information flow is large (κ is high) the price setting behavior is close to the profit-maximizing price setting behavior. When the decisionmaker allocates a large fraction of the information flow (his/her attention) to one variable, mistakes in the response to that variable become small. The decisionmaker needs to allocate more information flow to a variable with high variance or low serial correlation (for a given variance) in order to make small mistakes in the response to that variable.

We let firms choose the profit-maximizing allocation of attention. Formally, in period zero firm i solves

$$\max_{\{s_{it}\}\in\Gamma} E\left[\sum_{t=1}^{\infty}\beta^t \pi\left(P_{it}^*, P_t, Y_t, Z_{it}\right)\right], \tag{13}$$

subject to the information flow constraint (12) and

$$P_{it}^* = \arg\max_{P_{it}} E[\pi\left(P_{it}, P_t, Y_t, Z_{it}\right)|s_i^t]. \tag{14}$$

The firm chooses the stochastic process for the signal so as to maximize the expected discounted sum of profits. The firm has to respect the information flow constraint (12). The firm takes into account how the signal process affects its price setting behavior (14). For example, the firm knows that if it pays no attention to idiosyncratic conditions it will not respond to changes in idiosyncratic conditions.[16]

The firm can choose the stochastic process for the signal from the set Γ. The set Γ is the set of all signal processes that have the following four properties. First, the signal that firm i receives in period t contains no information about future innovations to nominal aggregate demand and future innovations to the idiosyncratic state variable, that is, the signal contains no information about shocks that nature has not drawn yet. Second, the

[15]See for example the papers cited in Footnote 6, the literature on the social value of information (e.g. Morris and Shin (2002)), the literature on global games (e.g. Morris and Shin (2003)) and the literature on forecasting the forecasts of others (e.g. Townsend (1983)).

[16]Here we assume that the decisionmaker chooses the signal process once and for all. In Section 8.3 we let the decisionmaker reconsider the choice of the signal process.

signal follows a stationary Gaussian process:

$$\{s_{it}, p_t, q_t, z_{it}\} \text{ is a stationary Gaussian process,} \tag{15}$$

where p_t denotes the log-deviation of the price level from its deterministic trend. We relax the Gaussianity assumption in Section 8.1. We show that Gaussian signals are optimal when the objective function is quadratic and we also study the optimal form of uncertainty when the objective function is not quadratic. Third, the signal that firm i receives in period t is a vector that can be partitioned into one subvector that only contains information about aggregate conditions and another subvector that only contains information about idiosyncratic conditions:

$$s_{it} = (s_{1it}, s_{2it}), \tag{16}$$

where

$$\{s_{1it}, p_t, q_t\} \text{ and } \{s_{2it}, z_{it}\} \text{ are independent.} \tag{17}$$

This assumption formalizes the idea that paying attention to aggregate conditions and paying attention to idiosyncratic conditions are separate activities. For example, attending to the price level or the state of monetary policy is a separate activity from attending to market-specific conditions. We relax this assumption in Section 8.2. Fourth, all noise in signals is idiosyncratic. This assumption accords well with the idea that the constraint is the decisionmakers' limited attention rather than the availability of information.[17]

Finally, we make a simplifying assumption. We assume that each firm receives a long sequence of signals in period one,

$$s_i^1 = \{s_{i-\infty}, \ldots, s_{i1}\}. \tag{18}$$

This assumption implies that the price set by each firm follows a stationary process. This simplifies the analysis.[18]

[17] Conditions (15) and (17) can only be satisfied when $\{p_t, q_t\}$ is a stationary Gaussian process and $\{p_t, q_t\}$ and $\{z_{it}\}$ are independent. We will verify that this is true in equilibrium.

[18] One can show that receiving a long sequence of signals in period one does not change the information flow in (12).

3.2 Equilibrium

An equilibrium of the model are stochastic processes for the signals, $\{s_{it}\}$, the prices, $\{P_{it}\}$, the price level, $\{P_t\}$, and real aggregate demand, $\{Y_t\}$, such that:

1. Given $\{P_t\}$, $\{Y_t\}$ and $\{Z_{it}\}$, each firm $i \in [0,1]$ chooses the stochastic process for the signal optimally in period $t = 0$ and sets the price for its good according to equation (14) in periods $t = 1, 2, \ldots$.

2. In every period $t = 1, 2, \ldots$ and in every state of nature, the price level satisfies (10) and real aggregate demand satisfies (9).

4 Price setting behavior

In this section, we derive the firms' price setting behavior for a given allocation of attention. In the following sections, we study the optimal allocation of attention. We work with a log-quadratic approximation to the profit function around the non-stochastic solution of the model. This yields a log-linear equation for the profit-maximizing price and a log-quadratic equation for the loss in profits due to a suboptimal price.

We start by deriving the non-stochastic solution of the model. Suppose that $Q_t = \bar{Q}$ for all t and $Z_{it} = \bar{Z}$ for all i, t. In this case, the price that firm i sets in period t is given by

$$\pi_1 \left(P_{it}, P_t, Y_t, \bar{Z}\right) = 0,$$

where π_1 denotes the derivative of the profit function π with respect to its first argument. Since all firms set the same price, in equilibrium

$$\pi_1 \left(P_t, P_t, Y_t, \bar{Z}\right) = 0.$$

Multiplying by $P_t > 0$ and using the fact that π is homogenous of degree zero in its first two arguments yields[19]

$$\pi_1 \left(1, 1, Y_t, \bar{Z}\right) = 0.$$

[19] It follows from Euler's theorem that π_1 is homogenous of degree minus one in its first two arguments.

The last equation characterizes equilibrium real aggregate demand, denoted $\bar{Y}$.[20] The equilibrium price level equals

$$\bar{P} = \frac{\bar{Q}}{\bar{Y}}.$$

Next we take a log-quadratic approximation to the profit function around the non-stochastic solution of the model. Let $x_t \equiv \ln X_t - \ln \bar{X}$ denote the log-deviation of a variable from its value at the non-stochastic solution. Let $\hat{\pi}$ denote the profit function expressed in terms of log-deviations, that is, $\hat{\pi}(p_{it}, p_t, y_t, z_{it}) = \pi\left(\bar{P}e^{p_{it}}, \bar{P}e^{p_t}, \bar{Y}e^{y_t}, \bar{Z}e^{z_{it}}\right)$. Let $\tilde{\pi}$ denote the second-order Taylor approximation to the profit function $\hat{\pi}$ at the origin

$$\begin{aligned}
\tilde{\pi}(p_{it}, p_t, y_t, z_{it}) \quad = \quad & \hat{\pi}(0,0,0,0) + \hat{\pi}_1 p_{it} + \hat{\pi}_2 p_t + \hat{\pi}_3 y_t + \hat{\pi}_4 z_{it} \\
& + \frac{\hat{\pi}_{11}}{2} p_{it}^2 + \frac{\hat{\pi}_{22}}{2} p_t^2 + \frac{\hat{\pi}_{33}}{2} y_t^2 + \frac{\hat{\pi}_{44}}{2} z_{it}^2 \\
& + \hat{\pi}_{12} p_{it} p_t + \hat{\pi}_{13} p_{it} y_t + \hat{\pi}_{14} p_{it} z_{it} \\
& + \hat{\pi}_{23} p_t y_t + \hat{\pi}_{24} p_t z_{it} + \hat{\pi}_{34} y_t z_{it}, \qquad (19)
\end{aligned}$$

where $\hat{\pi}_1$, for example, denotes the derivative of the profit function $\hat{\pi}$ with respect to its first argument evaluated at the origin. It is straightforward to show that $\hat{\pi}_1 = 0$, $\hat{\pi}_{11} < 0$ and $\hat{\pi}_{12} = -\hat{\pi}_{11}$.

After the log-quadratic approximation to the profit function, the price that firm i sets in period t is given by[21]

$$p_{it}^* = E\left[p_{it}^{\diamondsuit} | s_i^t\right], \qquad (20)$$

where $p_{it}^{\diamondsuit}$ denotes the profit-maximizing price of good i in period t

$$p_{it}^{\diamondsuit} = p_t + \frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|} y_t + \frac{\hat{\pi}_{14}}{|\hat{\pi}_{11}|} z_{it}. \qquad (21)$$

The price that the firm sets equals the conditional expectation of the profit-maximizing price. The profit-maximizing price is log-linear in the price level, real aggregate demand and the idiosyncratic state variable. The ratio $(\hat{\pi}_{13}/|\hat{\pi}_{11}|)$ determines the sensitivity of the profit-maximizing price to real aggregate demand. A low value of $(\hat{\pi}_{13}/|\hat{\pi}_{11}|)$ corresponds to

[20] Here we assume that this equation has a unique solution. For the profit function given in Footnote 11, a sufficient condition is $C_{11} + C_{12} > 0$.

[21] Set the derivative of $E\left[\tilde{\pi}(p_{it}, p_t, y_t, z_{it}) | s_i^t\right]$ with respect to p_{it} equal to zero and solve for p_{it}. Recall that $\hat{\pi}_1 = 0$, $\hat{\pi}_{11} < 0$ and $\hat{\pi}_{12} = -\hat{\pi}_{11}$. This yields equation (20).

a high degree of real rigidity. See Ball and Romer (1990). The ratio $(\hat{\pi}_{14}/|\hat{\pi}_{11}|)$ determines the sensitivity of the profit-maximizing price to idiosyncratic conditions.

Next we introduce some notation by stating the price-setting equations (20)-(21) as

$$p_{it}^{*} = \hat{\Delta}_{it} + \frac{\hat{\pi}_{14}}{|\hat{\pi}_{11}|}\hat{z}_{it}, \tag{22}$$

$$p_{it}^{\diamondsuit} = \Delta_t + \frac{\hat{\pi}_{14}}{|\hat{\pi}_{11}|} z_{it}, \tag{23}$$

where $\Delta_t \equiv p_t + (\hat{\pi}_{13}/|\hat{\pi}_{11}|)\, y_t$ denotes the profit-maximizing response to aggregate conditions and $\hat{\Delta}_{it} \equiv E\left[\Delta_t|s_i^t\right]$ and $\hat{z}_{it} \equiv E\left[z_{it}|s_i^t\right]$ denote conditional expectations.

Whenever the price (20) differs from the profit-maximizing price (21) there is a loss in profits due to a suboptimal price. The period t loss in profits due to a suboptimal price equals

$$\tilde{\pi}\left(p_{it}^{\diamondsuit}, p_t, y_t, z_{it}\right) - \tilde{\pi}\left(p_{it}^{*}, p_t, y_t, z_{it}\right) = \frac{|\hat{\pi}_{11}|}{2}\left(p_{it}^{\diamondsuit} - p_{it}^{*}\right)^2. \tag{24}$$

The allocation of attention affects the price (20) and thereby the loss (24).

If firms face no information flow constraint, all firms set the profit-maximizing price. Computing the integral over all i of the profit-maximizing price (21) and using $y_t = q_t - p_t$ as well as equation (11) yields the following price level

$$p_t^{\diamondsuit} = \left(1 - \frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}\right) p_t + \frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|} q_t. \tag{25}$$

The fixed point of this mapping is the equilibrium price level in the absence of an information flow constraint. Assuming $\hat{\pi}_{13} \neq 0$, the unique fixed point is

$$p_t^{\diamondsuit} = q_t. \tag{26}$$

Hence, if firms face no information flow constraint, the price level moves one-for-one with nominal aggregate demand.

5 Analytical solution when exogenous processes are white noise

Next we study the optimal allocation of attention and we derive the rational expectations equilibrium of the model. When q_t and z_{it} follow white noise processes the model can be solved analytically. In this section we illustrate the main mechanisms of the model with the help of this simple example. Afterwards we solve the model under more realistic assumptions concerning the exogenous processes.

In this section, we assume that q_t follows a white noise process with variance $\sigma_q^2 > 0$ and all the z_{it}, $i \in [0,1]$, follow a white noise process with variance $\sigma_z^2 > 0$. We guess that the equilibrium price level is a log-linear function of nominal aggregate demand

$$p_t = \alpha q_t. \tag{27}$$

The guess will be verified. Furthermore, for ease of exposition, we immediately restrict the firms' choice of signals to signals of the form "true state plus white noise error":

$$s_{1it} = \Delta_t + \varepsilon_{it}, \tag{28}$$

$$s_{2it} = z_{it} + \psi_{it}, \tag{29}$$

where $\Delta_t = p_t + (\hat{\pi}_{13}/|\hat{\pi}_{11}|)\, y_t$ is the profit-maximizing response to aggregate conditions and $\{\varepsilon_{it}\}$ and $\{\psi_{it}\}$ are idiosyncratic Gaussian white noise processes that are mutually independent and independent of $\{\Delta_t\}$ and $\{z_{it}\}$. Here we use a result that we prove in Section 6: When Δ_t and z_{it} follow white noise processes, signals of the form "true state plus white noise error" are optimal. See Proposition 4.[22]

Since the price level and the idiosyncratic state variable follow white noise processes and the signals have the form (28)-(29), the information flow constraint (12) reduces to

$$\frac{1}{2}\log_2\left(\frac{\sigma_\Delta^2}{\sigma_\varepsilon^2}+1\right)+\frac{1}{2}\log_2\left(\frac{\sigma_z^2}{\sigma_\psi^2}+1\right)\leq\kappa. \tag{30}$$

See Appendix B. Let $\kappa_1 = \frac{1}{2}\log_2\left(\left(\sigma_\Delta^2/\sigma_\varepsilon^2\right)+1\right)$ denote the information flow concerning aggregate conditions and let $\kappa_2 = \frac{1}{2}\log_2\left(\left(\sigma_z^2/\sigma_\psi^2\right)+1\right)$ denote the information flow concerning idiosyncratic conditions. A given allocation of attention (a pair κ_1 and κ_2 with

[22] Since $\Delta_t = \alpha q_t + (\hat{\pi}_{13}/|\hat{\pi}_{11}|)(1-\alpha)\, q_t$ one can make the signal (28) a signal concerning nominal aggregate demand, real aggregate demand or the price level by multiplying the signal with a constant. This yields a new signal that is associated with the same information flow, the same conditional expectation of Δ_t and the same price setting behavior.

$\kappa_1 + \kappa_2 \leq \kappa$) is associated with the following signal-to-noise ratios

$$\frac{\sigma_\Delta^2}{\sigma_\varepsilon^2} = 2^{2\kappa_1} - 1, \tag{31}$$

$$\frac{\sigma_z^2}{\sigma_\psi^2} = 2^{2\kappa_2} - 1. \tag{32}$$

When the information flow constraint is binding, firms face a trade-off: attending more carefully to aggregate conditions requires attending less carefully to idiosyncratic conditions.

Signals (28)-(29) with signal-to-noise ratios (31)-(32) result in the following price setting behavior

$$\begin{aligned} p_{it}^* &= \frac{\sigma_\Delta^2}{\sigma_\Delta^2 + \sigma_\varepsilon^2} s_{1it} + \frac{\hat{\pi}_{14}}{|\hat{\pi}_{11}|} \frac{\sigma_z^2}{\sigma_z^2 + \sigma_\psi^2} s_{2it} \\ &= \left(1 - 2^{-2\kappa_1}\right)(\Delta_t + \varepsilon_{it}) + \frac{\hat{\pi}_{14}}{|\hat{\pi}_{11}|}\left(1 - 2^{-2\kappa_2}\right)(z_{it} + \psi_{it}). \end{aligned} \tag{33}$$

This price setting behavior is associated with the following expected discounted sum of losses in profits due to suboptimal pricing

$$\begin{aligned} & E\left[\sum_{t=1}^{\infty} \beta^t \left\{\tilde{\pi}\left(p_{it}^{\diamondsuit}, p_t, y_t, z_{it}\right) - \tilde{\pi}\left(p_{it}^*, p_t, y_t, z_{it}\right)\right\}\right] \\ &= \sum_{t=1}^{\infty} \beta^t \frac{|\hat{\pi}_{11}|}{2} E\left[\left(p_{it}^{\diamondsuit} - p_{it}^*\right)^2\right] \\ &= \frac{\beta}{1-\beta} \frac{|\hat{\pi}_{11}|}{2} \left\{2^{-2\kappa_1}\sigma_\Delta^2 + \left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 2^{-2\kappa_2}\sigma_z^2\right\}. \end{aligned} \tag{34}$$

The first equality follows from (24). The second equality follows from (21) and (31)-(33).

When a firm chooses the allocation of attention (a pair κ_1 and κ_2 with $\kappa_1 + \kappa_2 \leq \kappa$) the firm trades off losses in profits due to imperfect tracking of aggregate conditions and losses in profits due to imperfect tracking of idiosyncratic conditions. The optimal allocation of attention is the solution to

$$\min_{\kappa_1 \in [0,\kappa]} \frac{\beta}{1-\beta} \frac{|\hat{\pi}_{11}|}{2} \left\{2^{-2\kappa_1}\sigma_\Delta^2 + \left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 2^{-2(\kappa-\kappa_1)}\sigma_z^2\right\}. \tag{35}$$

Assuming $\hat{\pi}_{14} \neq 0$, the unique solution for the attention allocated to aggregate conditions is

$$\kappa_1^* = \begin{cases} \kappa & \text{if } x \geq 2^{2\kappa} \\ \frac{1}{2}\kappa + \frac{1}{4}\log_2(x) & \text{if } x \in \left[2^{-2\kappa}, 2^{2\kappa}\right] \\ 0 & \text{if } x \leq 2^{-2\kappa} \end{cases}, \tag{36}$$

where $x \equiv \sigma_{\Delta}^2 / \left((\hat{\pi}_{14}/\hat{\pi}_{11})^2 \sigma_z^2 \right)$. The attention allocated to aggregate conditions is increasing in the ratio x – the variance of the profit-maximizing price due to aggregate shocks divided by the variance of the profit-maximizing price due to idiosyncratic shocks. See equation (23). The implications are straightforward. When idiosyncratic conditions are more variable or more important than aggregate conditions, firms pay more attention to idiosyncratic conditions than to aggregate conditions. In this case, the price (33) reacts strongly to idiosyncratic shocks but only weakly to aggregate shocks. This can explain why individual prices move around a lot and, at the same time, individual prices react little to nominal shocks.

Computing the integral over all i of the price (33) yields the price level under rational inattention

$$p_t^* = \left(1 - 2^{-2\kappa_1^*}\right) \Delta_t. \tag{37}$$

The equilibrium price level under rational inattention is the fixed point of the mapping between the guess (27) and the actual law of motion (37). Assuming $\hat{\pi}_{13} > 0$, the unique fixed point is

$$p_t^* = \begin{cases} \frac{\left(2^{2\kappa}-1\right)\frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}}{1+\left(2^{2\kappa}-1\right)\frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}} q_t & \text{if } \lambda \geq 2^{-\kappa} + \left(2^{\kappa} - 2^{-\kappa}\right)\frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|} \\ \left(1 - 2^{-\kappa}\lambda^{-1}\right) q_t & \text{if } \lambda \in \left[2^{-\kappa}, 2^{-\kappa} + \left(2^{\kappa} - 2^{-\kappa}\right)\frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}\right] \\ 0 & \text{if } \lambda \leq 2^{-\kappa} \end{cases}, \tag{38}$$

where $\lambda \equiv (\hat{\pi}_{13}\sigma_q / |\hat{\pi}_{14}| \sigma_z)$.[23] The response of the equilibrium price level to a nominal shock is increasing in the ratio λ. This ratio determines the optimal allocation of attention and the strength of feedback effects. For example, when idiosyncratic conditions become more variable, firms shift attention toward idiosyncratic conditions and away from aggregate conditions. This implies that prices respond less to changes in aggregate conditions and therefore the price level responds less to a nominal shock. In addition, there are feedback effects, because the profit-maximizing price depends on endogenous variables. Recall that the profit-maximizing response to aggregate conditions is given by $\Delta_t = p_t + (\hat{\pi}_{13}/|\hat{\pi}_{11}|)\, y_t$, which can be expressed as $\Delta_t = p_t + (\hat{\pi}_{13}/|\hat{\pi}_{11}|)(q_t - p_t)$. When firms pay limited attention to aggregate conditions, the price level responds less to a nominal shock than under perfect

[23]The derivation of equation (38) is in the Technical Appendix.

information while real aggregate demand responds more to a nominal shock than under perfect information. This changes the profit-maximizing response to a nominal shock. In particular, if prices are strategic complements, that is $(\hat{\pi}_{13}/|\hat{\pi}_{11}|) < 1$, the profit-maximizing response to a nominal shock decreases. Firms find it optimal to pay even less attention to aggregate conditions. The price level responds even less and so on. The feedback effects are stronger the smaller is $(\hat{\pi}_{13}/|\hat{\pi}_{11}|)$, that is, the higher is the degree of real rigidity.

When λ is very small, firms allocate no attention to aggregate conditions and the price level equals its deterministic trend in every period. In contrast, when λ is very large, firms allocate all attention to aggregate conditions. Note that there is always a unique linear rational expectations equilibrium.[24]

6 The firms' attention problem

Next we show how to solve the model when q_t and z_{it} follow arbitrary stationary Gaussian processes. In this section we study the optimal allocation of attention for a given process for the price level. In the next section we compute the rational expectations equilibrium.

In this section, we guess that the equilibrium price level follows a stationary Gaussian process that is driven only by the innovations to nominal aggregate demand

$$p_t = \sum_{l=0}^{\infty} \alpha_l \nu_{t-l}, \tag{39}$$

where the sequence $\{\alpha_l\}_{l=0}^{\infty}$ is absolutely summable and ν_t denotes the time t innovation to nominal aggregate demand, which follows a Gaussian white noise process. The guess (39) will be verified in the next section.

[24] There are similarities between this model and the setup studied in the literature on the social value of information. The price set by a firm is a linear function of the conditional expectation of nominal aggregate demand (an exogenous aggregate variable) and the conditional expectation of the price level (the average action of other firms). We solve for a linear equilibrium by making a guess concerning the price level process and by verifying this guess. This resembles the solution procedure in Section I.C in Morris and Shin (2002) and in Angeletos and Pavan (2006). Note that the price set by a firm can be expressed as a weighted average of first-order beliefs about q_t and p_t or as a weighted average of higher-order beliefs about q_t. Computing higher-order beliefs can be useful to show that the linear equilibrium is the unique equilibrium (see Section I.D in Morris and Shin (2002)) and to interpret equilibrium (see Section 3 in Woodford (2002)).

Firms choose the allocation of attention so as to maximize the expected discounted sum of profits (13) subject to the information flow constraint (12). The following two lemmata allow us to simplify the objective function and the constraint.

Lemma 1 *Let the profit function be given by (19). Suppose that (39) holds. Then*

$$E\left[\sum_{t=1}^{\infty}\beta^t\pi\left(P_{it}^*,P_t,Y_t,Z_{it}\right)\right]=E\left[\sum_{t=1}^{\infty}\beta^t\pi\left(P_{it}^\diamondsuit,P_t,Y_t,Z_{it}\right)\right]-\frac{\beta}{1-\beta}\frac{|\hat{\pi}_{11}|}{2}E\left[\left(p_{it}^\diamondsuit-p_{it}^*\right)^2\right]. \tag{40}$$

Proof. See Appendix C. ■

Profits at any price equal profits at the profit-maximizing price minus losses in profits due to a suboptimal price. When the profit function is given by (19), the price setting behavior is given by (20)-(21) and the loss in profits due to a suboptimal price is given by (24). Using equation (24) and the stationarity of the prices (20)-(21) yields Lemma 1. Furthermore, using equations (22)-(23) and the independence assumption (17) yields

$$E\left[\left(p_{it}^\diamondsuit-p_{it}^*\right)^2\right]=E\left[\left(\Delta_t-\hat{\Delta}_{it}\right)^2\right]+\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2E\left[\left(z_{it}-\hat{z}_{it}\right)^2\right]. \tag{41}$$

The mean squared error in price setting behavior equals the mean squared error in the response to aggregate conditions plus the mean squared error in the response to idiosyncratic conditions.

Lemma 2 *Suppose that (39) holds. Then*

$$\begin{aligned}\mathcal{I}\left(\{P_t,Z_{it}\};\{s_{it}\}\right) &= \mathcal{I}\left(\{p_t\};\{s_{1it}\}\right)+\mathcal{I}\left(\{z_{it}\};\{s_{2it}\}\right) && (42)\\ &\geq \mathcal{I}\left(\{p_t\};\left\{\hat{\Delta}_{it}\right\}\right)+\mathcal{I}\left(\{z_{it}\};\{\hat{z}_{it}\}\right) && (43)\\ &= \mathcal{I}\left(\{\Delta_t\};\left\{\hat{\Delta}_{it}\right\}\right)+\mathcal{I}\left(\{z_{it}\};\{\hat{z}_{it}\}\right). && (44)\end{aligned}$$

If s_{1it} and s_{2it} are scalars, inequality (43) holds with equality.

Proof. See Appendix D. ■

Equation (42) states that the information flow in (12) equals the information flow concerning aggregate conditions plus the information flow concerning idiosyncratic conditions. This follows from the independence assumption (17). Inequality (43) states that the signals contain weakly more information than the conditional expectations computed from the signals. Equation (44) states that the information flow between the p_t process and the $\hat{\Delta}_{it}$ process equals the information flow between the Δ_t process and the $\hat{\Delta}_{it}$ process. The reason is that all aggregate variables and all linear combinations of aggregate variables are driven by the same innovations – the innovations to nominal aggregate demand.

We solve the firms' attention problem by a two-step procedure that follows from Lemma 1, equation (41) and Lemma 2. In the first step we solve directly for the optimal price setting behavior subject to an information flow constraint on the price setting behavior. In the second step we solve for optimal signals.

Proposition 1 *Let the profit function be given by (19). Suppose that (39) holds. A signal process obtained by the following two-step procedure solves the attention problem (12)-(14).*

1. *Derive the optimal price setting behavior subject to an information flow constraint:*

$$\min_{\{\hat{\Delta}_{it},\hat{z}_{it}\}} \left\{ E\left[\left(\Delta_t - \hat{\Delta}_{it}\right)^2\right] + \left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 E\left[\left(z_{it} - \hat{z}_{it}\right)^2\right] \right\}, \tag{45}$$

   subject to

$$\mathcal{I}\left(\{\Delta_t\};\left\{\hat{\Delta}_{it}\right\}\right) + \mathcal{I}\left(\{z_{it}\};\{\hat{z}_{it}\}\right) \leq \kappa, \tag{46}$$

$$\left\{\Delta_t, \hat{\Delta}_{it}, z_{it}, \hat{z}_{it}\right\} \text{ is a stationary Gaussian process,} \tag{47}$$

$$\left\{\Delta_t, \hat{\Delta}_{it}\right\} \text{ and } \{z_{it}, \hat{z}_{it}\} \text{ are independent.} \tag{48}$$

2. *Denote the solution to step one by* $\left\{\hat{\Delta}_{it}^*, \hat{z}_{it}^*\right\}$. *Show that there exists a bivariate signal process* $\{s_{1it}, s_{2it}\} \in \Gamma$ *with the property*

$$\hat{\Delta}_{it}^* = E\left[\Delta_t | s_{1i}^t\right], \tag{49}$$

$$\hat{z}_{it}^* = E\left[z_{it} | s_{2i}^t\right]. \tag{50}$$

Proof. See Appendix E. ■

Step one consists of solving directly for the optimal price setting behavior, subject to a constraint on the information flow between the profit-maximizing price setting behavior (determined by Δ_t and z_{it}) and the actual price setting behavior (determined by $\hat{\Delta}_{it}$ and $\hat{z}_{it}$). See equations (22)-(23). The objective function (45) follows from Lemma 1 and equation

(41). The information flow constraint (46) is weaker than the information flow constraint (12). See Lemma 2. Step two consists of showing that there exist univariate signals that yield the solution to step one as conditional expectations. The requirement that the signals s_{1it} and s_{2it} are scalars ensures that the inequality (43) holds with equality. Then (46) implies (12).

For a given allocation of attention (a pair κ_1 and κ_2 with $\kappa_1 + \kappa_2 \leq \kappa$), the problem in step one is a collection of two problems of the form studied in Section 4 of Sims (2003):

$$\min_{b,c} E\left[(X_t - Y_t)^2\right], \tag{51}$$

subject to

$$X_t = \sum_{l=0}^{\infty} a_l u_{t-l}, \tag{52}$$

$$Y_t = \sum_{l=0}^{\infty} b_l u_{t-l} + \sum_{l=0}^{\infty} c_l \epsilon_{t-l}, \tag{53}$$

$$\mathcal{I}\left(\{X_t\}; \{Y_t\}\right) \leq \kappa_j, \tag{54}$$

where the sequences $\{a_l\}_{l=0}^{\infty}$, $\{b_l\}_{l=0}^{\infty}$ and $\{c_l\}_{l=0}^{\infty}$ are absolutely summable and u_t and ϵ_t follow independent Gaussian white noise processes with unit variance. The decisionmaker chooses a process for Y_t to track X_t. Equations (52)-(53) state that (X_t, Y_t) has to follow a stationary Gaussian process. Equation (54) restricts the information flow between X_t and Y_t. Here X_t and Y_t stand for Δ_t and $\hat{\Delta}_{it}$ or z_{it} and $\hat{z}_{it}$. In the first case, κ_j equals the information flow allocated to aggregate conditions. In the second case, κ_j equals the information flow allocated to idiosyncratic conditions. There are two differences between the problem studied in Section 4 of Sims (2003) and the firms' attention problem. First, the decisionmaker who has to set a price faces a multidimensional tracking problem. He or she has to decide how to allocate the available information flow across the problem of tracking aggregate conditions and the problem of tracking idiosyncratic conditions. Furthermore, the decisionmaker tracks an endogenous variable – the profit-maximizing response to aggregate conditions, $\Delta_t = p_t + (\hat{\pi}_{13}/|\hat{\pi}_{11}|)\, y_t$. This introduces the feedback effects.

In the next section we implement the two-step solution procedure given in Proposition 1 numerically. In the rest of this section we present analytical results.

Proposition 2 *A solution to the problem (51)-(54) satisfies:*

$$E\left[X_t - Y_t\right] = 0, \tag{55}$$

and, for all $k = 0, 1, 2, \ldots,$

$$E\left[\left(X_t - Y_t\right) Y_{t-k}\right] = 0. \tag{56}$$

Proof. See Appendix F. ■

The quadratic Gaussian tracking problem with an information flow constraint yields a solution that looks like behavior based on noisy observations: errors are zero on average and errors are orthogonal to current and past behavior. Thus step one of Proposition 1 yields price setting behavior that looks like price setting behavior based on noisy observations.

When Δ_t and z_{it} follow first-order autoregressive processes the firms' attention problem can be solved analytically. The next proposition characterizes the price setting behavior for a given allocation of attention. The following equation characterizes the optimal allocation of attention.

Proposition 3 *If*

$$X_t = \rho X_{t-1} + a u_t \tag{57}$$

with $\rho \in [0, 1)$ *then the following process is a solution to the problem (51)-(54):*[25]

$$Y_t^* = \sum_{l=0}^{\infty} \left(\rho^l - \frac{1}{2^{2\kappa_j}} \left(\frac{\rho}{2^{2\kappa_j}} \right)^l \right) a u_{t-l} + \sum_{l=0}^{\infty} \sqrt{\frac{1}{2^{2\kappa_j}} \frac{2^{2\kappa_j} - 1}{2^{2\kappa_j} - \rho^2}} \left(\frac{\rho}{2^{2\kappa_j}} \right)^l a \epsilon_{t-l}. \tag{58}$$

The value of the objective function at the solution equals

$$E\left[\left(X_t - Y_t^*\right)^2\right] = \sigma_X^2 \frac{1 - \rho^2}{2^{2\kappa_j} - \rho^2}. \tag{59}$$

Proof. See Appendix G. ■

The response to an innovation in X_t (that is Δ_t or z_{it}) is either hump-shaped or monotonically decreasing. This follows from the fact that the impulse response function is a difference between two exponentially decaying series.[26] See equation (58). Second, the mean squared

[25] If $\rho = 0$ we use the convention $0^0 = 1$.

[26] Specifically, the optimal response is hump-shaped if κ_j is less than $-\left(1/2 \ln 2\right) \ln\left(\left(1 - \rho\right)/\rho\right)$; in the opposite case, the optimal response is monotonically decreasing.

error is decreasing in the information flow allocated to that tracking problem, increasing in the variance of the variable being tracked and decreasing in the persistence of the variable being tracked (holding constant the variance of the variable being tracked). See equation (59). Third, the marginal value of information flow is decreasing in the information flow allocated to that tracking problem, increasing in the variance of the variable being tracked and may be increasing or decreasing in the persistence of the variable being tracked (holding constant the variance of the variable being tracked). This follows from differentiating (59) with respect to κ_j.

In the case of an interior solution, the optimal allocation of attention has the property that the marginal value of information flow concerning aggregate conditions equals the marginal value of information flow concerning idiosyncratic conditions. Equating the two and using $\kappa_1 + \kappa_2 = \kappa$ yields

$$\kappa_1^* = \frac{1}{2}\log_2\left(\frac{\frac{\sigma_\Delta}{\sigma_z}\sqrt{\frac{1-\rho_\Delta^2}{1-\rho_z^2}}2^\kappa + \frac{|\hat{\pi}_{14}|}{|\hat{\pi}_{11}|}\rho_\Delta^2}{\frac{|\hat{\pi}_{14}|}{|\hat{\pi}_{11}|} + \frac{\sigma_\Delta}{\sigma_z}\sqrt{\frac{1-\rho_\Delta^2}{1-\rho_z^2}}\rho_z^2 2^{-\kappa}}\right). \tag{60}$$

Equations (58) and (60) characterize the solution to step one of Proposition 1 in the AR(1) case. We still need to show that there exist univariate signals that yield the solution to step one as conditional expectations. It turns out that in the AR(1) case signals of the form "true state plus white noise error" have this property.

Proposition 4 *Let Y_t^* denote the process given by equation (58). The signal*

$$S_t = X_t + \sqrt{\frac{2^{2\kappa_j}}{\left(2^{2\kappa_j}-1\right)\left(2^{2\kappa_j}-\rho^2\right)}}a\epsilon_t \tag{61}$$

has the property

$$Y_t^* = E\left[X_t|S^t\right]. \tag{62}$$

Proof. See Appendix H. ■

Finally, there are many alternative formulations of the firms' attention problem that yield the same equilibrium. First, equation (44) holds for any aggregate variable (not only for p_t). We prove this at the end of Appendix D. The reason is that all aggregate variables are driven by the same innovations – the innovations to nominal aggregate demand. Hence, instead of including the price level in the information flow constraint we could have included any other macro variable in the information flow constraint.[27] Second, we solve the firms' attention problem by a two-step procedure. In the first step we solve directly for the optimal price setting behavior subject to an information flow constraint on the price setting behavior. Thus we could have also restricted from the start the information content of the price setting behavior.

7 Numerical solutions when exogenous processes are serially correlated

In this section we compute the rational expectations equilibrium for a variety of parameter values. We solve the model numerically, because in equilibrium the profit-maximizing response to aggregate conditions, $\Delta_t = p_t + (\hat{\pi}_{13}/|\hat{\pi}_{11}|)\, y_t$, in general does not follow an AR(1) process.[28] We compute the solution as follows. First, we make a guess concerning the process for the price level. Second, we solve the firms' attention problem by the two-step procedure given in Proposition 1. We solve directly for the optimal price setting behavior subject to an information flow constraint on the price setting behavior. This is a standard constrained minimization problem. The first-order conditions are given in Appendix I. We

[27] Including a set of macro variables in the information flow constraint (12) also yields the same equilibrium. For example, in an earlier version of the paper we included both P_t and Y_t instead of just P_t in the information flow constraint. We prefer the new formulation of the information flow constraint, because it simplifies some of the proofs without changing the equilibrium of the model. To see that both formulations of the information flow constraint yield the same equilibrium, note the following. In the earlier version of the paper we proved that $\mathcal{I}\left(\{P_t, Y_t, Z_{it}\}; \{s_{it}\}\right) \geq \mathcal{I}\left(\{\Delta_t\}; \left\{\hat{\Delta}_{it}\right\}\right) + \mathcal{I}\left(\{z_{it}\}; \{\hat{z}_{it}\}\right)$, with equality if (i) s_{1it} and s_{2it} are scalars and (ii) $s_{1it} = \Delta_t + \varepsilon_{it}$ where $\{\varepsilon_{it}\}$ is an error process. We solved the model by a two-step procedure that resembles the procedure given in Proposition 1. There was only one difference. In step two we had to show that there exists a signal of the form $s_{1it} = \Delta_t + \varepsilon_{it}$ that has the property (49). We continue to verify that such a signal exists to ensure that the two formulations of the information flow constraint yield the same equilibrium. For example, in the AR(1) case this signal is given by Proposition 4.

[28] There is one notable exception. If $(\hat{\pi}_{13}/|\hat{\pi}_{11}|) = 1$ the profit-maximizing response to aggregate conditions equals q_t. In this case, if q_t and z_{it} follow AR(1) processes, the equilibrium of the model can be computed directly from equations (22), (58), (60) and (61). Note that with $(\hat{\pi}_{13}/|\hat{\pi}_{11}|) = 1$ there are no feedback effects.

then show that there exist univariate signals with the property (49)-(50). Third, we compute the price level from equations (10) and (22). We compare the process for the price level that we obtain to our guess and we update the guess until a fixed point is reached.

7.1 The benchmark economy

In order to solve the model numerically, we must specify the exogenous process for nominal aggregate demand, the exogenous process for the idiosyncratic state variables and the parameters $(\hat{\pi}_{14}/|\hat{\pi}_{11}|)$, $(\hat{\pi}_{13}/|\hat{\pi}_{11}|)$ and κ. See Table 1 for the specification of the benchmark economy.

We calibrate the stochastic process for nominal aggregate demand using quarterly U.S. nominal GNP data from 1959:1 to 2004:1.[29] We take the natural log of the data and we detrend the data by fitting a second-order polynomial in time. We then estimate the equation $q_t = \rho q_{t-1} + \nu_t$, where q_t is the deviation of the log of nominal GNP from its fitted trend. The estimate of ρ that we obtain is, after rounding off, 0.95 and the standard deviation of the error term is 0.01. The moving average representation of the estimated process is $q_t = \sum_{l=0}^{\infty} \rho^l \nu_{t-l}$. Since with geometric decay shocks die out after a very large number of periods and computing time is fast increasing with the number of lags, we approximate the estimated process by a process that dies out after twenty periods: $q_t = \sum_{l=0}^{20} a_l \nu_{t-l}$, $a_0 = 1$ and $a_l = a_{l-1} - 0.05$, for $l = 1, ..., 20$.[30]

We calibrate the stochastic process for the idiosyncratic state variables so as to match the average absolute size of price changes in U.S. micro data. Bils and Klenow (2004) find that the median price changes every 4.3 months. Klenow and Kryvtsov (2005) find that, conditional on the occurrence of a price change, the average absolute size of the price change is 13.3%. These statistics are computed including price changes related to sales. When Klenow and Kryvtsov (2005) and Nakamura and Steinsson (2007b) exclude sales, both papers find that the average absolute size of price changes is 8.5%. We choose the standard deviation of z_{it} such that the average absolute size of price changes in our model equals 8.5% under perfect information. This yields a standard deviation of z_{it} that is ten

[29] The source is the National Income and Product Accounts of the United States.

[30] For the benchmark parameter values, we also solved the model with geometric decay and 80 lags, $q_t = \sum_{l=0}^{80} \rho^l \nu_{t-l}$. While computing time was many times larger, the results were affected little.

times the standard deviation of q_t. It is unclear whether one should exclude sales or not. We prefer to match the smaller of the two statistics reported in the literature only because this amounts to taking a conservative approach.[31] For now, we abstract from the fact that in the data prices remain fixed for longer than a quarter, whereas in our model prices change every quarter. Later we take into account that this change in frequency may affect the estimated size of idiosyncratic shocks for a given observed size of price changes. Finally, in the benchmark economy we assume the same rate of decay in the z_{it} process as in the q_t process.

The ratio $(\hat{\pi}_{14}/|\hat{\pi}_{11}|)$ determining the sensitivity of the profit-maximizing price to the idiosyncratic state variable has the same effects on equilibrium as the variance of the idiosyncratic state variable. Therefore we normalize $(\hat{\pi}_{14}/|\hat{\pi}_{11}|)$ to one and we only choose the variance of the idiosyncratic state variable.

The ratio $(\hat{\pi}_{13}/|\hat{\pi}_{11}|)$ determining the sensitivity of the profit-maximizing price to real aggregate demand is a standard parameter in models with monopolistic competition. Woodford (2003), chapter 3, recommends a value between 0.1 and 0.15. In the benchmark economy we set $(\hat{\pi}_{13}/|\hat{\pi}_{11}|) = 0.15$ and later we show how changes in $(\hat{\pi}_{13}/|\hat{\pi}_{11}|)$ affect the solution.

We choose the parameter that bounds the information flow such that firms set prices that are close to the profit-maximizing prices. Based on this reasoning we set $\kappa = 3$ bits.[32] The following calculations illustrate $\kappa = 3$ bits. Allocating 1, 2 and 3 bits of information flow to the problem of tracking a Gaussian white noise process yields a ratio of posterior variance to prior variance of 1/4, 1/16 and 1/64, respectively. Tracking autocorrelated processes is easier. Allocating 1, 2 and 3 bits of information flow to the problem of tracking a Gaussian AR(1) process with $\rho = 0.95$ yields a ratio of posterior variance to prior variance of 1/32, 1/155 and 1/647, respectively. These numbers follow from equation (59). Thus with $\kappa = 3$ bits the available information flow is large enough to track both aggregate and idiosyncratic conditions well. This implies that decisionmakers set prices that are close

[31] Matching an average absolute size of price changes of 13.3% instead of 8.5% would require a larger standard deviation of z_{it}. Matching a given average absolute size of price changes under rational inattention instead of under perfect information also would require a larger standard deviation of z_{it}.

[32] Information flow is measured in bits. This is explained in Sims (2003).

to the profit-maximizing prices. Therefore losses in profits due to suboptimal price setting behavior are small and the marginal value of information flow is low. Hence, decisionmakers have little incentive to increase the information flow.

To set the parameter that bounds the information flow one cannot query oneself about the information processing capacity of humans in the real world and endow decisionmakers in the model with the same capacity. This is because economic models are drastic simplifications of the real world. For example, in our model decisionmakers take no decision apart from the price-setting decision and they only need to track one firm-specific variable. One has to choose the parameter that bounds the information flow taking into account the simplicity of the model. We choose the parameter such that firms in the model do very well.

Table 1 and Figures 1-2 summarize the results for the benchmark economy. The average absolute size of price changes under rational inattention is 8.2%. Firms allocate 94% of their attention to idiosyncratic conditions. The optimal allocation of attention implies the following price setting behavior. Figure 1 shows the impulse response of an individual price to an innovation in the idiosyncratic state variable. Comparing the response under rational inattention (the line with squares) to the response under perfect information (the line with points), we see that the response to an idiosyncratic shock under rational inattention is almost as strong and quick as under perfect information. The line with crosses is the impulse response of an individual price to noise in the signal concerning idiosyncratic conditions.

Figure 2 shows the impulse response of an individual price to an innovation in nominal aggregate demand. The response to a nominal shock under rational inattention (the line with squares) is dampened and delayed relative to the response under perfect information (the line with points). The line with crosses in Figure 2 is the impulse response of an individual price to noise in the signal concerning aggregate conditions. Since the effect of idiosyncratic noise washes out in the aggregate, the line with squares is also the impulse response of the price level to an innovation in nominal aggregate demand. The price level under rational inattention responds weakly and slowly to a nominal shock. The reasons are as follows. First, to match the large average absolute size of price changes in the data, idiosyncratic volatility has to be one order of magnitude larger than aggregate volatility.

This implies that firms allocate almost all attention to idiosyncratic conditions. Therefore individual prices and the price level respond little to nominal shocks. Second, the profit-maximizing response to a nominal shock depends on the price setting behavior of other firms. Recall that $\Delta_t = p_t + (\hat{\pi}_{13}/|\hat{\pi}_{11}|)\, y_t$ is the profit-maximizing response to aggregate conditions. When all firms set the profit-maximizing price, the price level moves one-for-one with nominal aggregate demand and Δ_t equals q_t. In contrast, when firms face an information flow constraint, the price level moves less than one-for-one with nominal aggregate demand and this changes the profit-maximizing response to a nominal shock. The line with triangles in Figure 2 shows the impulse response of Δ_t to an innovation in nominal aggregate demand at the rational inattention fixed point. The feedback effects imply that the rational inattention equilibrium is far away from the perfect information equilibrium despite the fact that firms track the profit-maximizing response to aggregate conditions well.

The impulse response of real aggregate demand to an innovation in nominal aggregate demand equals the difference between the perfect information impulse response in Figure 2 and the rational inattention impulse response in Figure 2. It is apparent that nominal shocks have strong and persistent real effects.

Figures 3-4 show simulated price series. Figure 3 shows a sequence of prices set by an individual firm under rational inattention (crosses) and the sequence of profit-maximizing prices (diamonds). Since we have chosen a high value for κ, firms track the profit-maximizing price very well. For an individual firm, the ratio of posterior variance to prior variance of the profit-maximizing price is 1/300.[33] Therefore losses in profits due to suboptimal price setting behavior are small and the marginal value of information flow is low.[34] Figure 4

[33] Formally, $E\left[\left(p_{it}^{\diamondsuit} - p_{it}^{*}\right)^2\right] / E\left[\left(p_{it}^{\diamondsuit}\right)^2\right]$ equals 1/300.

[34] For the profit function and the parameter values given in Section 8.1 we have $|\hat{\pi}_{11}| = 27\bar{Y}$, which yields an expected per period loss in profits due to imperfect tracking of aggregate conditions equal to $0.0005\bar{Y}$ and an expected per period loss in profits due to imperfect tracking of idiosyncratic conditions equal to $0.0028\bar{Y}$. See equation (24) and Table 1. The marginal value of information flow equals $0.004\bar{Y}$. Here we assume a price elasticity of demand of 7. Thus a 10 percent price change induces a 70 percent change in demand. Assuming a smaller price elasticity of demand yields even smaller losses in profits and an even smaller marginal value of information flow.

shows sequences of aggregate price levels. The equilibrium price level under rational inattention (crosses) differs markedly from the equilibrium price level under perfect information (diamonds). The reason is the optimal allocation of attention in combination with the feedback effects. To illustrate that firms make fairly small mistakes in tracking the equilibrium price level, Figure 4 also shows the conditional expectation of the price level at the rational inattention fixed point (points).

The early New Keynesian literature emphasized that changes in real activity can be an order of magnitude larger than losses of individual firms. See, for example, Akerlof and Yelen (1985). We obtain a similar result in our model. The rational inattention equilibrium is far away from the perfect information equilibrium, despite the fact that losses in profits due to suboptimal price setting behavior are small. Firms do not take into account how their attention affects aggregate variables.

In the benchmark economy, prices react strongly and quickly to idiosyncratic shocks, but prices react only weakly and slowly to nominal shocks. Therefore the model can explain the combination of observations that individual prices move around a lot and, at the same time, the price level responds slowly to monetary policy shocks. The model is also consistent with the finding by Boivin, Giannoni and Mihov (2006) that sectoral prices respond quickly to sector-specific shocks and slowly to monetary policy shocks.

We turn to examining how changes in parameter values affect the optimal allocation of attention and the dynamics of the economy.

7.2 Varying parameter values

Table 2 and Figure 5 show how changes in idiosyncratic volatility affect the solution. When the variance of the idiosyncratic state variables increases, firms shift attention toward idiosyncratic conditions and away from aggregate conditions. Therefore the response of the price level to a nominal shock becomes more dampened and delayed. The model makes a prediction about how prices in different sectors respond to nominal shocks: If the degree of real rigidity is the same across sectors, firms operating in more volatile sectors respond more slowly to nominal shocks.

Table 2 and Figure 6 illustrate the effects of a large increase in the variance of nominal aggregate demand to a level one may expect in an economy with high and variable inflation. When the variance of nominal aggregate demand increases, firms shift attention toward aggregate conditions and away from idiosyncratic conditions. Since firms allocate more attention to aggregate conditions, a given nominal shock has smaller real effects. However, the reallocation of attention is not large enough to compensate fully for the fact that the size of nominal shocks has increased. On average firms make larger absolute mistakes in tracking aggregate conditions and the variance of real aggregate demand increases. In addition, since firms allocate less attention to idiosyncratic conditions, firms also make larger mistakes in tracking idiosyncratic conditions. The prediction that real volatility increases when nominal shocks become larger differs markedly from the Lucas model. At the same time, our model is consistent with the empirical finding of Lucas (1973) that the Phillips curve becomes steeper as the variance of nominal aggregate demand increases.

The predictions described above would continue to hold in a model with an endogenous κ. Suppose that firms can choose the information flow facing an increasing, strictly convex cost function, $C(\kappa)$. Consider again the effects of increasing the variance of nominal aggregate demand. Since the marginal value of information flow about aggregate conditions increases, firms choose a higher κ and the marginal cost of information flow increases. This implies that the marginal value of information flow about both aggregate and idiosyncratic conditions has to increase. Firms track both aggregate and idiosyncratic conditions less well.

Reducing the persistence of nominal aggregate demand (holding constant the variance of nominal aggregate demand) has an unambiguous effect on the quality of tracking and an ambiguous effect on the allocation of attention. Firms track the profit-maximizing price less well and therefore losses in profits due to suboptimal price setting behavior increase. This suggests that there is a payoff from "interest rate smoothing" by central banks. The attention allocated to aggregate conditions may increase or decrease, because reducing the persistence of nominal aggregate demand makes firms track aggregate conditions less well (for a given allocation of attention), but also lowers the improvement in tracking that can be achieved by reallocating attention to aggregate conditions.[35] These numerical findings

[35] When we changed the persistence of the idiosyncratic state variables (holding constant the variance of the

are consistent with the analytical results in the AR(1) case. See Proposition 3.

Table 2 and Figure 7 illustrate how the ratio $(\hat{\pi}_{13}/|\hat{\pi}_{11}|)$ affects the solution. When the profit-maximizing price becomes less sensitive to real aggregate demand, the response of the price level to a nominal shock becomes more dampened and delayed. The reason is that the feedback effects become stronger.

Figure 8 illustrates how changes in κ affect the solution. With $\kappa = 3$ nominal shocks have real effects for about 19 quarters. With $\kappa = 4$ and $\kappa = 5$ nominal shocks have real effects for about 7 quarters and 5 quarters, respectively. With $\kappa = 2$ firms allocate all attention to idiosyncratic conditions. Real aggregate demand moves one-for-one with nominal aggregate demand.[36] Hence, the model's prediction that nominal shocks have real effects is robust to changes in the value of κ. The values $\kappa = 2, 3, 4, 5$ imply a ratio of posterior variance to prior variance of the profit-maximizing price of 1/100, 1/300, 1/750 and 1/1250, respectively, and expected per period losses in profits (as a fraction of steady state real output) of 1%, one third of 1%, one seventh of 1% and one twelfth of 1%, respectively. We find numbers in this range reasonable.

We have chosen the variance of the idiosyncratic state variables so as to match the average absolute size of price changes in U.S. micro data. So far we have abstracted from the fact that in the data prices remain fixed for longer than a quarter, whereas in our model prices change every quarter. Let us now see how this change in frequency may affect the estimated variance of idiosyncratic shocks for a given observed size of price changes. Consider the following simple model: Firms can adjust prices every T periods as in Taylor (1980), firms have perfect information, and the profit-maximizing price follows a random walk without drift, where the innovation has a normal distribution with mean zero and variance σ^2. In this simple model, a firm that can adjust its price in a given period sets this period's profit-maximizing price, because this period's profit-maximizing price also equals

idiosyncratic state variables), we also obtained ambiguous predictions concerning the allocation of attention. However, for the benchmark economy, we found that decreasing the persistence of the idiosyncratic state variables (holding constant the variance of the idiosyncratic state variables) always increased the attention allocated to idiosyncratic conditions.

[36] This result is due to the fact that in our model all firms are identical. If a non-negligible fraction of firms operate in stable idiosyncratic environments, the price level responds to nominal shocks also when $\kappa = 2$.

the conditional expectation of the profit-maximizing price in future periods. The expected absolute price adjustment after T periods equals

$$\begin{aligned} E\left[\left|p_{it+T}-p_{it}\right|\right] &= \frac{1}{2}E\left[p_{it+T}-p_{it}|p_{it+T}-p_{it}>0\right]+\frac{1}{2}E\left[-\left(p_{it+T}-p_{it}\right)|p_{it+T}-p_{it}<0\right] \\ &= \frac{2}{\sqrt{2\pi}}\sqrt{T\sigma^2}, \end{aligned}$$

because $p_{it+T} - p_{it}$ has a normal distribution with mean zero and variance $T\sigma^2$. The second line follows from the formula for the expectation of a truncated normal distribution. The expected absolute price adjustment is increasing linearly in $\sqrt{T}$. Thus increasing the price duration from 3 months to T months raises the expected absolute price adjustment by a factor of $\sqrt{T/3}$. When the profit-maximizing price follows a stationary process, the formula for the expected absolute price adjustment is more complicated, but one can show that increasing the price duration from 3 months to T months raises the expected absolute price adjustment by *less* than a factor of $\sqrt{T/3}$. For example, when the profit-maximizing price follows a white noise process, the expected absolute price adjustment is independent of T. Motivated by these observations we computed the equilibrium of our model matching an average absolute price adjustment of 6.3%, because $\sqrt{5.5/3} = (8.5\%)/(6.3\%)$ where 5.5 months is the median price duration excluding sales reported in Bils and Klenow (2004) and 8.5% is the average absolute size of price changes excluding sales reported in Klenow and Kryvtsov (2005) and Nakamura and Steinsson (2007b). We also computed the equilibrium of our model matching an average absolute price adjustment of 4.7%, because $\sqrt{10/3} = (8.5\%)/(4.7\%)$ where 10 months is in the range of the median price durations excluding sales reported in Nakamura and Steinsson (2007b). See Figure 9. Real effects of nominal shocks decrease but remain sizable. Note that we take a conservative approach. We use the median price durations excluding sales (5.5 months and 10 months) and the average absolute size of price changes excluding sales (8.5%).[37] Furthermore, we do not take into account that in our model the profit-maximizing price follows a stationary process.

[37] If we decided to match the U.S. data including sales the average absolute size of price changes in our model would have to equal 11.1%, because $\sqrt{4.3/3} = (13.3\%)/(11.1\%)$ where 4.3 months is the median price duration including sales reported in Bils and Klenow (2004) and 13.3% is the average absolute size of price changes including sales reported in Klenow and Kryvtsov (2005).

7.3 Optimal signals

We always verify that there exist univariate signals that have the property (49)-(50). These are optimal signals. Figures 10 and 11 show optimal signals for the benchmark economy. Here we have computed optimal signals of the form "z_{it} plus a moving average error process" and "Δ_t plus a moving average error process." The signal concerning idiosyncratic conditions turns out to have white noise errors. The signal concerning aggregate conditions has autocorrelated errors. Taking a closer look at the signal concerning aggregate conditions helps understand why autocorrelated errors can be optimal. In the case of this signal, eliminating autocorrelation in the errors (for a given variance of the error process) improves tracking of aggregate conditions but also requires a higher information flow. The value of the improvement in tracking turns out to be smaller than the opportunity cost of the higher information flow.

While the optimal price setting behavior is unique, optimal signals are not unique. One pair of optimal signals are the conditional expectations themselves, $s_{1it} = \hat{\Delta}_{it}^*$ and $s_{2it} = \hat{z}_{it}^*$. This follows from Proposition 2. Once we have an optimal signal it is easy to construct a new optimal signal. For example, applying a one-sided linear filter yields a new signal that is an element of the set Γ and is associated with the same information flow. Typically, applying a one-sided linear filter also does not change the conditional expectation computed from the signal.[38] Thus an optimal signal about aggregate conditions can be a signal concerning the price level, real aggregate demand, nominal aggregate demand, Δ_t or the firms' favorite linear combination of these variables. It does not matter for the equilibrium whether firms pay attention to the price level or real aggregate demand. What matters for the equilibrium is the attention allocated to macro variables. Optimal signals are also indeterminate in the sense that the number of signals is not unique. Signals of any dimension that yield the same conditional covariance matrix of the variables of interest imply the same price setting behavior and are associated with the same information flow. See equations (1)-(4).

[38] One exception is a filter with a zero coefficient on the period t signal.

8 Extensions and shortcomings

8.1 The Gaussianity assumption

So far we have solved the model assuming that signals have to follow a Gaussian process. Now we drop this assumption. When the objective function in the firms' attention problem is quadratic and the variables being tracked follow a Gaussian process, Gaussian signals are optimal. The proof of this result is in the Technical Appendix. Hence, after the log-quadratic approximation to the profit function, Gaussian signals are optimal and dropping the Gaussianity assumption has no effect on the equilibrium. The following questions arise. What is the optimal form of uncertainty without the log-quadratic approximation to the profit function? What is the optimal allocation of attention without the log-quadratic approximation to the profit function? Solving for the equilibrium of the model without the log-quadratic approximation is very difficult. We approach this problem by making two simplifications. First, we focus on the white noise case. Second, we only study the attention problem of an individual firm. We do not derive the rational expectations equilibrium.

Once we move away from the log-quadratic approximation, we must specify a particular profit function. We assume that the demand for good i is $Y_{it} = Y_t \left(P_{it}/P_t\right)^{-\theta}$ with $\theta > 1$. We assume that the output of firm i is $Y_{it} = Z_{it} L_{it}^{\phi}$ where L_{it} is labor input and $\phi \in (0, 1]$. Furthermore, we assume that the cost of labor expressed in consumption units equals χY_t^{γ} with $\chi > 0$ and $\gamma \geq 0$.[39] Then the profit function of firm i is

$$\pi\left(P_{it}, P_t, Y_t, Z_{it}\right) = \frac{P_{it}}{P_t} Y_t \left(\frac{P_{it}}{P_t}\right)^{-\theta} - \chi Y_t^{\gamma} \left(\frac{Y_t \left(\frac{P_{it}}{P_t}\right)^{-\theta}}{Z_{it}}\right)^{\frac{1}{\phi}}. \tag{63}$$

Expressing the profit function in terms of log-deviations from the non-stochastic solution of the model and using $\pi_1\left(1, 1, \bar{Y}, \bar{Z}\right) = 0$ yields

$$\hat{\pi}\left(p_{it}, p_t, y_t, z_{it}\right) = \bar{Y}\left[e^{y_t + (1-\theta)(p_{it} - p_t)} - \frac{(\theta - 1)\,\phi}{\theta} e^{\left(\gamma + \frac{1}{\phi}\right) y_t - \frac{\theta}{\phi}(p_{it} - p_t) - \frac{1}{\phi} z_{it}}\right]. \tag{64}$$

[39] Consider a yeoman farmer model or a model with a labor market. Suppose that utility is additively separable in consumption and labor. With constant relative risk aversion equal to γ and constant disutility of labor equal to χ the cost of labor expressed in consumption units equals χY_t^{γ}.

The profit-maximizing price of firm i in period t is

$$p_{it}^{\diamondsuit} = p_t + \frac{\phi\gamma + 1 - \phi}{\phi + \theta(1-\phi)} y_t - \frac{1}{\phi + \theta(1-\phi)} z_{it}. \tag{65}$$

We study the attention problem of an individual firm assuming that the aggregate variables are given by $p_t = \alpha q_t$ and $y_t = (1-\alpha) q_t$. Then the profit-maximizing price (65) becomes a function of q_t and z_{it} only. We denote the actual price set by the firm by

$$p_{it}^* = p_{it}^A + p_{it}^I,$$

where p_{it}^A denotes the response to aggregate conditions and p_{it}^I denotes the response to idiosyncratic conditions. We assume as before that the responses to aggregate and idiosyncratic conditions are independent. We solve for the optimal joint distribution of $\left(p_{it}^A, p_{it}^I, q_t, z_{it}\right)$ by discretizing the distribution and maximizing the expectation of (64) subject to[40]

$$\mathcal{I}\left(\{q_t\};\{p_{it}^A\}\right) + \mathcal{I}\left(\{z_{it}\};\{p_{it}^I\}\right) \leq \kappa.$$

Figures 12 and 14 show the solution for $\theta = 7$, $\phi = 2/3$, $\gamma = 2$, $\alpha = 1/4$, $\sigma_q = 0.025$, $\sigma_z = 0.25$ and $\kappa = 3$.[41] Figure 12 shows the joint distribution of p_{it}^I and z_{it} under rational inattention. Figure 14 shows the joint distribution of p_{it}^A and q_t under rational inattention. Figures 13 and 15 depict the corresponding joint distributions without the information flow constraint. The firm allocates 90% of the information flow to idiosyncratic conditions. We compare this allocation of attention to the allocation of attention that we obtain when we take a log-quadratic approximation to the profit function (63). In the discrete case analyzed here, the attention allocated to idiosyncratic conditions equals 90%. In the continuous case analyzed in Section 5, the attention allocated to idiosyncratic conditions equals 89%. Hence, the log-quadratic approximation to the profit function seems to have no noticeable effect on the allocation of attention.

[40] The definition of mutual information between two discrete random variables is given in Cover and Thomas (1991), Chapter 2.

[41] This means that we set the standard deviation of q_t approximately equal to the empirical standard deviation of nominal aggregate demand and we set the standard deviation of z_{it} to the number that is required to obtain an average absolute size of price changes of about 10% in the white noise case.

The distributions depicted in Figures 12 and 14 have non-Gaussian features. The firm decides to make smaller mistakes when productivity is high.[42] In addition, the conditional distribution of the response to aggregate conditions has two peaks for some values of nominal aggregate demand. However, the departures from normality are small for our choice of κ. Furthermore, our choice of κ implies a low marginal value of information flow. In contrast, when we decrease κ by a lot and thereby we raise the marginal value of information flow by a lot, the marginal distribution of the response to idiosyncratic conditions and the marginal distribution of the response to aggregate conditions have only a few mass points. Hence, our findings resemble the findings in Sims (2006). Gaussian uncertainty is a good approximation when the marginal value of information flow is low. Gaussian uncertainty is a bad approximation when the marginal value of information flow is high.[43]

8.2 The independence assumption

So far we have assumed that attending to aggregate conditions and attending to idiosyncratic conditions are separate activities. This implies that a manager is able to explain what has caused the change in the profit-maximizing price (a change in aggregate conditions and/or a change in idiosyncratic conditions). In contrast, without the independence assumption the manager would decide to get a signal directly concerning the profit-maximizing price and would not be able to explain what has caused the change in the profit-maximizing price. We think that the model with the independence assumption is more realistic.

Furthermore, the main predictions of the model are robust to small deviations from the independence assumption. Suppose that firm i can choose signals of the form

$$s_{1it} = \Delta_t + \omega \frac{\hat{\pi}_{14}}{|\hat{\pi}_{11}|} z_{it} + \varepsilon_{it}, \tag{66}$$

$$s_{2it} = \omega \Delta_t + \frac{\hat{\pi}_{14}}{|\hat{\pi}_{11}|} z_{it} + \psi_{it}, \tag{67}$$

where $\omega \geq 0$ is a parameter. In contrast to the signals (28)-(29) the signals (66)-(67) have the property that each signal contains information about both aggregate and idiosyncratic

[42] In contrast, a multivariate Gaussian distribution has the property that the conditional variance is independent of the realization.

[43] See Sims (2006), p. 161.

conditions. The signals (66)-(67) can be interpreted as attending to pieces of data that reveal information about both aggregate and idiosyncratic conditions. We study the attention problem of an individual firm in the white noise case by letting the decisionmaker choose the variances σ_ε^2 and σ_ψ^2 subject to the information flow constraint (12). If $\omega < 1$ the decisionmaker chooses a higher precision for signal two. If $\omega > 1$ the decisionmaker chooses a higher precision for signal one. Hence, so long as $\omega \neq 1$ the decisionmaker chooses a higher precision for the signal that contains more information about idiosyncratic conditions and the price set by the firm responds more to idiosyncratic shocks than to aggregate shocks. See Figure 16 and the Technical Appendix.[44] As $\omega \to 0$ or $\omega \to \infty$ the solution converges to the solution presented in Section 5. Only if the decisionmaker can attend directly to a sufficient statistic concerning the profit-maximizing price ($\omega = 1$) the price responds strongly to idiosyncratic shocks and to aggregate shocks.[45] Hence, the main predictions of the model are robust to small deviations from the independence assumption: decisionmakers pay more attention to idiosyncratic conditions and prices respond more to idiosyncratic shocks.[46]

8.3 Reconsidering the allocation of attention

Suppose that in some period $t > 0$ a decisionmaker can reconsider the allocation of attention. The realization of the signal process up to period t affects conditional means but does not affect conditional variances, because in a Gaussian environment conditional variances are independent of realizations. The conditional variance of the profit-maximizing response to aggregate conditions and the conditional variance of the profit-maximizing response to idiosyncratic conditions are deterministic. In fact, due to assumption (18) the conditional variances are constant over time. Hence, in period zero the decisionmaker anticipates correctly the conditional variances in period t and has no incentive to reoptimize in period t.

[44] Figure 16 is drawn assuming the parameter values used in Section 8.1.

[45] The shape of Figure 16 follows from the fact that the decisionmaker decides to receive only one signal in a neighborhood of $\omega = 1$ and switches from signal two to signal one at $\omega = 1$.

[46] Even if some firms can attend directly to a sufficient statistic concerning the profit-maximizing price, it seems unlikely that all firms have this opportunity. If all firms had this opportunity, it would be difficult to explain why managers spend a non-negligible fraction of their time making price setting decisions. See Footnote 4.

8.4 Shortcomings

The model has some shortcomings. The model cannot explain why prices remain fixed for some time. In the model prices change every period. One could add a menu cost. It may be that reality is a combination of a menu cost model and the model presented here. Adding a menu cost is likely to increase the real effects of nominal shocks even further. For a given allocation of attention, the menu cost will make the response of the price level to a nominal shock even more dampened and delayed. If prices are strategic complements, this implies that firms shift attention toward idiosyncratic conditions and away from aggregate conditions. In addition, firms may also shift attention towards idiosyncratic conditions and away from aggregate conditions, because changes in idiosyncratic conditions are more likely to move the price outside the inaction band. These observations suggest that there may be interesting interactions between a menu cost and rational inattention.

In this paper we try to make progress modeling how agents take decisions in complex environments. In this respect we think that the model has two shortcomings. First, we do not spell out all factors that make the price setting decision complicated. We assume a general profit function. We summarize the market-specific factors by the idiosyncratic state variable. We choose a value for the information flow parameter such that firms take good but not perfect decisions. We focus on the tension between attending to aggregate conditions and attending to idiosyncratic conditions. In many models of price setting used in macroeconomics the optimal decision is so simple that it may be unclear why firms make mistakes at all. For example, in a model with monopolistic competition, Dixit-Stiglitz preferences and linear technology in homogeneous labor the profit-maximizing price equals a constant markup times the nominal wage divided by labor productivity. We think that in reality setting the profit-maximizing price is substantially more complicated, e.g., the optimal markup may vary, there may be decreasing returns, there may be different types of labor, there may be various other inputs, the interaction with competitors may be complex, the interaction with customers may be complex, etc. In the future it could be desirable to spell out all factors that make the price setting decision complicated. Second, rational inattention captures some (but certainly not all) aspects of decisionmaking in complex environments. Rational inattention captures the idea that making good decisions is more complicated when firms operate in a more volatile and less persistent environment. Rational inattention does not capture the idea that the size of mistakes also depends on how complex the actual computation is that leads to the decision. The latter aspect of decisionmaking has been emphasized by Gabaix and Laibson (2000).

9 Conclusions and further research

We have studied a model in which price-setting firms decide what to pay attention to. If idiosyncratic conditions are more variable or more important than aggregate conditions, firms pay more attention to idiosyncratic conditions than to aggregate conditions. Prices respond strongly and quickly to idiosyncratic shocks, but prices respond only weakly and slowly to nominal shocks. The model can explain the combination of observations that individual prices move around a lot and, at the same time, the price level responds slowly to monetary policy shocks.

It matters how we model price stickiness. Rational inattention suggests different lessons for monetary policy than standard sticky price models. Rational inattention suggests that stabilizing monetary policy is good because it allows the private sector to focus on market-specific conditions. Interest rate smoothing is good because it makes the firms' tracking problem easier. The allocation of attention changes as monetary policy changes.

It will be interesting to develop a dynamic stochastic general equilibrium model in which firms and households choose their allocation of attention and to compare the predictions to, for example, Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2003). Furthermore, it will be interesting to study the interactions between a menu cost and rational inattention. In addition, it may be interesting to apply this modeling approach to other areas in economics, where it has been noted that idiosyncratic uncertainty dominates aggregate uncertainty.[47]

It will also be interesting to compare the predictions of the model to micro and macro data. The model predicts that, if the degree of real rigidity is the same across sectors, firms operating in more volatile sectors respond more slowly to nominal shocks. Furthermore,

[47] See, for example, Pischke (1995).

the model predicts that as the variance of nominal aggregate demand increases the Phillips curve becomes steeper and, at the same time, real volatility increases.

A Information flow in the white noise case

The definition of information flow (4) and the equation for mutual information (3) yield

$$\mathcal{I}\left(\{X_t\};\{Y_t\}\right)=\lim_{T\to\infty}\frac{1}{T}\left[H\left(X_1,...,X_T\right)-H\left(X_1,...,X_T|Y_1,...,Y_T\right)\right].$$

Using equation (1) for the entropy of a multivariate normal distribution and the fact that the random variables $X_1,...,X_T$ are independent yields

$$H\left(X_1,...,X_T\right)=T\frac{1}{2}\log_2\left(2\pi e\sigma_X^2\right).$$

Using equation (2) for the conditional entropy of a multivariate normal distribution and the fact that the random variables $X_1,...,X_T$ are conditionally independent given $Y_1,...,Y_T$ yields

$$H\left(X_1,...,X_T|Y_1,...,Y_T\right)=T\frac{1}{2}\log_2\left(2\pi e\sigma_{X|Y}^2\right).$$

Combining results yields

$$\begin{aligned}\mathcal{I}\left(\{X_t\};\{Y_t\}\right)&=\frac{1}{2}\log_2\left(2\pi e\sigma_X^2\right)-\frac{1}{2}\log_2\left(2\pi e\sigma_{X|Y}^2\right)\\&=\frac{1}{2}\log_2\left(\frac{\sigma_X^2}{\sigma_{X|Y}^2}\right).\end{aligned}$$

Finally, since (X_t,Y_t) has a multivariate normal distribution,

$$\sigma_{X|Y}^2=\left(1-\rho_{X,Y}^2\right)\sigma_X^2.$$

We arrive at

$$\mathcal{I}\left(\{X_t\};\{Y_t\}\right)=\frac{1}{2}\log_2\left(\frac{1}{1-\rho_{X,Y}^2}\right).$$

B Information flow constraint in the white noise case

The independence assumption (17) implies that

$$\mathcal{I}\left(\{P_t,Z_{it}\};\{s_{it}\}\right)=\mathcal{I}\left(\{p_t\};\{s_{1it}\}\right)+\mathcal{I}\left(\{z_{it}\};\{s_{2it}\}\right).$$

See Lemma 2. The fact that (p_t,s_{1it}) follows a bivariate Gaussian white noise process and (z_{it},s_{2it}) follows a bivariate Gaussian white noise process implies that

$$\mathcal{I}\left(\{p_t\};\{s_{1it}\}\right)+\mathcal{I}\left(\{z_{it}\};\{s_{2it}\}\right)=\frac{1}{2}\log_2\left(\frac{1}{1-\rho_{p,s_{1i}}^2}\right)+\frac{1}{2}\log_2\left(\frac{1}{1-\rho_{z_i,s_{2i}}^2}\right).$$

See equation (5). Using $p_t = \alpha q_t$, $\Delta_t = \alpha q_t + (\hat{\pi}_{13}/|\hat{\pi}_{11}|)(1-\alpha) q_t$ and the equation for the aggregate signal (28) yields

$$\rho^2_{p,s_{1i}} = \frac{1}{1+\frac{\sigma^2_\varepsilon}{\sigma^2_\Delta}}.$$

Using the equation for the idiosyncratic signal (29) yields

$$\rho^2_{z_i,s_{2i}} = \frac{1}{1+\frac{\sigma^2_\psi}{\sigma^2_z}}.$$

Combining results yields

$$\mathcal{I}\left(\{P_t, Z_{it}\};\{s_{it}\}\right) = \frac{1}{2}\log_2\left(\frac{\sigma^2_\Delta}{\sigma^2_\varepsilon}+1\right) + \frac{1}{2}\log_2\left(\frac{\sigma^2_z}{\sigma^2_\psi}+1\right).$$

C Proof of lemma 1

When the profit function is given by (19) then

$$\begin{aligned}
& E\left[\sum_{t=1}^{\infty}\beta^t\pi\left(P^\diamondsuit_{it}, P_t, Y_t, Z_{it}\right)\right] - E\left[\sum_{t=1}^{\infty}\beta^t\pi\left(P^*_{it}, P_t, Y_t, Z_{it}\right)\right] \\
= \; & E\left[\sum_{t=1}^{\infty}\beta^t\tilde{\pi}\left(p^\diamondsuit_{it}, p_t, y_t, z_{it}\right)\right] - E\left[\sum_{t=1}^{\infty}\beta^t\tilde{\pi}\left(p^*_{it}, p_t, y_t, z_{it}\right)\right] \\
= \; & E\left[\sum_{t=1}^{\infty}\beta^t\frac{|\hat{\pi}_{11}|}{2}\left(p^\diamondsuit_{it} - p^*_{it}\right)^2\right],
\end{aligned}$$

where the second equality follows from equation (24). Equation (21), $y_t = q_t - p_t$ and assumption (15) imply that $\left(s_{it}, p^\diamondsuit_{it}\right)$ follows a stationary Gaussian process. Furthermore, assumption (18) implies that at each point in time a long sequence of signals is available. It follows that

$$E\left[p^\diamondsuit_{it}|s^t_i\right] = \mu + \gamma(L)\, s_{it},$$

where μ is a constant and $\gamma(L)$ is an infinite order vector lag polynomial. Hence, $p^\diamondsuit_{it} - p^*_{it}$ follows a stationary process, implying that $E\left[\left(p^\diamondsuit_{it} - p^*_{it}\right)^2\right]$ is independent of t. Combining results yields

$$E\left[\sum_{t=1}^{\infty}\beta^t\pi\left(P^\diamondsuit_{it}, P_t, Y_t, Z_{it}\right)\right] - E\left[\sum_{t=1}^{\infty}\beta^t\pi\left(P^*_{it}, P_t, Y_t, Z_{it}\right)\right] = \frac{\beta}{1-\beta}\frac{|\hat{\pi}_{11}|}{2}E\left[\left(p^\diamondsuit_{it} - p^*_{it}\right)^2\right].$$

D Proof of lemma 2

First, P_t and Z_{it} can be calculated from p_t and z_{it} and vice versa. Therefore

$$\mathcal{I}\left(\{P_t, Z_{it}\};\{s_{it}\}\right) = \mathcal{I}\left(\{p_t, z_{it}\};\{s_{it}\}\right).$$

The definition of information flow (4) and the equation for mutual information (3) yield

$$\mathcal{I}\left(\{p_t, z_{it}\};\{s_{it}\}\right) = \lim_{T\to\infty}\frac{1}{T}\left[H\left(p^T, z_i^T\right) - H\left(p^T, z_i^T | s_i^T\right)\right],$$

where $p^T \equiv (p_1, \ldots, p_T)$, $z_i^T \equiv (z_{i1}, \ldots, z_{iT})$ and $s_i^T \equiv \left(s_i^1, s_{i2}, \ldots, s_{iT}\right)$. The guess (39) implies that the random vectors p^T and z_i^T are independent. It follows that

$$H\left(p^T, z_i^T\right) = H\left(p^T\right) + H\left(z_i^T\right).$$

The assumption (16)-(17) implies that the random vectors p^T and z_i^T are also conditionally independent given the sequence of signals. It follows that

$$H\left(p^T, z_i^T | s_i^T\right) = H\left(p^T | s_{1i}^T\right) + H\left(z_i^T | s_{2i}^T\right).$$

Combining results yields

$$\begin{aligned}
\mathcal{I}\left(\{P_t, Z_{it}\};\{s_{it}\}\right) &= \lim_{T\to\infty}\frac{1}{T}\left[H\left(p^T\right) - H\left(p^T|s_{1i}^T\right) + H\left(z_i^T\right) - H\left(z_i^T|s_{2i}^T\right)\right] \\
&= \mathcal{I}\left(\{p_t\};\{s_{1it}\}\right) + \mathcal{I}\left(\{z_{it}\};\{s_{2it}\}\right).
\end{aligned}$$

Second, the definition of information flow (4) and the equation for mutual information (3) yield

$$\mathcal{I}\left(\{p_t\};\{s_{1it}\}\right) = \lim_{T\to\infty}\frac{1}{T}\left[H\left(p^T\right) - H\left(p^T|s_{1i}^T\right)\right].$$

Furthermore

$$H\left(p^T\right) - H\left(p^T|s_{1i}^T\right) = H\left(p^T\right) - H\left(p^T|\hat{\Delta}_i^T\right) + H\left(p^T|\hat{\Delta}_i^T\right) - H\left(p^T|s_{1i}^T\right).$$

Combining results yields

$$\begin{aligned}
\mathcal{I}\left(\{p_t\};\{s_{1it}\}\right) &= \lim_{T\to\infty}\frac{1}{T}\left[H\left(p^T\right) - H\left(p^T|\hat{\Delta}_i^T\right) + H\left(p^T|\hat{\Delta}_i^T\right) - H\left(p^T|s_{1i}^T\right)\right] \\
&= \mathcal{I}\left(\{p_t\};\left\{\hat{\Delta}_{it}\right\}\right) + \lim_{T\to\infty}\frac{1}{T}\left[H\left(p^T|\hat{\Delta}_i^T\right) - H\left(p^T|s_{1i}^T\right)\right].
\end{aligned}$$

Conditioning reduces entropy. See Cover and Thomas (1991), p. 232. It follows that

$$H\left(p^T|\hat{\Delta}_i^T\right) \geq H\left(p^T|\hat{\Delta}_i^T, s_{1i}^T\right).$$

Furthermore $\hat{\Delta}_i^T = \left(\hat{\Delta}_{i1}, \ldots, \hat{\Delta}_{iT}\right)$ can be calculated from $s_{1i}^T = \left(s_{1i}^1, s_{1i2}, \ldots, s_{1iT}\right)$. Thus

$$H\left(p^T|\hat{\Delta}_i^T, s_{1i}^T\right) = H\left(p^T|s_{1i}^T\right).$$

Combining results yields

$$\mathcal{I}\left(\{p_t\};\{s_{1it}\}\right) \geq \mathcal{I}\left(\{p_t\};\left\{\hat{\Delta}_{it}\right\}\right).$$

The same arguments yield

$$\mathcal{I}\left(\{z_{it}\};\{s_{2it}\}\right) \geq \mathcal{I}\left(\{z_{it}\};\{\hat{z}_{it}\}\right).$$

Third, consider the special case of a univariate signal s_{1it}. Assumption (15) implies that (p_t, s_{1it}) follows a bivariate stationary Gaussian process. The definition of Δ_t, $y_t = q_t - p_t$, assumption (15) and assumption (18) imply that

$$E\left[\Delta_t|s_{1i}^t\right] = \mu + F(L)\, s_{1it},$$

where μ is a constant and $F(L)$ is an infinite order lag polynomial. Thus $\left(p_t, \hat{\Delta}_{it}\right)$ also follows a bivariate stationary Gaussian process. It follows from equation (6) that, if

$$\mathcal{C}_{p,s_{1i}}(\omega) = \mathcal{C}_{p,\hat{\Delta}_i}(\omega)$$

then

$$\mathcal{I}\left(\{p_t\};\{s_{1it}\}\right) = \mathcal{I}\left(\{p_t\};\left\{\hat{\Delta}_{it}\right\}\right).$$

The coherence between two stochastic processes equals the product of the two cross spectra divided by the product of the two spectra.

$$\begin{aligned}
\mathcal{C}_{p,\hat{\Delta}_i}(\omega) &= \frac{S_{p,\hat{\Delta}_i}(\omega)\, S_{\hat{\Delta}_i,p}(\omega)}{S_p(\omega)\, S_{\hat{\Delta}_i}(\omega)} \\
&= \frac{F\left(e^{i\omega}\right) S_{p,s_{1i}}(\omega)\, F\left(e^{-i\omega}\right) S_{s_{1i},p}(\omega)}{S_p(\omega)\, F\left(e^{-i\omega}\right) F\left(e^{i\omega}\right) S_{s_{1i}}(\omega)} \\
&= \frac{S_{p,s_{1i}}(\omega)\, S_{s_{1i},p}(\omega)}{S_p(\omega)\, S_{s_{1i}}(\omega)} \\
&= \mathcal{C}_{p,s_{1i}}(\omega),
\end{aligned}$$

where the second equality follows from the fact that the $\hat{\Delta}_{it}$ process is obtained from the s_{1it} process by applying a linear filter. See Hamilton (1994), pages 277-278. The same arguments yield that, if s_{2it} is a univariate signal, then

$$\mathcal{I}\left(\{z_{it}\};\{s_{2it}\}\right)=\mathcal{I}\left(\{z_{it}\};\{\hat{z}_{it}\}\right).$$

Fourth, one can state the guess (39) as

$$p_t=\alpha\left(L\right)\nu_t,$$

where $\alpha\left(L\right)$ is an infinite order lag polynomial. Let the moving average representation for q_t be given by

$$q_t=A\left(L\right)\nu_t,$$

where $A\left(L\right)$ is an infinite order lag polynomial. The definition of Δ_t and $y_t=q_t-p_t$ yield

$$\Delta_t=\left[\left(1-\frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}\right)\alpha\left(L\right)+\frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}A\left(L\right)\right]\nu_t.$$

The p_t process is obtained from the ν_t process by applying a linear filter. Thus

$$\mathcal{C}_{p,\hat{\Delta}_i}\left(\omega\right)=\mathcal{C}_{\nu,\hat{\Delta}_i}\left(\omega\right).$$

The Δ_t process is also obtained from the ν_t process by applying a linear filter. Thus

$$\mathcal{C}_{\Delta,\hat{\Delta}_i}\left(\omega\right)=\mathcal{C}_{\nu,\hat{\Delta}_i}\left(\omega\right).$$

It follows from equation (6) that

$$\begin{aligned}\mathcal{I}\left(\{p_t\};\left\{\hat{\Delta}_{it}\right\}\right)&=\mathcal{I}\left(\{\Delta_t\};\left\{\hat{\Delta}_{it}\right\}\right)\\&=\mathcal{I}\left(\{\nu_t\};\left\{\hat{\Delta}_{it}\right\}\right).\end{aligned}$$

This argument applies to p_t, q_t, y_t and all linear combinations of these variables. Hence, it does not matter which macro variable one includes in the information flow constraint.

E Proof of proposition 1

First, when the profit function is given by (19) and (39) holds, the objective function (45) is a monotonic transformation of the objective function (13). This follows from Lemma 1 and equation (41). Hence, one can use either objective function to evaluate decisions.

Second, the information flow constraint (12) implies (46). This follows from Lemma 2. Furthermore, the definition of the set Γ, $y_t = q_t - p_t$ and assumption (18) imply (47)-(48). Hence, expected profits at a solution to (12)-(14) cannot be strictly larger than expected profits at a solution to (45)-(48).

Third, suppose that there exists a bivariate signal process $\{s_{1it}, s_{2it}\} \in \Gamma$ that has the property (49)-(50). Since s_{1it} and s_{2it} are scalars, inequality (43) holds with equality and therefore the fact that $\left\{\hat{\Delta}_{it}^{*}, \hat{z}_{it}^{*}\right\}$ satisfies (46) implies that $\{s_{1it}, s_{2it}\}$ satisfies (12). Furthermore, the fact that $\left\{\hat{\Delta}_{it}^{*}, \hat{z}_{it}^{*}\right\}$ is a solution to the problem (45)-(48) implies that $\{s_{1it}, s_{2it}\}$ must be a solution to the problem (12)-(14).

F Proof of proposition 2

First, the mean of Y_t affects the objective function (51) but does not affect the information flow in (54). Thus a solution to the problem (51)-(54) has to satisfy

$$E\left[Y_t\right] = E\left[X_t\right].$$

Second, a solution to the problem (51)-(54) has to satisfy, for all $k = 0, 1, 2, \ldots$,

$$E\left[\left(X_t - Y_t\right) Y_{t-k}\right] = 0.$$

Take a process Y_t' that does not have this property. Formally, for some $k \in \{0, 1, 2, \ldots\}$,

$$E\left[\left(X_t - Y_t'\right) Y_{t-k}'\right] \neq 0.$$

Then one can define a new process Y_t'' as follows

$$Y_t'' = Y_t' + \gamma Y_{t-k}',$$

where γ is the projection coefficient in the linear projection of $X_t - Y_t'$ on Y_{t-k}'. It is easy to verify that the new process has the property

$$E\left[\left(X_t - Y_t''\right)^2\right] < E\left[\left(X_t - Y_t'\right)^2\right].$$

Furthermore the new process is of the form (53). Finally the new process has the property

$$\mathcal{I}\left(\{X_t\}; \{Y_t''\}\right) = \mathcal{I}\left(\{X_t\}; \{Y_t'\}\right),$$

because applying a linear filter does not change the information flow. See proof of Lemma 2. Hence, the process Y_t' cannot be a solution to the problem (51)-(54).

G Proof of proposition 3

We first establish a lower bound for the mean squared error at the solution. We then show that the process Y_t^* attains this bound and satisfies the information flow constraint. These results imply that the process Y_t^* is a solution.

First, the equation for mutual information (3) and the symmetry of mutual information yield

$$I\left(X^T;Y^T\right)=H\left(Y^T\right)-H\left(Y^T|X^T\right),$$

where $X^T\equiv(X_1,\ldots,X_T)$ and $Y^T\equiv(Y_1,\ldots,Y_T)$. Furthermore let $Y^{t-1}\equiv(Y_1,\ldots,Y_{t-1})$. The chain rule for entropy yields

$$H\left(Y^T\right)=H\left(Y_1\right)+\sum_{t=2}^{T}H\left(Y_t|Y^{t-1}\right).$$

See Cover and Thomas (1991), p. 232. The chain rule for entropy also yields

$$H\left(Y^T|X^T\right)=H\left(Y_1|X^T\right)+\sum_{t=2}^{T}H\left(Y_t|Y^{t-1},X^T\right).$$

Conditioning reduces entropy. See again Cover and Thomas (1991), p. 232. Thus

$$H\left(Y_t|Y^{t-1},X_t\right)\geq H\left(Y_t|Y^{t-1},X^T\right).$$

Combining results yields

$$I\left(X^T;Y^T\right)\geq H\left(Y_1\right)-H\left(Y_1|X^T\right)+\sum_{t=2}^{T}\left[H\left(Y_t|Y^{t-1}\right)-H\left(Y_t|Y^{t-1},X_t\right)\right].$$

Furthermore the chain rule for entropy yields

$$\begin{aligned}
& H\left(Y_t|Y^{t-1}\right)-H\left(Y_t|Y^{t-1},X_t\right)\\
= & H\left(Y_t,Y^{t-1}\right)-H\left(Y^{t-1}\right)-\left[H\left(Y_t,Y^{t-1},X_t\right)-H\left(Y^{t-1},X_t\right)\right]\\
= & H\left(Y^{t-1},X_t\right)-H\left(Y^{t-1}\right)-\left[H\left(Y_t,Y^{t-1},X_t\right)-H\left(Y_t,Y^{t-1}\right)\right]\\
= & H\left(X_t|Y^{t-1}\right)-H\left(X_t|Y_t,Y^{t-1}\right).
\end{aligned}$$

The fact that X^T and Y^T have a multivariate normal distribution yields

$$H\left(X_t|Y^{t-1}\right)-H\left(X_t|Y^t\right)=\frac{1}{2}\log_2\left[\frac{Var\left(X_t|Y^{t-1}\right)}{Var\left(X_t|Y^t\right)}\right].$$

The fact that X_t follows an AR(1) process yields

$$Var\left(X_t|Y^{t-1}\right) = \rho^2 Var\left(X_{t-1}|Y^{t-1}\right) + a^2.$$

Proposition 2 implies that, at a solution,

$$Var\left(X_t|Y^t\right) = E\left[\left(X_t - Y_t\right)^2\right].$$

Stationarity implies that

$$E\left[\left(X_t - Y_t\right)^2\right] = E\left[\left(X_{t-1} - Y_{t-1}\right)^2\right].$$

Combining results yields that, at a solution,

$$I\left(X^T;Y^T\right) \geq H\left(Y_1\right) - H\left(Y_1|X^T\right) + \sum_{t=2}^{T} \frac{1}{2}\log_2\left[\rho^2 + \frac{a^2}{E\left[\left(X_t - Y_t\right)^2\right]}\right].$$

Dividing by T on both sides and taking the limit as $T \to \infty$ yields

$$\mathcal{I}\left(\{X_t\};\{Y_t\}\right) \geq \frac{1}{2}\log_2\left[\rho^2 + \frac{a^2}{E\left[\left(X_t - Y_t\right)^2\right]}\right].$$

It follows that, at a solution,

$$E\left[\left(X_t - Y_t\right)^2\right] \geq \frac{a^2}{2^{2\kappa_j} - \rho^2}.$$

Second, the process Y_t^* has the property

$$X_t - Y_t^* = \frac{\rho}{2^{2\kappa_j}}\left(X_{t-1} - Y_{t-1}^*\right) + \left(\frac{1}{2^{2\kappa_j}} a u_t - \sqrt{\frac{1}{2^{2\kappa_j}}\frac{2^{2\kappa_j} - 1}{2^{2\kappa_j} - \rho^2}} a\epsilon_t\right).$$

Using $X_t = \rho X_{t-1} + a u_t$ to substitute for $a u_t$ in the last equation and solving for Y_t^* yields

$$Y_t^* = \frac{\rho}{2^{2\kappa_j}} Y_{t-1}^* + \left(1 - \frac{1}{2^{2\kappa_j}}\right) X_t + \sqrt{\frac{1}{2^{2\kappa_j}}\frac{2^{2\kappa_j} - 1}{2^{2\kappa_j} - \rho^2}} a\epsilon_t.$$

The first of the last two equations yields

$$E\left[\left(X_t - Y_t^*\right)^2\right] = \frac{a^2}{2^{2\kappa_j} - \rho^2}.$$

The second equation yields

$$Var\left(Y_t^*|Y_1^*,\ldots,Y_{t-1}^*,X^T\right) = Var\left(Y_t^*|Y_1^*,\ldots,Y_{t-1}^*,X_t\right),$$

implying

$$H\left(Y_t^*|Y_1^*,\ldots,Y_{t-1}^*,X^T\right)=H\left(Y_t^*|Y_1^*,\ldots,Y_{t-1}^*,X_t\right).$$

Hence, for the process Y_t^* all the weak inequalities in the first half of this proof hold with equality. Thus

$$\begin{aligned}\mathcal{I}\left(\{X_t\};\{Y_t^*\}\right) &= \frac{1}{2}\log_2\left(\rho^2+\frac{a^2}{E\left[\left(X_t-Y_t^*\right)^2\right]}\right)\\ &= \kappa_j.\end{aligned}$$

H Proof of proposition 4

The formula for updating a linear projection yields

$$E\left[X_t|S^t\right]=E\left[X_t|S^{t-1}\right]+\theta\left(S_t-E\left[S_t|S^{t-1}\right]\right),$$

with

$$\theta=\frac{E\left[\left(X_t-E\left[X_t|S^{t-1}\right]\right)\left(S_t-E\left[S_t|S^{t-1}\right]\right)\right]}{E\left[\left(S_t-E\left[S_t|S^{t-1}\right]\right)^2\right]}.$$

Using equation (61) to substitute for S_t yields

$$E\left[X_t|S^t\right]=\left(1-\theta\right)E\left[X_t|S^{t-1}\right]+\theta\left(X_t+\sqrt{\frac{2^{2\kappa_j}}{\left(2^{2\kappa_j}-1\right)\left(2^{2\kappa_j}-\rho^2\right)}}a\epsilon_t\right),$$

$$\theta=\frac{E\left[\left(X_t-E\left[X_t|S^{t-1}\right]\right)^2\right]}{E\left[\left(X_t-E\left[X_t|S^{t-1}\right]\right)^2\right]+\frac{2^{2\kappa_j}}{\left(2^{2\kappa_j}-1\right)\left(2^{2\kappa_j}-\rho^2\right)}a^2}.$$

Using equation (57) to substitute for X_t yields

$$E\left[X_t|S^t\right]=\left(1-\theta\right)\rho E\left[X_{t-1}|S^{t-1}\right]+\theta\left(X_t+\sqrt{\frac{2^{2\kappa_j}}{\left(2^{2\kappa_j}-1\right)\left(2^{2\kappa_j}-\rho^2\right)}}a\epsilon_t\right),$$

$$\theta=\frac{\rho^2E\left[\left(X_{t-1}-E\left[X_{t-1}|S^{t-1}\right]\right)^2\right]+a^2}{\rho^2E\left[\left(X_{t-1}-E\left[X_{t-1}|S^{t-1}\right]\right)^2\right]+a^2+\frac{2^{2\kappa_j}}{\left(2^{2\kappa_j}-1\right)\left(2^{2\kappa_j}-\rho^2\right)}a^2}.$$

Next we plug in the guess

$$\begin{aligned}E\left[X_t|S^t\right] &= Y_t^*,\\ E\left[\left(X_t-E\left[X_t|S^t\right]\right)^2\right] &= \frac{a^2}{2^{2\kappa_j}-\rho^2}.\end{aligned}$$

We obtain

$$Y_t^* = 2^{-2\kappa_j} \rho Y_{t-1}^* + \left(1 - 2^{-2\kappa_j}\right) \left(X_t + \sqrt{\frac{2^{2\kappa_j}}{\left(2^{2\kappa_j} - 1\right)\left(2^{2\kappa_j} - \rho^2\right)}} a \epsilon_t \right).$$

Computing the moving average representation for Y_t^* from the last equation yields equation (58). Hence, the guess is correct.

I Numerical solution procedure

Let the moving average representations for q_t and z_{it} be given by

$$q_t = \sum_{l=0}^{\infty} a_l \nu_{t-l},$$

$$z_{it} = \sum_{l=0}^{\infty} b_l \xi_{it-l},$$

where $\{\nu_t\}$ and $\{\xi_{it}\}$ are independent Gaussian white noise processes. The definition of Δ_t, $y_t = q_t - p_t$ and guess (39) yield

$$\Delta_t = \left(1 - \frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}\right) \sum_{l=0}^{\infty} \alpha_l \nu_{t-l} + \frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|} \sum_{l=0}^{\infty} a_l \nu_{t-l}.$$

Applying Proposition 1, we solve the following constrained minimization problem

$$\min_{c,d,f,g} \left\{ E\left[\left(\Delta_t - \hat{\Delta}_{it}\right)^2\right] + \left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 E\left[\left(z_{it} - \hat{z}_{it}\right)^2\right] \right\},$$

subject to the equation for Δ_t, the equation for z_{it},

$$\hat{\Delta}_{it} = \sum_{l=0}^{\infty} c_l \nu_{t-l} + \sum_{l=0}^{\infty} d_l \varepsilon_{it-l},$$

$$\hat{z}_{it} = \sum_{l=0}^{\infty} f_l \xi_{it-l} + \sum_{l=0}^{\infty} g_l \psi_{it-l},$$

and

$$-\frac{1}{4\pi} \int_{-\pi}^{\pi} \log_2 \left[1 - \mathcal{C}_{\Delta,\hat{\Delta}_i}(\omega)\right] d\omega - \frac{1}{4\pi} \int_{-\pi}^{\pi} \log_2 \left[1 - \mathcal{C}_{z_i,\hat{z}_i}(\omega)\right] d\omega \leq \kappa,$$

where $\{\varepsilon_{it}\}$ and $\{\psi_{it}\}$ are idiosyncratic Gaussian white noise processes that are mutually independent and independent of $\{\nu_t\}$ and $\{\xi_{it}\}$. Here we make use of equation (6) to express information flow as a function of coherence.

Consider, as an example, the choice of the f_l and g_l, for all $l = 0, 1, \ldots$. Observe that in the objective

$$\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 E\left[(z_{it} - \hat{z}_{it})^2\right] = \left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 \left[\sum_{l=0}^{\infty}(b_l - f_l)^2 + \sum_{l=0}^{\infty} g_l^2\right],$$

and in the constraint

$$\mathcal{C}_{z_i,\hat{z}_i}(\omega) = \frac{\frac{F\left(e^{-i\omega}\right)F\left(e^{i\omega}\right)}{G\left(e^{-i\omega}\right)G\left(e^{i\omega}\right)}}{\frac{F\left(e^{-i\omega}\right)F\left(e^{i\omega}\right)}{G\left(e^{-i\omega}\right)G\left(e^{i\omega}\right)} + 1},$$

where the polynomials $F\left(e^{i\omega}\right)$ and $G\left(e^{i\omega}\right)$ are defined as $F\left(e^{i\omega}\right) \equiv f_0 + f_1 e^{i\omega} + f_2 e^{i2\omega} + \ldots$ and $G\left(e^{i\omega}\right) \equiv g_0 + g_1 e^{i\omega} + g_2 e^{i2\omega} + \ldots$. The first-order condition with respect to f_l is

$$\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 2(b_l - f_l) = -\frac{\mu}{4\pi \ln(2)} \int_{-\pi}^{\pi} \frac{\partial \ln\left[1 - \mathcal{C}_{z_i,\hat{z}_i}(\omega)\right]}{\partial f_l} d\omega,$$

where μ is the Lagrange multiplier. The first-order condition with respect to g_l is

$$\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 2g_l = \frac{\mu}{4\pi \ln(2)} \int_{-\pi}^{\pi} \frac{\partial \ln\left[1 - \mathcal{C}_{z_i,\hat{z}_i}(\omega)\right]}{\partial g_l} d\omega.$$

We obtain a system of nonlinear equations in c, d, f, g and μ that we solve numerically.

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Table 1: Parameters and main results for the benchmark economy	
Parameter	**Interpretation**
$q_t = \sum_{l=0}^{20} a_l \nu_{t-l}, \sigma_\nu = 0.01$ with $a_0 = 1, a_l = a_{l-1} - 0.05, l = 1, ..., 20$	The MA representation of nominal aggregate demand q_t
$\sigma_q = 0.0268$	The standard deviation of nominal aggregate demand q_t
$z_{it} = \sum_{l=0}^{20} b_l \xi_{it-l}, \sigma_\xi = 0.1$ with $b_l = a_l, l = 0, 1, ..., 20$	The MA representation of the idiosyncratic state variables z_{it}
$\sigma_z = 0.268$	The standard deviation of the idiosyncratic state variables z_{it}
$(\hat{\pi}_{14}/\lvert\hat{\pi}_{11}\rvert) = 1$	Determines the sensitivity of prices to the idiosyncratic state variables z_{it}
$(\hat{\pi}_{13}/\lvert\hat{\pi}_{11}\rvert) = 0.15$	Determines the sensitivity of prices to real aggregate demand y_t
$\kappa = 3$	The upper bound on the information flow
Result	**Interpretation**
8.2%	The average absolute size of price changes
$\kappa_1^* = 0.19, \kappa_2^* = 2.81$	94% of attention is allocated to idiosyncratic conditions
$E\left[\left(\Delta_t - \hat{\Delta}_{it}^*\right)^2\right] = 0.000039$	Expected per period loss from imperfect tracking of Δ_t
$\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 E\left[(z_{it} - \hat{z}_{it}^*)^2\right] = 0.00021$	Expected per period loss from imperfect tracking of z_{it}

Table 2: Varying parameter values	
Change in parameter values relative to the benchmark economy in Table 1	**Changes in the results**
$\sigma_{\xi} = 0.12$ Larger variance of the idiosyncratic state variables	The average absolute size of price changes equals 10% κ_1^* decreases to 4% of κ $E\left[\left(\Delta_t - \hat{\Delta}_{it}^*\right)^2\right] = 0.000044, \left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 E\left[\left(z_{it} - \hat{z}_{it}^*\right)^2\right] = 0.00027$
$\sigma_{\nu} = 0.5$ Larger variance of nominal aggregate demand	The average absolute size of price changes equals 35% κ_1^* increases to 76% of κ $E\left[\left(\Delta_t - \hat{\Delta}_{it}^*\right)^2\right] = 0.0076, \left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 E\left[\left(z_{it} - \hat{z}_{it}^*\right)^2\right] = 0.0054$
$(\hat{\pi}_{13}/\|\hat{\pi}_{11}\|) = 0.1$ Higher degree of real rigidity	The average absolute size of price changes equals 8.2% κ_1^* decreases to 5% of κ $E\left[\left(\Delta_t - \hat{\Delta}_{it}^*\right)^2\right] = 0.000031, \left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 E\left[\left(z_{it} - \hat{z}_{it}^*\right)^2\right] = 0.00019$
$(\hat{\pi}_{13}/\|\hat{\pi}_{11}\|) = 0.95$ Lower degree of real rigidity	The average absolute size of price changes equals 8.2% κ_1^* increases to 12% of κ $E\left[\left(\Delta_t - \hat{\Delta}_{it}^*\right)^2\right] = 0.0001, \left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 E\left[\left(z_{it} - \hat{z}_{it}^*\right)^2\right] = 0.00027$

Figure 1: Impulse response of an individual price to an innovation in the idiosycratic state variable, benchmark economy

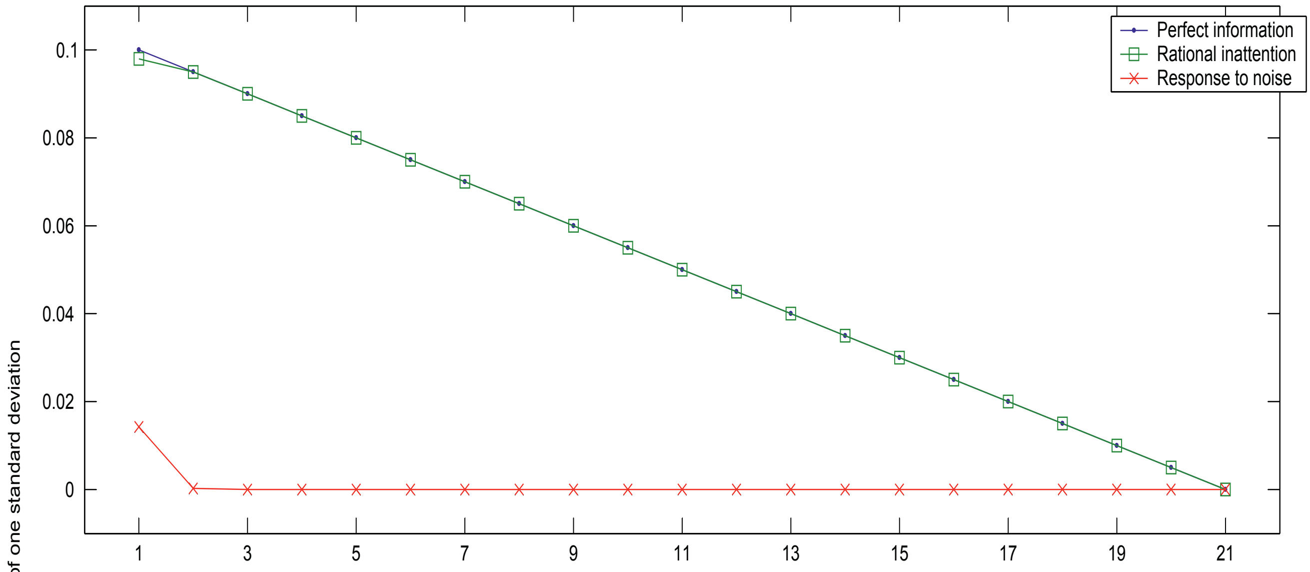


Figure 2: Impulse response of an individual price to an innovation in nominal aggregate demand, benchmark economy

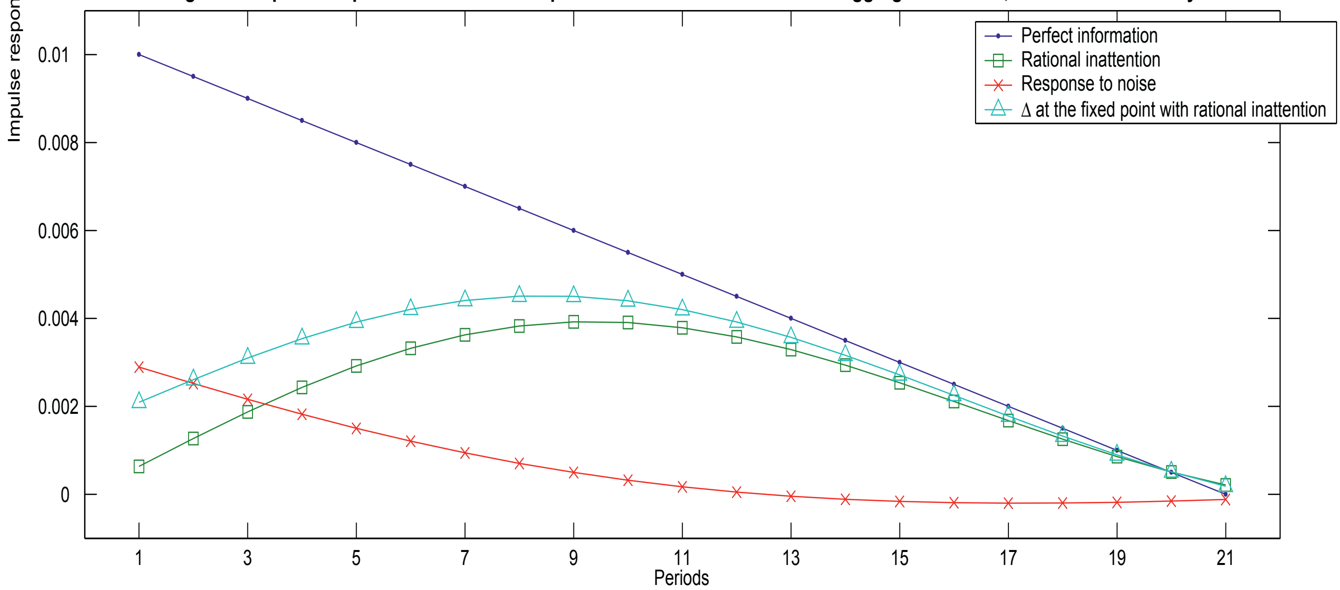


Figure 3: Simulated price set by an individual firm in the benchmark economy

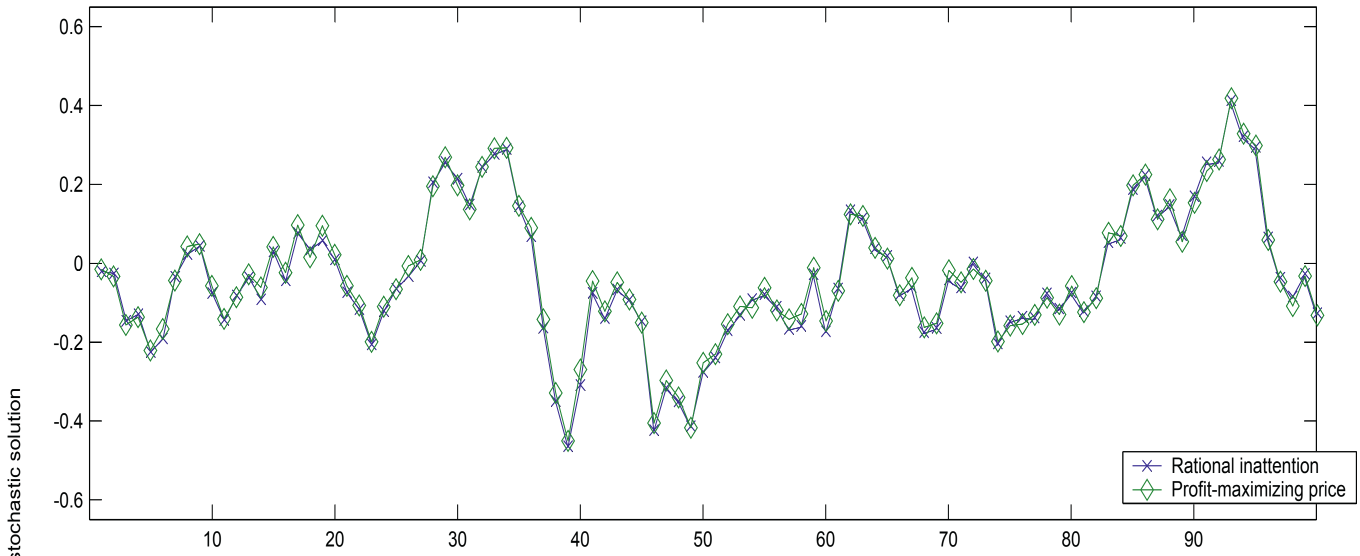


Figure 4: Simulated aggregate price level

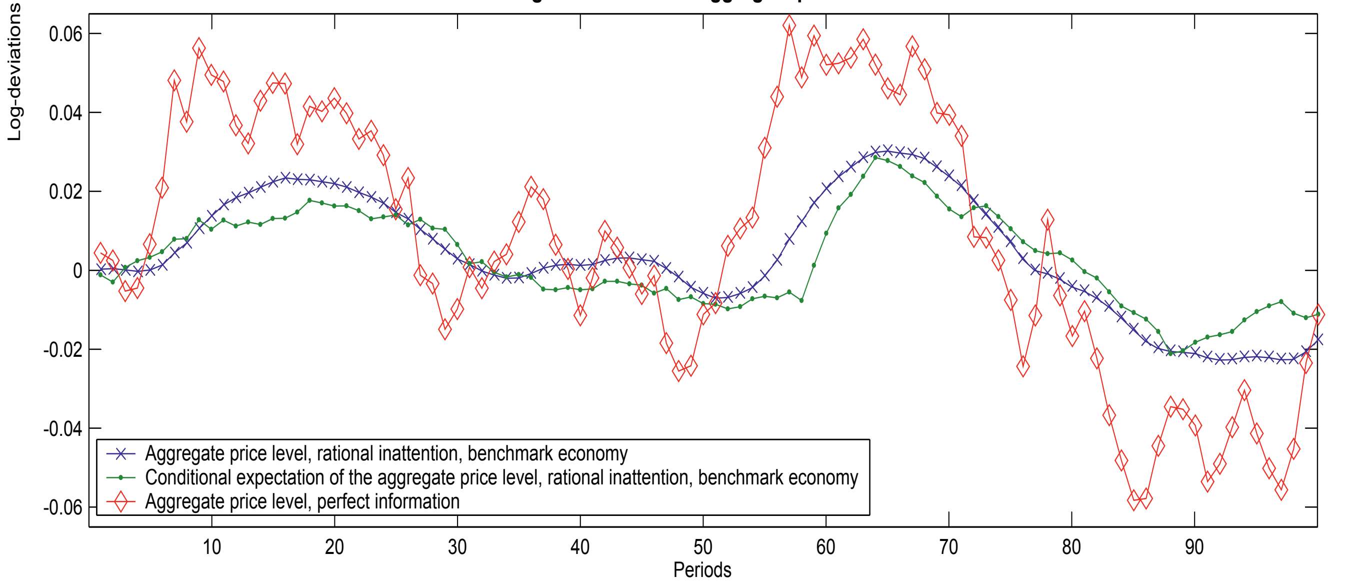


Figure 5: Impulse response of the aggregate price level to an innovation in nominal aggregate demand

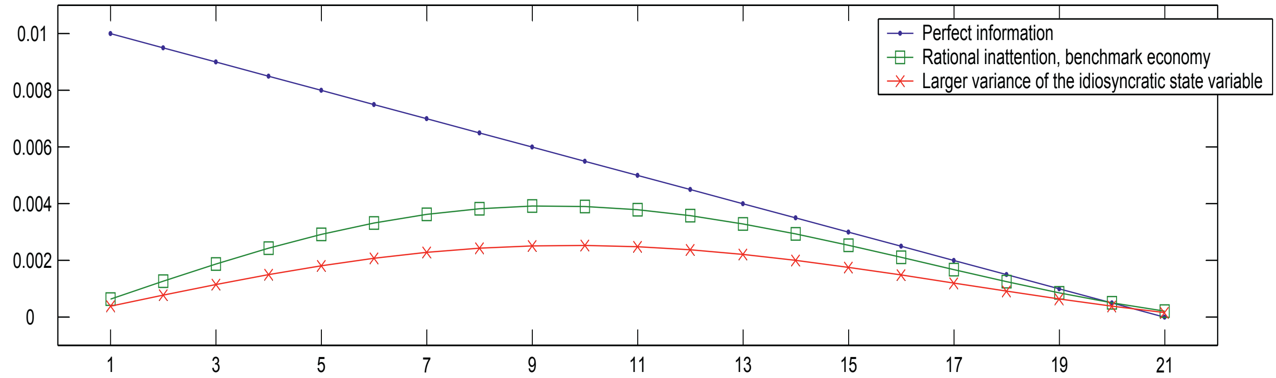


Figure 6: Impulse response of an individual price to an innovation in the idiosycratic state variable

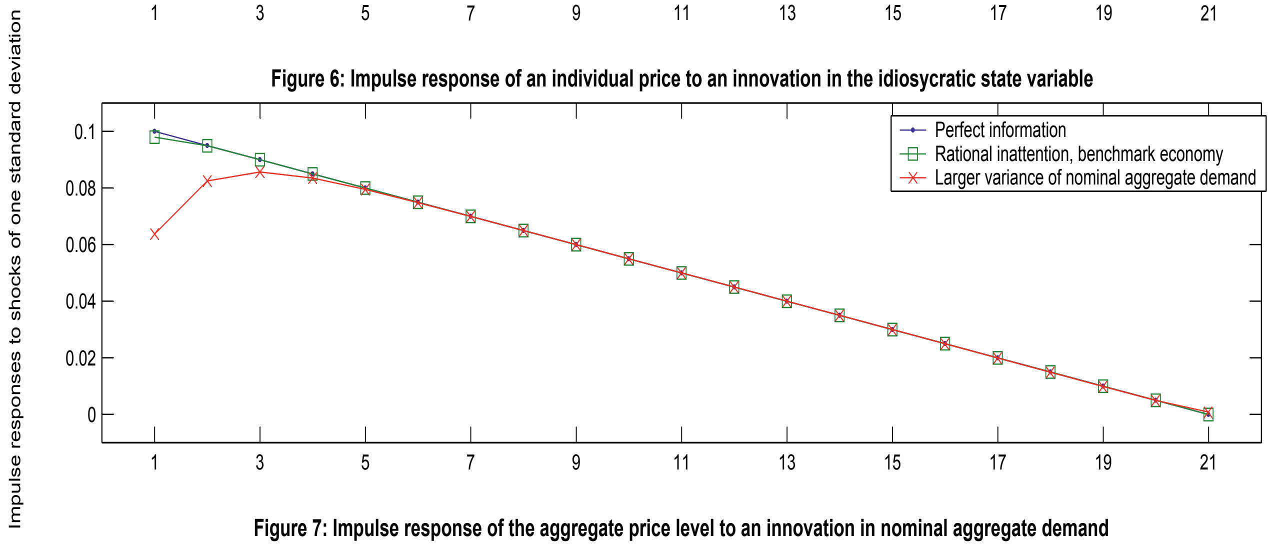


Figure 7: Impulse response of the aggregate price level to an innovation in nominal aggregate demand

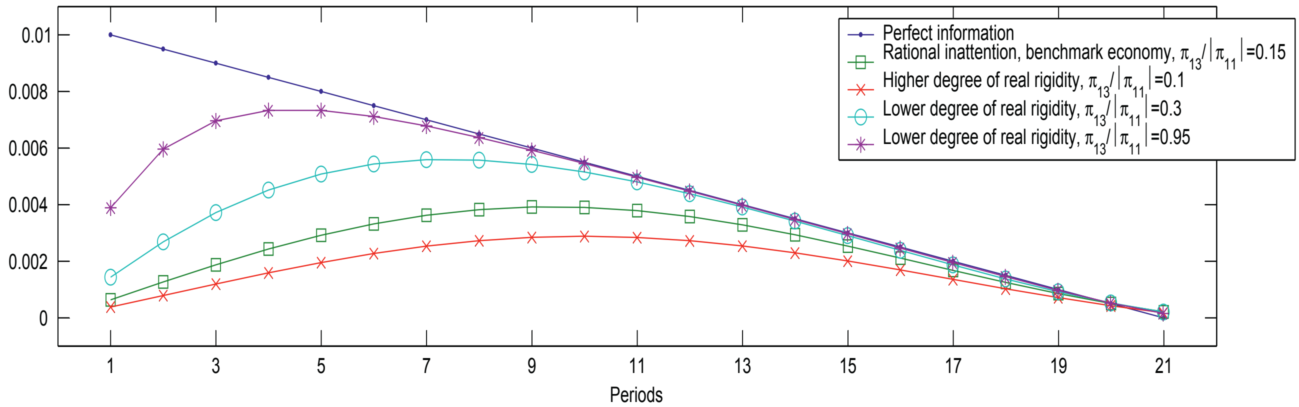


Figure 8: Impulse response of the aggregate price level to an innovation in nominal aggregate demand

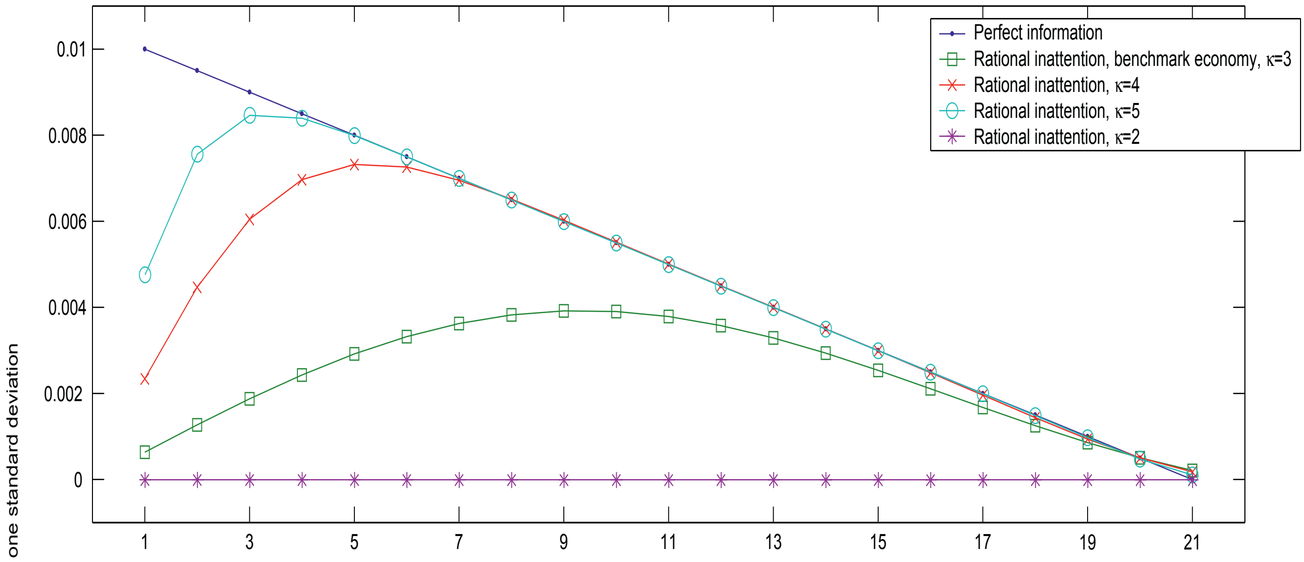


Figure 9: Impulse response of the aggregate price level to an innovation in nominal aggregate demand

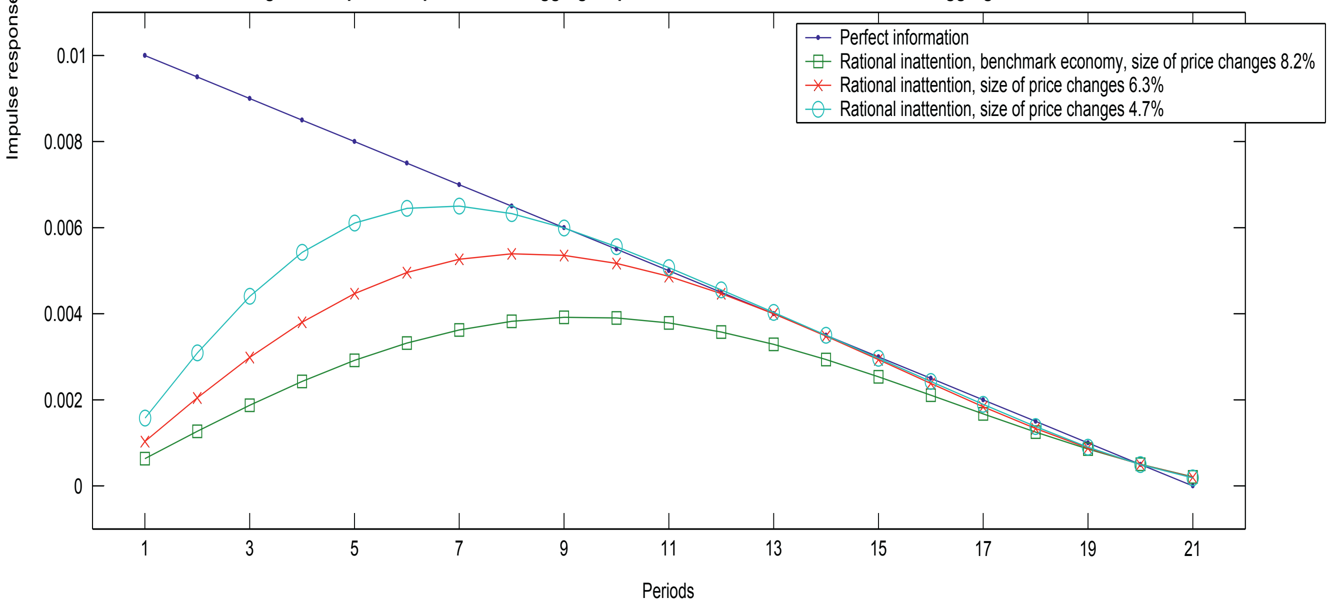


Figure 10: An optimal signal about idiosyncratic conditions, benchmark economy

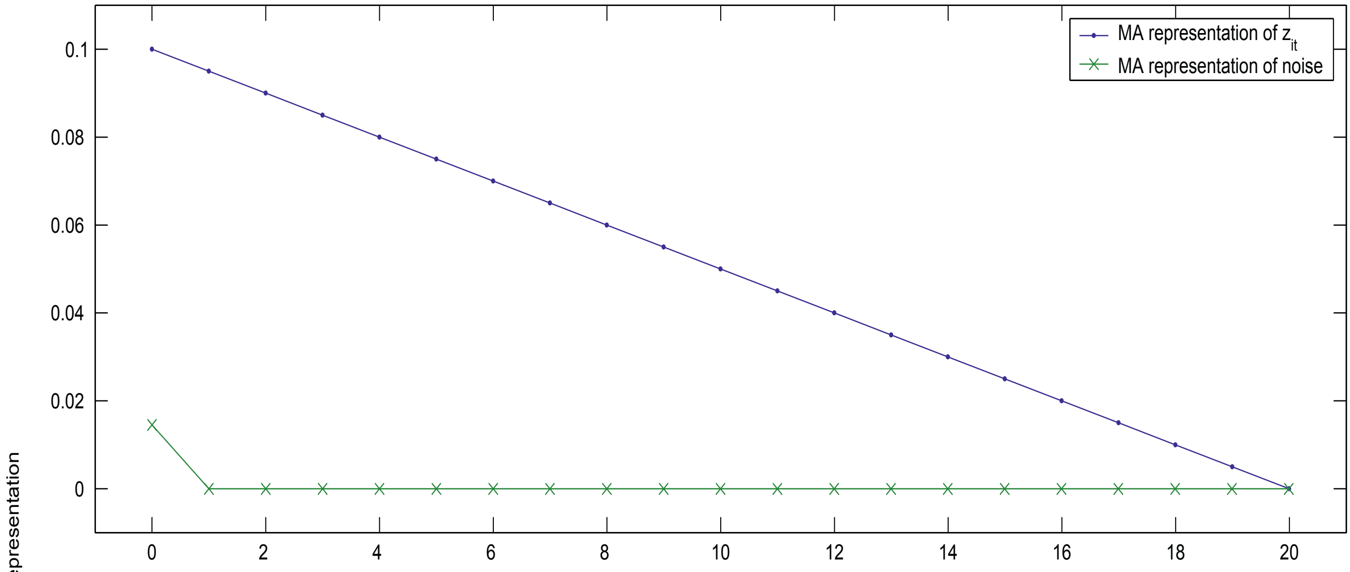


Figure 11: An optimal signal about aggregate conditions, benchmark economy

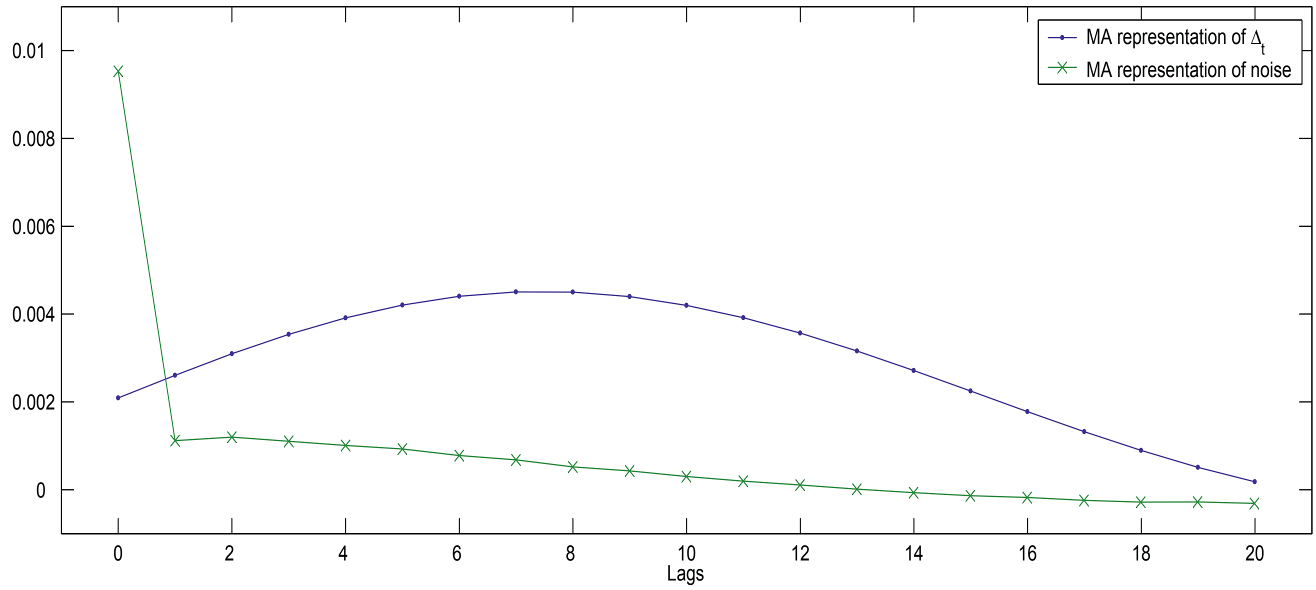


Figure 12: The joint distribution of the optimal price response and the idiosyncratic state variable, rational inattention

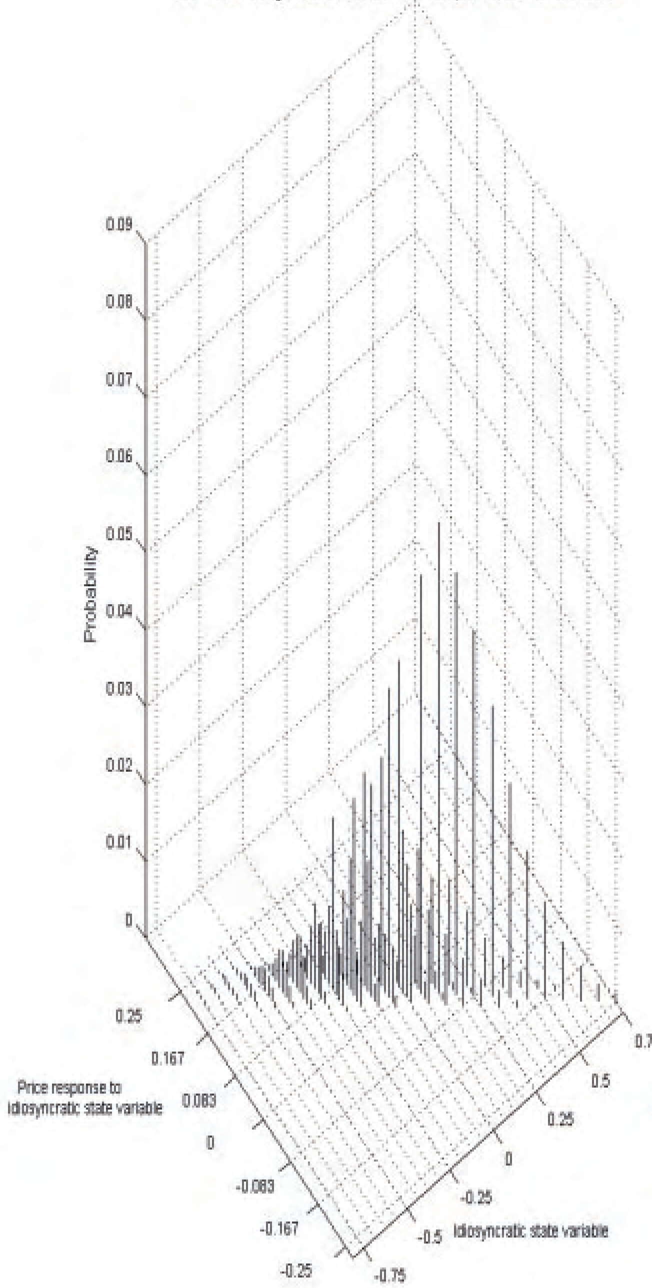


Figure 13: The joint distribution of the optimal price response and the idiosyncratic state variable, perfect information

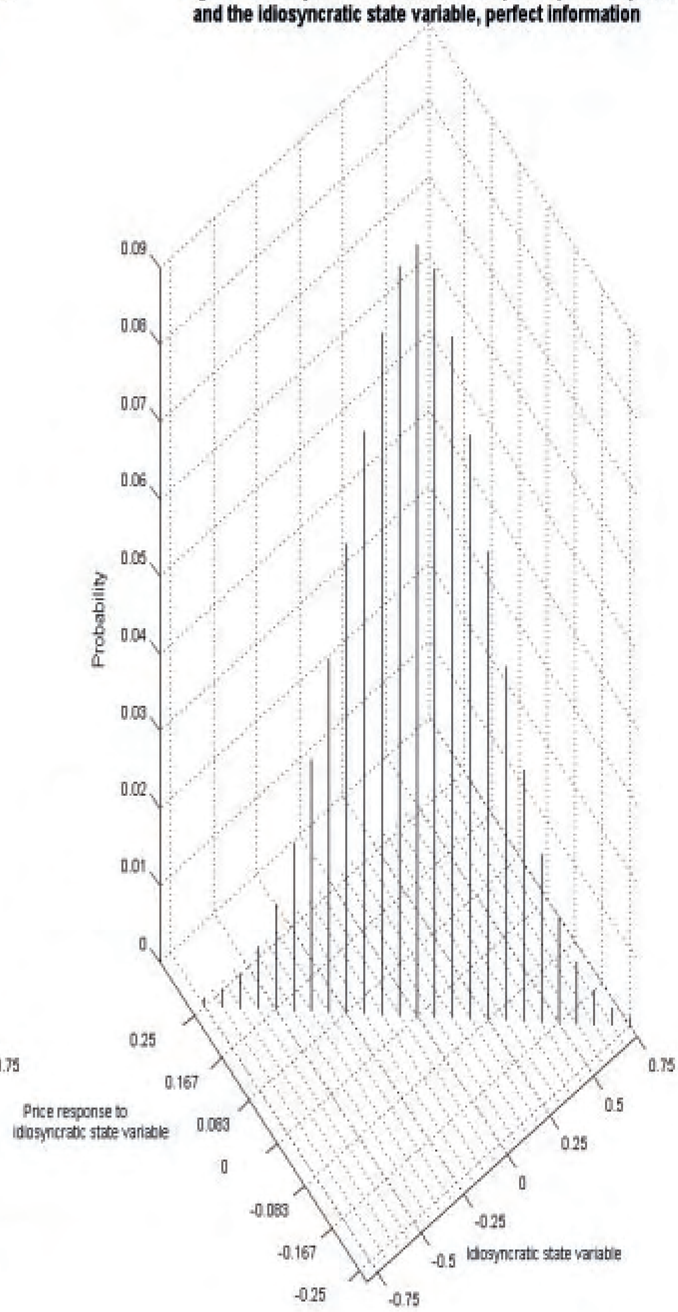


Figure 14: The joint distribution of the optimal price response and nominal aggregate demand, rational inattention

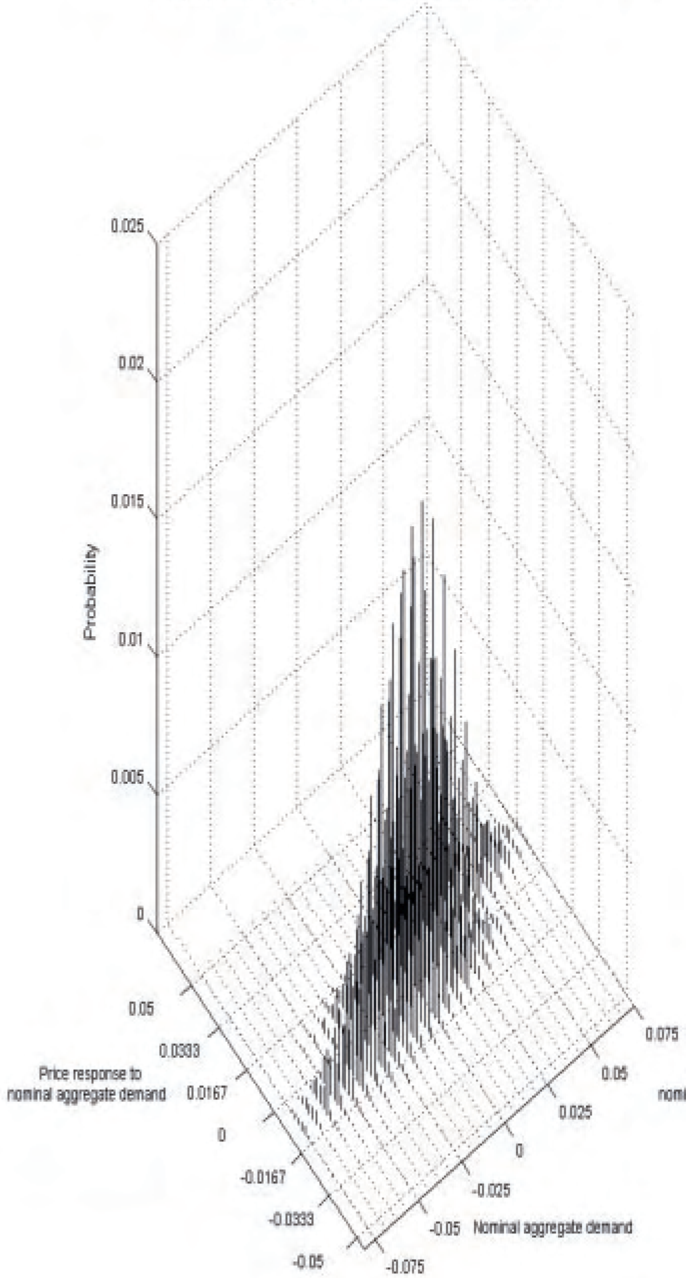


Figure 15: The joint distribution of the optimal price response and nominal aggregate demand, perfect information

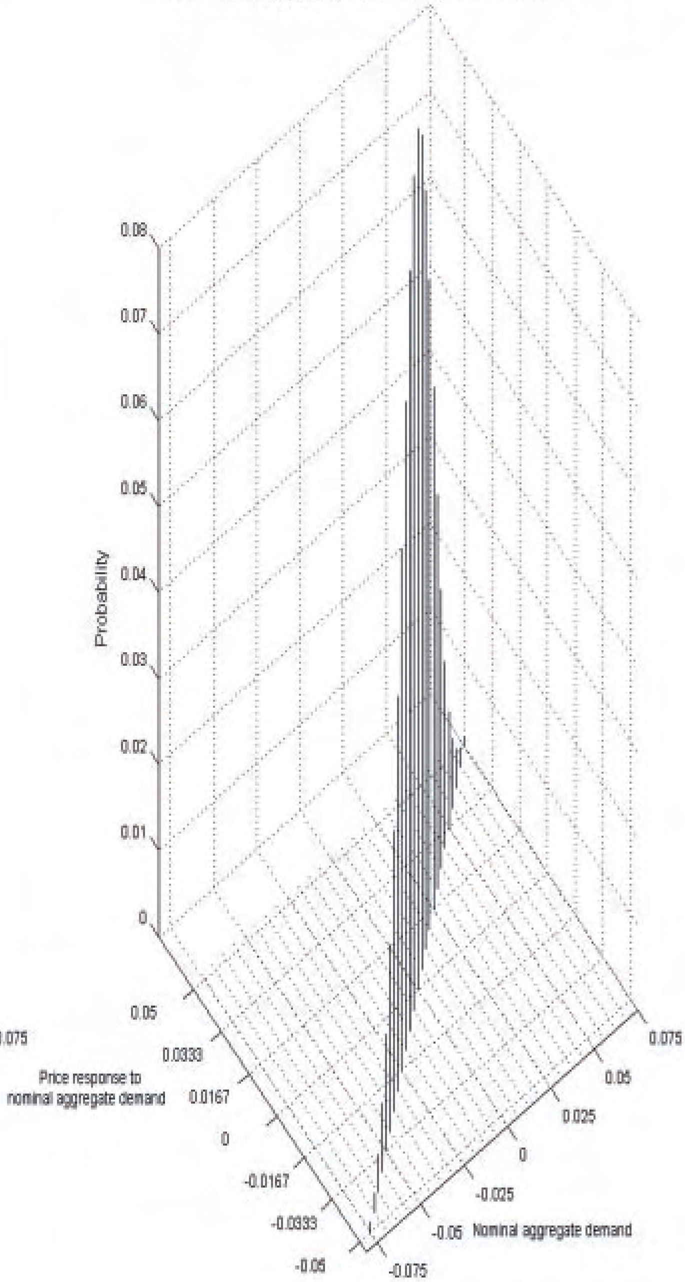
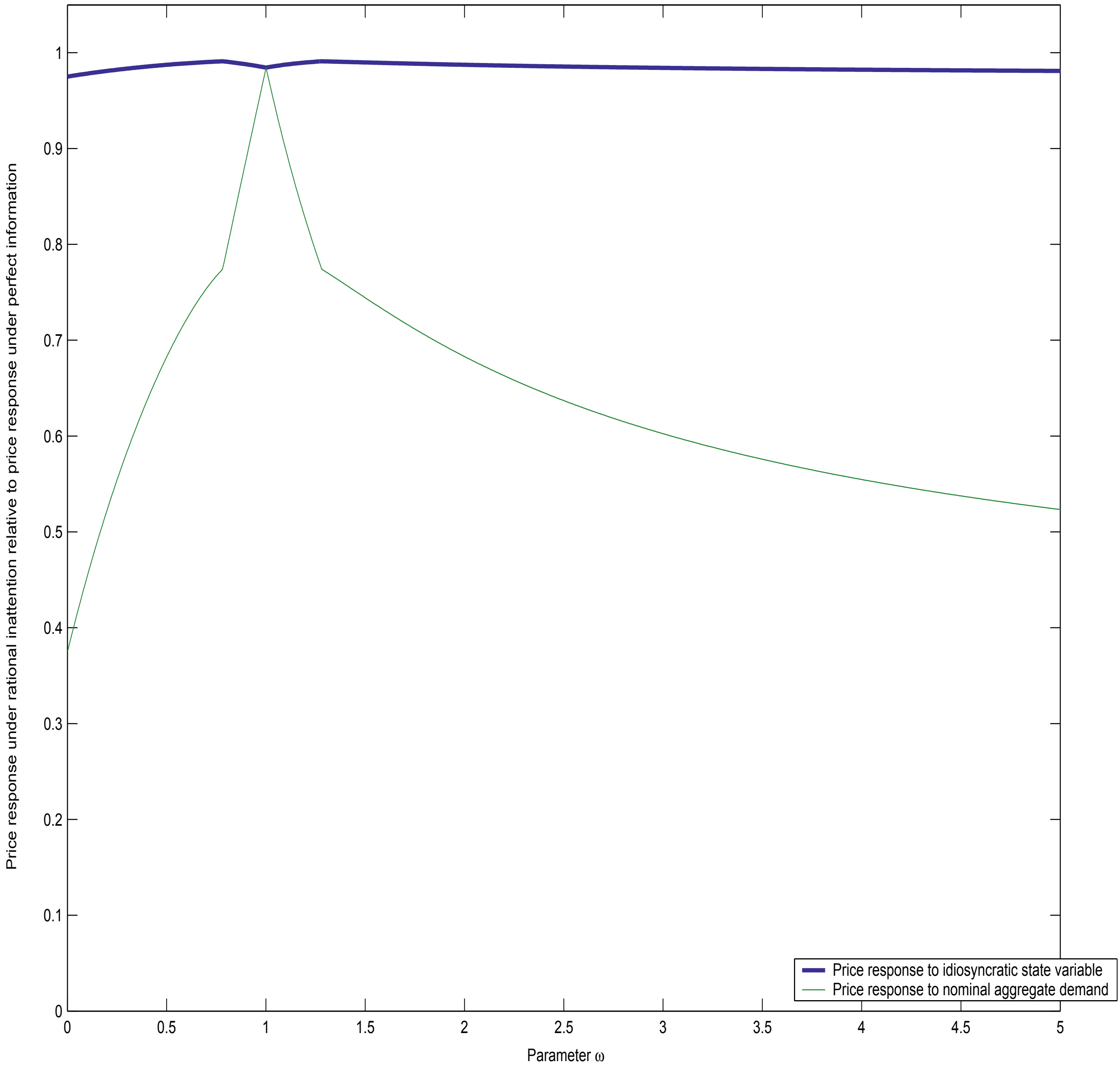


Figure 16: Price responses as a function of the parameter ω, see Section 8.2



Technical Appendix to

Optimal Sticky Prices under Rational Inattention

1 Introduction

This Technical Appendix contains proofs of three results that appear in the paper "Optimal Sticky Prices under Rational Inattention." In Section 2 of the Technical Appendix we derive equation (38). In Section 3 of the Technical Appendix we prove that, after the log-quadratic approximation to the profit function, Gaussian signals are optimal. In Section 4 of the Technical Appendix we derive the relevant equations for the problem studied in Section 8.2.

2 Equilibrium price level in the white noise case

In Section 5, we start from the guess

$$p_t = \alpha q_t \tag{1}$$

and we obtain the actual law of motion

$$p_t^* = \left(1 - 2^{-2\kappa_1^*}\right) \Delta_t, \tag{2}$$

where

$$\kappa_1^* = \begin{cases} \kappa & \text{if } \frac{\sigma_\Delta^2}{\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 \sigma_z^2} \geq 2^{2\kappa} \\ \frac{1}{2}\kappa + \frac{1}{4}\log_2\left(\frac{\sigma_\Delta^2}{\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 \sigma_z^2}\right) & \text{if } \frac{\sigma_\Delta^2}{\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 \sigma_z^2} \in \left[2^{-2\kappa}, 2^{2\kappa}\right] \\ 0 & \text{if } \frac{\sigma_\Delta^2}{\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 \sigma_z^2} \leq 2^{-2\kappa} \end{cases}$$

and

$$\begin{aligned}\Delta_t &= p_t + \frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|} y_t \\ &= \left[\alpha + \frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}(1-\alpha)\right] q_t.\end{aligned}$$

The equilibrium price level is the fixed point of the mapping between the perceived law of motion (1) and the actual law of motion (2). Since the optimal allocation of attention can be a corner solution we have to distinguish three possible cases.

First, suppose that in equilibrium firms allocate no attention to aggregate conditions, $\kappa_1^* = 0$. Then the actual law of motion for the price level is

$$p_t^* = 0.$$

The fixed point of the mapping between the perceived law of motion and the actual law of motion is

$$\alpha = 0. \tag{3}$$

At the fixed point

$$\Delta_t = \frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|} q_t,$$

which implies that $\kappa_1^* = 0$ is an optimal choice at the fixed point if and only if

$$\frac{\sigma_\Delta^2}{\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 \sigma_z^2} = \frac{\left(\frac{\hat{\pi}_{13}}{\hat{\pi}_{11}}\right)^2 \sigma_q^2}{\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 \sigma_z^2} \leq 2^{-2\kappa}.$$

Assuming $\hat{\pi}_{13} > 0$, the weak inequality can also be expressed as

$$\frac{\hat{\pi}_{13}\sigma_q}{|\hat{\pi}_{14}|\,\sigma_z} \leq 2^{-\kappa}. \tag{4}$$

Hence, there exists an equilibrium with $\kappa_1^* = 0$ if and only if the parameters satisfy (4). The equilibrium is given by (3).

Second, suppose that in equilibrium firms allocate all attention to aggregate conditions, $\kappa_1^* = \kappa$. Then the actual law of motion for the price level is

$$p_t^* = \left(1 - 2^{-2\kappa}\right)\left[\alpha + \frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}(1-\alpha)\right] q_t.$$

The fixed point of the mapping between the perceived law of motion and the actual law of motion is characterized by the equation

$$\alpha = \left(1 - 2^{-2\kappa}\right)\left[\alpha + \frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}(1-\alpha)\right],$$

which has the unique solution

$$\alpha = \frac{\left(2^{2\kappa}-1\right)\frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}}{1+\left(2^{2\kappa}-1\right)\frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}}. \tag{5}$$

At the fixed point

$$\Delta_t = \frac{2^{2\kappa}\frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}}{1+\left(2^{2\kappa}-1\right)\frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}} q_t,$$

which implies that $\kappa_1^* = \kappa$ is an optimal choice at the fixed point if and only if

$$\frac{\sigma_\Delta^2}{\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 \sigma_z^2} = \frac{\left(\frac{2^{2\kappa}\frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}}{1+\left(2^{2\kappa}-1\right)\frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}}\right)^2 \sigma_q^2}{\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 \sigma_z^2} \geq 2^{2\kappa}.$$

Assuming $\hat{\pi}_{13} > 0$, the weak inequality can also be expressed as

$$\frac{\hat{\pi}_{13}\sigma_q}{|\hat{\pi}_{14}|\,\sigma_z} \geq 2^{-\kappa} + \left(2^{\kappa} - 2^{-\kappa}\right)\frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}. \tag{6}$$

Hence, there exists an equilibrium with $\kappa_1^* = \kappa$ if and only if the parameters satisfy (6). The equilibrium is given by (5).

Third, suppose that in equilibrium firms allocate attention to aggregate and idiosyncratic conditions, $\kappa_1^* \in (0,1)$. Then the actual law of motion for the price level is

$$\begin{aligned}
p_t^* &= \left(1 - 2^{-2\kappa_1^*}\right)\left[\alpha + \frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}(1-\alpha)\right] q_t \\
&= \left(1 - 2^{-\kappa}\sqrt{\frac{\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 \sigma_z^2}{\sigma_\Delta^2}}\right)\left[\alpha + \frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}(1-\alpha)\right] q_t \\
&= \left(1 - 2^{-\kappa}\sqrt{\frac{\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 \sigma_z^2}{\left[\alpha + \frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}(1-\alpha)\right]^2 \sigma_q^2}}\right)\left[\alpha + \frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}(1-\alpha)\right] q_t.
\end{aligned}$$

The fixed point of the mapping between the perceived law of motion and the actual law of motion is characterized by the equation

$$\alpha=\left(1-2^{-\kappa}\sqrt{\frac{\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^{2}\sigma_z^2}{\left[\alpha+\frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}(1-\alpha)\right]^{2}\sigma_q^2}}\right)\left[\alpha+\frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}(1-\alpha)\right],$$

which can also be written as

$$\alpha=\left[\alpha+\frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}(1-\alpha)\right]-2^{-\kappa}\sqrt{\frac{\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^{2}\sigma_z^2}{\sigma_q^2}}\frac{\left[\alpha+\frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}(1-\alpha)\right]}{\left|\alpha+\frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}(1-\alpha)\right|}. \tag{7}$$

Now there are two possibilities. The first possibility is $\alpha+\frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}(1-\alpha)>0$. In this case, equation (7) becomes

$$\alpha=\left[\alpha+\frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}(1-\alpha)\right]-2^{-\kappa}\sqrt{\frac{\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^{2}\sigma_z^2}{\sigma_q^2}},$$

which has the unique solution

$$\alpha=1-2^{-\kappa}\sqrt{\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{13}}\right)^{2}\frac{\sigma_z^2}{\sigma_q^2}}. \tag{8}$$

At the fixed point

$$\Delta_t=\left[1-2^{-\kappa}\sqrt{\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{13}}\right)^{2}\frac{\sigma_z^2}{\sigma_q^2}}+\frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}2^{-\kappa}\sqrt{\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{13}}\right)^{2}\frac{\sigma_z^2}{\sigma_q^2}}\right]q_t$$

and $\kappa_1^*\in(0,1)$ is an optimal choice at the fixed point if and only if

$$2^{-2\kappa}<\frac{\left[1-2^{-\kappa}\sqrt{\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{13}}\right)^{2}\frac{\sigma_z^2}{\sigma_q^2}}+\frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}2^{-\kappa}\sqrt{\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{13}}\right)^{2}\frac{\sigma_z^2}{\sigma_q^2}}\right]^{2}\sigma_q^2}{\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^{2}\sigma_z^2}<2^{2\kappa}.$$

Assuming $\hat{\pi}_{13}>0$, these inequalities can also be expressed as

$$2^{-\kappa}<\frac{\hat{\pi}_{13}\sigma_q}{|\hat{\pi}_{14}|\,\sigma_z}<2^{-\kappa}+\left(2^{\kappa}-2^{-\kappa}\right)\frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}. \tag{9}$$

The second possibility is $\alpha+\frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}(1-\alpha)<0$. In this case, equation (7) becomes

$$\alpha=\left[\alpha+\frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}(1-\alpha)\right]+2^{-\kappa}\sqrt{\frac{\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^{2}\sigma_z^2}{\sigma_q^2}},$$

which has the unique solution

$$\alpha = 1 + 2^{-\kappa}\sqrt{\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{13}}\right)^2 \frac{\sigma_z^2}{\sigma_q^2}}.$$

At the fixed point

$$\Delta_t = \left[1 + 2^{-\kappa}\sqrt{\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{13}}\right)^2 \frac{\sigma_z^2}{\sigma_q^2}} - \frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|} 2^{-\kappa}\sqrt{\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{13}}\right)^2 \frac{\sigma_z^2}{\sigma_q^2}}\right] q_t$$

and $\kappa_1^* \in (0,1)$ is an optimal choice at the fixed point if and only if

$$2^{-2\kappa} < \frac{\left[1 + 2^{-\kappa}\sqrt{\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{13}}\right)^2 \frac{\sigma_z^2}{\sigma_q^2}} - \frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|} 2^{-\kappa}\sqrt{\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{13}}\right)^2 \frac{\sigma_z^2}{\sigma_q^2}}\right]^2 \sigma_q^2}{\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 \sigma_z^2} < 2^{2\kappa}.$$

Assuming $\hat{\pi}_{13} > 0$, these inequalities can also be expressed as

$$2^{-\kappa} < -\frac{\hat{\pi}_{13}\sigma_q}{|\hat{\pi}_{14}|\,\sigma_z} < 2^{-\kappa} + \left(2^{\kappa} - 2^{-\kappa}\right)\frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}. \tag{10}$$

The first inequality in (10) can never be satisfied. Hence, there exists an equilibrium with $\kappa_1^* \in (0,1)$ if and only if the parameters satisfy (9). The equilibrium is given by (8).

Collecting results yields equation (38) in the paper. Note that there is always a unique linear rational expectations equilibrium.

3 Optimality of Gaussian signals

3.1 The white noise case

So far we have only allowed Gaussian signals. Now we relax this assumption. We assume that the conditional distribution of the variables of interest up to time t given the signals up to time t has a density function. We continue to assume that the joint distribution of the variables of interest up to time t and the signals up to time t is independent of time. In this subsection, we assume that the variables of interest follow a white noise process. After the log-quadratic approximation to the profit function, Gaussian signals are optimal.

Let κ_2 denote the information flow allocated to idiosyncratic conditions

$$\begin{aligned}\kappa_2 &= \mathcal{I}\left(\{z_{it}\};\{s_{2it}\}\right)\\ &= \lim_{T\to\infty}\frac{1}{T} I\left(z_i^T; s_{2i}^T\right).\end{aligned}$$

The mutual information can be expressed as

$$
\begin{aligned}
I\left(z_i^T; s_{2i}^T\right) &= H\left(z_{i1},\ldots,z_{iT}\right) - H\left(z_{i1},\ldots,z_{iT}|s_{2i}^T\right) \\
&= H\left(z_{i1}\right) + \ldots + H\left(z_{iT}\right) - H\left(z_{i1},\ldots,z_{iT}|s_{2i}^T\right) \\
&= H\left(z_{i1}\right) + \ldots + H\left(z_{iT}\right) - \left[H\left(z_{i1}|s_{2i}^T\right) + \ldots + H\left(z_{iT}|z_{i1},\ldots,z_{iT-1},s_{2i}^T\right)\right] \\
&\geq H\left(z_{i1}\right) + \ldots + H\left(z_{iT}\right) - \left[H\left(z_{i1}|s_{2i}^1\right) + \ldots + H\left(z_{iT}|s_{2i}^T\right)\right] \\
&= TI\left(z_{it}; s_{2i}^t\right).
\end{aligned}
$$

The first equality follows from the fact that mutual information equals the difference between entropy and conditional entropy. The second equality follows from the fact that the entropy of independent random variables equals the sum of their entropies. The third equality follows from the chain rule for entropy. The weak inequality follows from the fact that conditioning reduces entropy. See Cover and Thomas (1991), p. 232, for these results. The last equality follows from the stationarity assumption. Furthermore,

$$
\begin{aligned}
I\left(z_{it}; s_{2i}^t\right) &= H\left(z_{it}\right) - H\left(z_{it}|s_{2i}^t\right) \\
&= H\left(z_{it}\right) - E\left[H\left(z_{it}|s_{2i}^t = \tilde{s}_{2i}^t\right)\right] \\
&\geq H\left(z_{it}\right) - E\left[\frac{1}{2}\log_2\left(2\pi e \sigma^2_{z|s_2^t=\tilde{s}_2^t}\right)\right] \\
&\geq H\left(z_{it}\right) - \frac{1}{2}\log_2\left(2\pi e E\left[\sigma^2_{z|s_2^t=\tilde{s}_2^t}\right]\right) \\
&= \frac{1}{2}\log_2\left(2\pi e \sigma_z^2\right) - \frac{1}{2}\log_2\left(2\pi e E\left[\sigma^2_{z|s_2^t=\tilde{s}_2^t}\right]\right).
\end{aligned}
$$

The first equality follows from the fact that mutual information equals the difference between entropy and conditional entropy. The second equality follows from the definition of conditional entropy, where $\tilde{s}_{2i}^t$ denotes a realization of s_{2i}^t. The first weak inequality follows from the fact that the normal density maximizes entropy over all densities with the same variance. See Cover and Thomas (1991), chapter 11. The second weak inequality follows from Jensen's inequality. The last equality follows from the equation for the entropy of a normal distribution. Together these results imply

$$
\kappa_2 \geq \frac{1}{2}\log_2\left(\frac{\sigma_z^2}{E\left[\sigma^2_{z|s_2^t=\tilde{s}_2^t}\right]}\right).
$$

After the log-quadratic approximation to the profit function, the expected period t loss in profits due to imperfect tracking of idiosyncratic conditions equals

$$\frac{|\hat{\pi}_{11}|}{2}\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 E\left[\left(z_{it}-E[z_{it}|s_{2i}^t]\right)^2\right]=\frac{|\hat{\pi}_{11}|}{2}\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 E\left[\sigma^2_{z|s_2^t=\tilde{s}_2^t}\right].$$

The weak inequality given above implies that

$$E\left[\sigma^2_{z|s_2^t=\tilde{s}_2^t}\right]\geq 2^{-2\kappa_2}\sigma_z^2.$$

It is easy to verify that a Gaussian white noise signal of the form $s_{2it}=z_{it}+\psi_{it}$ attains this bound. See Section 5 of the paper or simply note that in this case all the weak inequalities given above hold with equality. Hence, a Gaussian white noise signal is optimal.

The same arguments yield that, after the log-quadratic approximation to the profit function, a Gaussian white noise signal of the form $s_{1it}=\Delta_t+\varepsilon_{it}$ is optimal.

3.2 The general case

We now turn to the general case where the variables being tracked follow arbitrary stationary Gaussian processes. We again assume that the conditional distribution of the variables of interest up to time t given the signals up to time t has a density function. Furthermore, we continue to assume that the joint distribution of the variables of interest up to time t and the signals up to time t is independent of time. We also continue to assume that firms receive a long sequence of signals in period one. We prove the following result. After the log-quadratic approximation to the profit function, Gaussian signals are optimal.

Let κ_2 denote the information flow allocated to idiosyncratic conditions

$$\begin{aligned}\kappa_2 &= \mathcal{I}\left(\{z_{it}\};\{s_{2it}\}\right)\\ &= \lim_{T\to\infty}\frac{1}{T}I\left(z_i^T;s_{2i}^T\right),\end{aligned}$$

where $z_i^T\equiv(z_{i1},\ldots,z_{iT})$ and $s_{2i}^T\equiv\left(s_{2i}^1,s_{2i2},\ldots,s_{2iT}\right)$. The mutual information can be expressed as

$$
\begin{aligned}
I\left(z_i^T; s_{2i}^T\right) &= H\left(z_i^T\right) - H\left(z_i^T | s_{2i}^T\right) \\
&= H\left(z_i^T\right) - E\left[H\left(z_i^T | s_{2i}^T = \tilde{s}_{2i}^T\right)\right] \\
&\geq H\left(z_i^T\right) - E\left[\frac{1}{2}\log_2\left\{(2\pi e)^T \det \Omega_{zz|s_2^T=\tilde{s}_2^T}\right\}\right] \qquad (11) \\
&= H\left(z_i^T\right) - \frac{1}{2}\log_2\left\{(2\pi e)^T\right\} - \frac{1}{2}E\left[\log_2\left\{\det \Omega_{zz|s_2^T=\tilde{s}_2^T}\right\}\right] \\
&\geq H\left(z_i^T\right) - \frac{1}{2}\log_2\left\{(2\pi e)^T\right\} - \frac{1}{2}\log_2\left\{\det E\left[\Omega_{zz|s_2^T=\tilde{s}_2^T}\right]\right\}. \qquad (12)
\end{aligned}
$$

The first equality follows from the fact that mutual information equals the difference between entropy and conditional entropy. The second equality follows from the definition of conditional entropy, where $\tilde{s}_{2i}^T$ denotes a realization of s_{2i}^T. The weak inequality (11) follows from the fact that the multivariate normal density maximizes entropy over all densities with the same covariance matrix. See Cover and Thomas (1991), chapter 11. The weak inequality (12) follows from Ky Fan's inequality which states that the log of the determinant of a symmetric nonnegative definite matrix is a concave function. See Cover and Thomas (1991), page 501. If z_i^T and s_{2i}^T have a multivariate normal distribution then the conditional distribution of z_i^T given s_{2i}^T is a normal distribution and the conditional covariance matrix of z_i^T given s_{2i}^T is independent of the realization of s_{2i}^T. In this case, the weak inequalities (11) and (12) hold with equality. Hence, for a given expected conditional covariance matrix $E\left[\Omega_{zz|s_2^T=\tilde{s}_2^T}\right]$, the mutual information $I\left(z_i^T; s_{2i}^T\right)$ is minimized by a multivariate normal distribution for z_i^T and s_{2i}^T.

After the log-quadratic approximation to the profit function, the expected discounted sum of losses in profits due to imperfect tracking of idiosyncratic conditions equals

$$
\begin{aligned}
& E\left[\sum_{t=1}^{\infty}\beta^t \frac{|\hat{\pi}_{11}|}{2}\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 \left(z_{it} - E\left[z_{it}|s_{2i}^t\right]\right)^2\right] \\
&= \sum_{t=1}^{\infty}\beta^t \frac{|\hat{\pi}_{11}|}{2}\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 E\left[\left(z_{it} - E\left[z_{it}|s_{2i}^t\right]\right)^2\right] \qquad (13) \\
&= \sum_{t=1}^{\infty}\beta^t \frac{|\hat{\pi}_{11}|}{2}\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 E\left[E\left[\left(z_{it} - E\left[z_{it}|s_{2i}^t\right]\right)^2 |s_{2i}^t\right]\right] \\
&= \sum_{t=1}^{\infty}\beta^t \frac{|\hat{\pi}_{11}|}{2}\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 E\left[\sigma^2_{z_t|s_2^t=\tilde{s}_2^t}\right]. \qquad (14)
\end{aligned}
$$

The assumption that the joint distribution of z_i^t and s_{2i}^t is independent of t in combination with the assumption that firms receive a long sequence of signals in period one implies that $E\left[\sigma^2_{z_t|s_2^t=\tilde{s}_2^t}\right]$ is independent of t for all $t \geq 1$. Equation (13) becomes

$$E\left[\sum_{t=1}^{\infty}\beta^t \frac{|\hat{\pi}_{11}|}{2}\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2\left(z_{it} - E\left[z_{it}|s_{2i}^t\right]\right)^2\right] = \frac{\beta}{1-\beta}\frac{|\hat{\pi}_{11}|}{2}\left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 E\left[\sigma^2_{z_T|s_2^T=\tilde{s}_2^T}\right], \quad (15)$$

for any $T \geq 1$.

Equation (15) implies that the expected discounted sum of losses in profits due to imperfect tracking of idiosyncratic conditions only depends on the expected conditional variance $E\left[\sigma^2_{z_T|s_2^T=\tilde{s}_2^T}\right]$. Furthermore, the expected conditional variance $E\left[\sigma^2_{z_T|s_2^T=\tilde{s}_2^T}\right]$ is the (T,T) element of the expected conditional covariance matrix $E\left[\Omega_{zz|s_2^T=\tilde{s}_2^T}\right]$. Finally, we proved above that, for any expected conditional covariance matrix $E\left[\Omega_{zz|s_2^T=\tilde{s}_2^T}\right]$, the mutual information $I\left(z_i^T; s_{2i}^T\right)$ is minimized by a multivariate normal distribution for z_i^T and s_{2i}^T. Hence, Gaussian signals are optimal.

The same arguments yield that Gaussian signals about aggregate conditions are optimal.

4 Attending to variables that reveal information about both aggregate and idiosyncratic conditions

Let the profit function be given by equation (19) in the paper. Then the price set by firm i in period t is given by equation (20) in the paper and the profit-maximizing price is given by equation (21) in the paper. For simplicity, consider the case where q_t and z_{it} follow Gaussian white noise processes and $p_t = \alpha q_t$. For ease of exposition, assume that $(\hat{\pi}_{14}/|\hat{\pi}_{11}|) = 1$. Suppose that firm i can choose signals of the form

$$\begin{aligned} s_{1it} &= \Delta_t + \omega z_{it} + \varepsilon_{it}, \\ s_{2it} &= \omega\Delta_t + z_{it} + \psi_{it}, \end{aligned}$$

where the parameter $\omega \geq 0$ and $\{\varepsilon_{it}\}$ and $\{\psi_{it}\}$ are idiosyncratic Gaussian white noise processes that are mutually independent and independent of $\{\Delta_t\}$ and $\{z_{it}\}$. The price set by firm i in period t equals

$$\begin{aligned}
p_{it}^{*} &= E\left[p_{it}^{\diamondsuit}|s_i^t\right] \\
&= \frac{\left(\omega^2-1\right)\left(\omega-1\right)\frac{\sigma_\Delta^2}{\sigma_\varepsilon^2}\frac{\sigma_z^2}{\sigma_\psi^2}+\left(1+\omega\frac{\sigma_z^2}{\sigma_\Delta^2}\right)\frac{\sigma_\Delta^2}{\sigma_\varepsilon^2}}{\left(\omega^2-1\right)^2\frac{\sigma_\Delta^2}{\sigma_\varepsilon^2}\frac{\sigma_z^2}{\sigma_\psi^2}+\left(1+\omega^2\frac{\sigma_z^2}{\sigma_\Delta^2}\right)\frac{\sigma_\Delta^2}{\sigma_\varepsilon^2}+\left(\omega^2\frac{\sigma_\Delta^2}{\sigma_z^2}+1\right)\frac{\sigma_z^2}{\sigma_\psi^2}+1}s_{1it} \\
&\quad +\frac{\left(\omega^2-1\right)\left(\omega-1\right)\frac{\sigma_\Delta^2}{\sigma_\varepsilon^2}\frac{\sigma_z^2}{\sigma_\psi^2}+\left(\omega\frac{\sigma_\Delta^2}{\sigma_z^2}+1\right)\frac{\sigma_z^2}{\sigma_\psi^2}}{\left(\omega^2-1\right)^2\frac{\sigma_\Delta^2}{\sigma_\varepsilon^2}\frac{\sigma_z^2}{\sigma_\psi^2}+\left(1+\omega^2\frac{\sigma_z^2}{\sigma_\Delta^2}\right)\frac{\sigma_\Delta^2}{\sigma_\varepsilon^2}+\left(\omega^2\frac{\sigma_\Delta^2}{\sigma_z^2}+1\right)\frac{\sigma_z^2}{\sigma_\psi^2}+1}s_{2it}.
\end{aligned}$$

The expected period t loss in profits equals

$$\frac{|\hat{\pi}_{11}|}{2}E\left[\left(p_{it}^{\diamondsuit}-p_{it}^{*}\right)^2\right]=\frac{|\hat{\pi}_{11}|}{2}\frac{\left(\omega-1\right)^2\sigma_\Delta^2\frac{\sigma_z^2}{\sigma_\psi^2}+\left(\omega-1\right)^2\sigma_z^2\frac{\sigma_\Delta^2}{\sigma_\varepsilon^2}+\sigma_\Delta^2+\sigma_z^2}{\left(\omega^2-1\right)^2\frac{\sigma_\Delta^2}{\sigma_\varepsilon^2}\frac{\sigma_z^2}{\sigma_\psi^2}+\left(1+\omega^2\frac{\sigma_z^2}{\sigma_\Delta^2}\right)\frac{\sigma_\Delta^2}{\sigma_\varepsilon^2}+\left(\omega^2\frac{\sigma_\Delta^2}{\sigma_z^2}+1\right)\frac{\sigma_z^2}{\sigma_\psi^2}+1}. \tag{16}$$

The information flow equals

$$\begin{aligned}
&\mathcal{I}\left(\{P_t,Z_{it}\};\{s_{it}\}\right) \\
&= \mathcal{I}\left(\{p_t,z_{it}\};\{s_{it}\}\right) \\
&= I\left(p_t,z_{it};s_{it}\right) \\
&= H\left(s_{it}\right)-H\left(s_{it}|p_t,z_{it}\right) \\
&= H\left(s_{1it},s_{2it}\right)-H\left(s_{1it},s_{2it}|\Delta_t,z_{it}\right) \\
&= \frac{1}{2}\log_2\left[\left(\omega^2-1\right)^2\frac{\sigma_\Delta^2}{\sigma_\varepsilon^2}\frac{\sigma_z^2}{\sigma_\psi^2}+\left(1+\omega^2\frac{\sigma_z^2}{\sigma_\Delta^2}\right)\frac{\sigma_\Delta^2}{\sigma_\varepsilon^2}+\left(\omega^2\frac{\sigma_\Delta^2}{\sigma_z^2}+1\right)\frac{\sigma_z^2}{\sigma_\psi^2}+1\right]. \qquad (17)
\end{aligned}$$

The second equality follows from the assumption that p_t, z_{it} and $s_{it}=(s_{1it},s_{2it})$ follow white noise processes. The third equality follows from the fact that mutual information equals the difference between entropy and conditional entropy. The fourth equality follows from the fact that p_t and Δ_t contain the same information in the white noise case. The fifth equality follows from the expressions for the entropy and the conditional entropy of a multivariate normal distribution.

Choosing the signal-to-noise ratios $\left(\sigma_\Delta^2/\sigma_\varepsilon^2\right)$ and $\left(\sigma_z^2/\sigma_\psi^2\right)$ so as to minimize the expected period t loss in profits (16) subject to a constraint on the information flow (17) is a standard constrained minimization problem. The solution depends on ω and $(\sigma_\Delta^2/\sigma_z^2)$. Suppose that $(\sigma_\Delta^2/\sigma_z^2)<1$. Then there is a critical value $\bar{\omega}\in[0,1)$. For $\omega\in[0,\bar{\omega})$ the firm decides to receive both signals. For $\omega \in [\bar{\omega}, 1)$ the firm decides to receive only signal two. At $\omega = 1$ the firm is indifferent between receiving only signal two and receiving only signal one. For $\omega \in \left(1, \frac{1}{\bar{\omega}}\right]$ the firm decides to receive only signal one. For $\omega > \frac{1}{\bar{\omega}}$ the firm decides to receive both signals. So long as $\omega \neq 1$, the price set by firm i responds more to idiosyncratic conditions than to aggregate conditions. As $\omega \to 0$ or $\omega \to \infty$ the solution converges to the solution presented in Section 5 of the paper.

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