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MODEL OF THE UNITED STATES ECONOMY WITH LEARNING MUSEL

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Abstract

The model presented here is an estimated medium-scale model for the United States (US) economy developed to forecast and analyse policy issues for the US. The model is specified to track the deviation of the medium-run developments from the balanced-growth-path via an estimated CES production function for the private sector, where factor augmenting technical progress is not constrained to evolve at a constant rate. The short-run deviations from the medium run are estimated based on three optimising private sector decision making units: firms, trade unions and households. We assume agents optimise under limited-information model-consistent learning, where each agent knows the parameters related to his/her optimization problem. Under this learning approach the effect of a monetary policy shock on output and inflation is more muted but persistent than under rational expectations, but both specifications are broadly comparable to other US macro models. Using the learning version, we find stronger expansionary effects of an increase in government expenditure during periods of downturns compared to booms.

Keywords: Macro model, Open-economy macroeconomics, Rational expectations, Learning.

JEL Classification: C51, C6, E5

1 Non-Technical Summary

In this paper we present an estimated medium-scale model of the US economy that forms part of the ECB's New Multi-Country Model and is used for projections and scenario analysis. The model is specified to track both the short-run dynamics of economic variables around the medium-run equilibrium and also the deviations of the medium-run developments from the balanced-growth-path. In the medium-run, the supply-side is modelled allowing for a non-unitary elasticity of substitution between labour and capital and factor augmenting technical progress that is not constrained to evolve at a constant rate. We estimate the supply side by non-linear Seemingly Unrelated Regression (SUR), which allows for cross-equation restrictions in the medium-run. The dynamic equations are estimated by the Generalised Methods of Moments (GMM) estimator, which allows for limited information bounded rational expectations and as such does not require the model to be fully specified.

The model has firm micro-economic foundations with the theoretic core containing one privately produced exportable domestic good, one imported good and a government sector good for public consumption. All central behavioural relations are based on the optimisation behaviour of three private sector decision making units, namely utility maximising households, profit maximising firms and trade unions, as well as reaction functions for the government sector and the central bank. Monopolistically competing firms set prices, inventories, fixed investment and employment under the assumptions of indivisible labour. Output is in the short run demand-determined. Overlapping generation households make consumption/saving decisions and monopolistic unions set wages by minimising the quadratic loss function under the staggered wage adjustment assumption. There are assumed to be two type of trade unions. The first type is utilitarian, maximising the utility of member households, whilst the rest are non-utilitarian, keeping wage developments in line with productivity developments, coupled with a high desired employment rate. The theoretical framework for the US economy is slightly different to that of the euro area countries of the New Multi-Country Model. This reflects the availability of a richer dataset that allows for modelling the housing market behaviour and also the separate modelling of private and government sector production, employment, capital formation as well as price and wage setting.

The formation of expectations is treated explicitly, although we drop the typical assumption that all optimizing agents know fully the whole structure and all parameter values of the model, including the stochastic processes generating exogenous shocks. Instead the framework is based on bounded rationality, where each agent knows only the parameters related to her optimization problem but does not need to know the rest of the model, i.e. other agent's optimising problems, nor, more importantly, the stochastic processes driving the variables predetermined to each agent. This modelling strategy allows the use of the model under rational expectations and to flexibly vary the degree to which the exact stochastic nature of the shock is correctly anticipated, as well as the use of the model under boundedly rational learning, where the true stochastic nature of the shock is gradually learned.

In this paper, indicative simulations are undertaken under the assumption that agents' expectations are based on this learning approach or under rational, model consistent expectations. These simulations suggest plausible impulse responses of the model to exogenous shocks. Under learning, the effect of a monetary policy shock on output and inflation is more muted, but at the same time more persistent than under rational expectations, but the responses under both assumptions are broadly comparable to other US macro models. Using the learning version, we find stronger expansionary effects of an increase in government expenditure during periods of downturns compared to booms.

2 Introduction

We present an estimated medium-scale model of the US economy developed at the European Central Bank. Given the need by central bankers to analyse the interaction of a large set of variables in the economy, small models, by their nature, have limited policy use. Therefore, we have aimed to build a model with economic coherence that matches the key characteristics of the data and that is useful to build projections and to analyse policy issues via scenario analysis. The model builds upon the structure of Dieppe et al (2012 and 2013) and is intended to form part of the New Multi-Country Model (NMCM) which covers the 5 biggest euro area countries (Germany, France, Italy, Spain and the Netherlands) and a rest-of-the euro area block. As such it could be used either on a single country basis or as a linked multi-country model.[1] The model enables us to analyse the interaction of fiscal, monetary, and competitiveness issues, just to name a few of the possibilities.

The model has firm micro-economic foundations with the theoretic core containing one privately produced exportable domestic good, one imported good and a government sector good for public consumption. All central behavioural relations are based on the optimisation behaviour of three private sector decision making units, namely utility maximising households, profit maximising firms and trade unions, as well as reaction functions for the government sector and the central bank. Monopolistically competing firms set prices, inventories, fixed investment and employment under the assumptions of indivisible labour. Output is in the short run demand-determined. Overlapping generation households make consumption/saving decisions and monopolistic unions set wages by minimising the quadratic loss function under the staggered wage adjustment assumption. There are assumed to be two type of trade unions. The first type is utilitarian, maximising the utility of member households, whilst the rest are non-utilitarian keeping wage developments in line with productivity developments, coupled with a high desired employment rate.

The theoretical framework for the US economy is slightly different to that of the euro area countries as introduced in Dieppe et al (2012 and 2013). This reflects the availability of a richer dataset that allows for modelling the housing market behaviour and also the separate modelling of the private and government sector production, employment, capital formation and price and wage setting.

The model is specified to track both the short-run dynamics around the medium-run equilibrium and also deviations of the medium-run developments from the balanced-growth-path. This is executed by a careful modelling of the supply-side by using a CES structure, which allows for a non-unitary elasticity of substitution between labour and capital and factor augmenting technical progress that is not constrained to evolve at a constant rate over time. We estimate the supply side by non-linear SUR, which allows for cross-equation

[1] This aspect will be explored in a separate paper, here we consider the model as a single country.

restrictions in the medium-run. The dynamic equations are estimated by GMM, which does not require the model to be fully specified and therefore allows for limited information bounded rational expectations.

The formation of expectations is treated explicitly. As such, the model can be characterised as a micro-founded New Keynesian model. However, unlike typically in the DSGE models the optimizing agents are not assumed to have full information on the structure and the parameter values of the model including the stochastic processes generating exogenous shocks. Instead, in estimating the model bounded rationality is assumed where each agent knows only the parameters related to her optimization problem but does not need to know the rest of the model, i.e. other agents' optimising problems, nor, more importantly, the stochastic processes driving the variables predetermined to each agent. This modelling strategy allows the use of the model under rational expectations and to flexibly vary the degree to which the exact stochastic nature of the shock is correctly anticipated, as well as the use of the model under boundedly rational learning, where the true stochastic nature of the shock is gradually learned.

We start by specifying the production function and the medium-run supply system which forms the core of the model and then go into some detail on the framework and estimation of the model, starting with firm and union behaviour, followed by consumer behaviour before turning to the foreign trade block. Thereafter, we present some indicative simulations undertaken with agents' expectations based on either rational, model consistent expectations or on learning. These simulations suggest that the impulse responses of the model to exogenous shocks depend strongly on the underlying expectations assumptions.

3 Production function and the medium-run supply system

As discussed by Dieppe et al (2012 and 2013), McAdam and Willman (2013a), Solow (2000) and Blanchard (1997), we adopt a *medium-run view* regarding the underlying "trend" developments of our data, which *is* allowed to deviate from the Balanced Growth Path (BGP). However, this view does not exclude the possibility that many processes eventually converge to the BGP. Accordingly, we allow for a non-unitary elasticity of substitution and factor augmenting technical progress that is not constrained to evolve at a constant rate. We achieve this following McAdam and Willman (2010 and 2013), by assuming a normalised *Constant Elasticity of Substitution (CES) production functio*n and factor augmenting technical progress that is modelled in terms of Box-Cox functions (Box and Cox 1964).

In this section we start by specifying the explicit form of the private sector production function. Thereafter we present the 5-equation medium-run supply-system implied by the first order conditions of the profit maximisation of the firm and the utility maximisation of labour unions. The parameter estimates of this system defines empirically the production function, the marginal products of inputs and the mark-up needed in estimating dynamic private sector labour demand, capital formation, inventory, export, price and wage equations. Accordingly, the medium run developments define the path towards which the short-run dynamics converges.

3.1 The Normalised private sector CES production function

The output of a firm is defined by the general production function,

$$Y_t^P = F(K_t, H_t) \tag{1}$$

Y_t^P = private sector output[2], $K_t = \kappa_t K_t^P$, private non-housing sector capital input in efficiency units, κ_t = capital utilisation rate, K_t^P= private sector capital stock, H_t= $N_t^P h_t$ = private sector labour input, which is measured in terms of 'effective' labour hours, N_t^P = the number of employees in private sector, and h_t = 'effective' working hours per employee. In the following we assume that labour and capital utilisation rates are equal; which under constant returns to scale also corresponds to the capacity utilisation rate $u_t = \kappa_t/\kappa_0 = h_t/h_0$.[3]

Our production function is assumed to take the form of a "normalized" CES function, allowing for time-varying factor augmenting technical progress. The importance of explicitly normalizing CES functions was discovered by La Grandville (1989) and first implemented empirically by Klump, McAdam and Willman (2007a). Normalization starts from the observation that a family of CES functions whose members are distinguished only by different substitution elasticities need a common benchmark point. Since the elasticity of substitution is defined as a point elasticity, one needs to fix benchmark values for the level of production, factor inputs and marginal rate of substitution, or equivalently for per-capita production, capital deepening and factor income shares. The normalized CES production function, corresponding to the general function (1), is given by[4]:

$$\frac{Y_t^P}{Y_0^P} = \left\{ \pi_0 \left[\Gamma_K(t,t_0)\, u_t \frac{K_t^P}{K_0^P} \right]^{\frac{\sigma-1}{\sigma}} + (1-\pi_0) \left[\Gamma_N(t,t_0)\, u_t \frac{N_t^P}{N_0^P} \right]^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1}} \tag{2}$$

where σ is the elasticity of substitution between private non-residential capital and private sector labour, π_0 the distribution parameter equalling the capital share evaluated at the normalization point (subscript 0) and $\Gamma_i(t,t_0)$ defines the (indexed) level of technical progress associated *with* factor i (with $\Gamma_i(t_0,t_0) = 1$).

As it is not obvious that the growth rate of technical progress should be constant over time, we follow an agnostic approach and model technical progress drawing on a well-known flexible, functional form (Box and Cox, 1964):

$$\log\left[\Gamma_i(t,t_0,\gamma_i,\lambda_i)\right] = \frac{\gamma_i t_0}{\lambda_i}\left[\left(\frac{t}{t_0}\right)^{\lambda_i} - 1\right] \tag{3}$$

where $i = N, K$. The log level of technical progress, $\Gamma_i(\bullet)$ is, therefore, a function of time, t (around its normalization point t_0), a curvature parameter, λ_i, and has a growth rate of γ_i at the representative point of normalization.[5] When λ_i =1 (=0) [<0], technical progress displays linear (log-linear) [hyperbolic] dynamics:

$$\log \Gamma_i(t) \Rightarrow \begin{cases} \lim_{t\to\infty}\left[\log \Gamma_i(t)\right] = \infty & \text{if } \lambda_i \geqslant 0 \\ \lim_{t\to\infty}\left[\log \Gamma_i(t)\right] = -\frac{\gamma_i t_0}{\lambda_i} > 0 & \text{if } \lambda_i < 0 \end{cases} \tag{4}$$

[2]For notation we use superscript ^{P} to denote private and ^{G} to denote government sector. Where there is no superscript it is either implicit or refers to the whole economy.

[3]McAdam and Willman (2011) discuss the case where this constraint is relaxed.

[4]León-Ledesma, McAdam and Willman (2010) and Klump, McAdam and Willman (2012) discuss and evaluate normalization more extensively.

[5]Note that we scaled the Box-Cox specification by t0 to interpret γ_N and γ_K as the rates of labour- and capital-augmenting technical change at the fixed (i.e., representative) point.

$$\frac{\partial \log \Gamma_i(\mathrm{t})}{\partial \mathrm{t}} = \gamma_i \left(t/t_0\right)^{\lambda_i - 1} \tag{5}$$

$$\Rightarrow \begin{cases} = \gamma_i \left(t/t_0\right)^{\lambda_i - 1} > 0; \lim_{t\to\infty} \frac{\partial \log \Gamma_i(\mathrm{t})}{\partial \mathrm{t}} = \infty & \text{if } \lambda_i > 1 \\ = \gamma_i, \forall t & \text{if } \lambda_i = 1 \\ \geqslant 0\,; \lim_{t\to\infty} \frac{\partial \log \Gamma_i(\mathrm{t})}{\partial \mathrm{t}} = 0 & \text{if } \lambda_i < 1 \end{cases} \tag{6}$$

Thus, if $\lambda_i \geq 0$, the level of technical progress accruing from factor i tends to infinity but is bounded otherwise (4). If $\lambda_i = 1$ the growth of technical progress is constant (i.e. the "text-book" case) but asymptotes to zero from above for any $\lambda_i < 1$, (5).

3.2 The supply system framework

All central behavioural relations are based on the optimisation behaviour of three private sector decision making units, namely utility maximising households, profit maximising firms and trade unions. The steady-state form of the profit maximizing firm can be reduced to a four-equation supply-side system. Further, by incorporating into the system the wage setting equation of the unions that maximise the utility of its member households, the steady-state system can be transformed into the following fully-fledged five-equation system:

$$\log\left(\frac{P_t^y Y_t^P}{w_t^P N_t^P + q_t K_t^P}\right) - \log\left(1+\mu\right) = 0 \tag{7}$$

$$\log\left(\frac{w_t^P N_t^P}{P_t^y Y_t^P}\right) - \log\left(1-\bar{\pi}\right) - \frac{1-\sigma}{\sigma}\left[\log\left(\frac{Y_t^P/\bar{Y}^P}{N_t^P/\bar{N}^P}\right) - \log\xi - \frac{\bar{t}\gamma_N}{\lambda_N}\left(\left(\frac{t}{\bar{t}}\right)^{\lambda_N} - 1\right)\right] + \log\left(1+\mu\right) = 0 \tag{8}$$

$$\log\left(\frac{q_t K_t^P}{P_t^y Y_t^P}\right) - \log\left(\bar{\pi}\right) - \frac{1-\sigma}{\sigma}\left[\log\left(\frac{Y_t^P/\bar{Y}^P}{K_t^P/\bar{K}^P}\right) - \log\xi - \frac{\bar{t}\gamma_K}{\lambda_K}\left(\left(\frac{t}{\bar{t}}\right)^{\lambda_K} - 1\right)\right] + \log\left(1+\mu\right) = 0 \tag{9}$$

$$\log\left(\frac{Y_t^P/\bar{Y}^P}{N_t^P/\bar{N}^P}\right) - \log\left(\xi\right) - \frac{\bar{t}\gamma_N}{\lambda_N}\left(\left(\frac{t}{\bar{t}}\right)^{\lambda_N} - 1\right) + \frac{\sigma}{1-\sigma}\log\left[\bar{\pi} e^{\frac{1-\sigma}{\sigma}\left[\frac{\bar{t}\gamma_N}{\lambda_N}\left(\left(\frac{t}{\bar{t}}\right)^{\lambda_N}-1\right) - \frac{\bar{t}\gamma_K}{\lambda_K}\left(\left(\frac{t}{\bar{t}}\right)^{\lambda_K}-1\right)\right]}\left(\frac{K_t^P/\bar{K}^P}{N_t^P/\bar{N}^P}\right)^{\frac{\sigma-1}{\sigma}} + \left(1-\bar{\pi}\right)\right] = 0 \tag{10}$$

$$\log\left(\frac{N_t^F w_t}{P_t^c C_t}\right) + \log\left\{\sigma - 1 + \left(1-\bar{\pi}\right)\left[\frac{Y_t^P/\left(\xi\cdot\bar{Y}^P\right)}{\frac{N_t^P}{\bar{N}^P} e^{\frac{\bar{t}\gamma_N}{\lambda_N}\left(\left(\frac{t}{\bar{t}}\right)^{\lambda}-1\right)}}\right]^{\frac{\sigma-1}{\sigma}}\right\} - \log\left(\frac{\sigma\kappa}{\bar{h}}\right) = 0 \tag{11}$$

where Y^P, N^P and K^P refer to private sector output, private sector employment and private non-residential capital. P^y, w^P and q are their respective private sector prices; C, P_c and N^F are consumption, consumption deflator and labour force. Bars above the variables refer to the sample averages.[6] *The n*ormalised production function implies that $\overline{\pi} = \frac{\bar{q}\bar{K}^P}{\bar{w}^P\bar{N}^P+\bar{q}\bar{K}^P}$ is the capital share evaluated at the fixed point (sample mean).

The task of the first equation (7) is to control for the common mark-up component in equations (8) and (9), which are the steady state forms of the first order conditions of profit maximisation with respect to labour and capital. Equation (10) corresponds to the production function (2) after taking logs on its both sides. Equation (11), determining the optimal frictionless wage rate, is the first order maximisation condition of the utilitarian trade union under the right-to-manage structure. It is part of the supply side system, as it is conditional on the same production technology as the firm's maximisation conditions. Parameter $\bar{h}$ refers to average normal working hours per employee that was assumed to be constant over time and in the following is normalised to one.

The steady state form of the first-order conditions of the profit maximising firm and unions maximising the utility of member households imply the 5-equation medium-run supply system (7)-(11) that allows a consistent two-step estimation of the underlying deep parameters of the model. As the supply system contains cross-equation parameter constraints, it is estimated with the method of non-linear SUR, which has proven to be a very efficient estimation approach outperforming all single-equation methods, see León-Ledesma et al. (2010). This system defines all parameters related to technology, production function and the mark-up, allowing to define optimal frictionless prices, wages, labour input, marginal cost and marginal product concepts needed in estimating, in the second stage, the dynamic first order optimisation conditions of firms and unions.

Our estimation results support the general form of factor augmenting technical progress, with the labor augmenting component dominating. The estimate of the key technology parameter, the elasticity of substitution between capital and labour, is shown below. It is close to 0.6 implying a stark deviation from the unit elasticity of the Cobb Douglas production function, and is well in line with estimates from the euro area NMCM. This has important implications for the transmission mechanism of the whole model, where the elasticity of substitution is one of the key parameters.

Production Function Estimates

	US
Elasticity of Substitution (1951Q1:2011Q4)	0.562 (0.019)

*Standard error of estimate in brackets

In the model, we distinguish between the concepts of full capacity output, denoted as Y_t^{P*}, and potential output, denoted as $\widetilde{Y}_t$. Full capacity output is a firm-level concept derived from the firms' optimisation framework, implied by costs related to input changes and variations in their utilisation rates (see McAdam and Willman, 2013b). This, together with capacity utilisation that is derived from full capacity, are the concepts that are used in the behavioural equations of the model. By contrast, potential output is a purely macroeconomic concept, measuring the unused resources in the economy relevant for policymakers. This latter concept is used to derive the output gap, which is used in the monetary policy rule, but also as reporting variable in order to gauge the

[6] We have defined the point of normalisation so that $t_0 = \bar{t}$, $N_0 = \bar{N}$, $K_0 = \bar{K}$, $Y_0 = \xi\bar{Y}$ and $\pi_0 = \bar{\pi}$. Parameter ξ is a normalization constant (close to unity) reflecting that, due to nonlinearities, the sample average of production need not exactly coincide with the level of production implied by the sample averages of inputs and time, Klump et al. (2007b).

extent to which free resources are available. Both concepts, whole-economy full capacity and potential output, are based on their private-sector counterparts, while assuming that the government sector is continuously at full use.

4 Dynamic equations

In addition to the relaxations concerning the medium-run development, the optimisation frameworks of agents contain a lot of frictional elements which are needed to match the observed short-run dynamics of the data. In the subsequent sections we report the key equations, but more detail on the theoretical underpinnings are provided in McAdam and Willman (2013a, 2013b) and Dieppe et al (2012). All dynamic equations containing the leads of variables are estimated by the generalised method of moment (GMM), Hansen (1982). GMM is a limited information estimation approach which uses only a subset of the restrictions and structure imposed by full information approaches. As such, GMM has the advantage that it can be applied without a full statistical specification of all variables of the model and GMM estimates are not contaminated by possible misspecifications in the rest of the model. Furthermore, GMM is useful when a likelihood is difficult to derive or evaluate, which is the case in large scale models.Therefore, in the present context, where optimizing agents are assumed to be boundedly rational in knowing only the parameters related to their problem but not necessarily the rest of the model, GMM is an especially appealing estimation method. Moreover, using the law of iterated expectations, GMM estimation also allows for rational expectations or the case that agents in the model have more information than the econometrician. However, as a partial, single equation method, GMM is not as efficient as full information maximum likelihood.

Below we report the key equations and estimated parameters, which were estimated with data up to 2011q4.

4.1 Firm and union behaviour

4.1.1 Labour Demand - Private Sector

Labour is considered as indivisible, which has important implications for the behaviour of optimising agents. In the profit maximising framework of the *firm,* the assumptions of indivisible labour, adjustment costs with respect to the number of workers and convex costs with respect to work intensity introduce the discrepancy between paid hours and efficient hours. This explains the observed pro-cyclicality in labour productivity, when labour input is measured in heads or paid hours. It also introduces the ratio of efficient hours (per worker) to normal hours into an argument of optimal price setting on top of the conventionally defined marginal cost of labour.

From the point of view of employment, the desired (optimal) number of workers, working normal hours at full intensity, is derived from the inverted production function equation (2) such that:

$$N_t^{P*} = \frac{\bar{N}\left(1-\bar{\pi}\right)^{\frac{\sigma}{\sigma-1}}}{\Gamma_N\left(t\right)}\left[\left(\frac{Y_t^P}{\xi\bar{Y}^P}\right)^{\frac{\sigma-1}{\sigma}} - \bar{\pi}\left(\Gamma_K\left(t\right)\cdot\frac{Y_t^P}{Y_t^{P*}}\cdot\frac{K_{t-1}^P}{\bar{K}^P}\right)^{\frac{\sigma}{\sigma-1}}\right]^{\frac{\sigma}{\sigma-1}} \tag{12}$$

Both capital and labor utilization rates are allowed to be time-varying as in McAdam and Willman (2013b). Hence in the above equation, $\frac{Y_t^P}{Y_t^{P*}}$ is defined as the time-varying factor utilisation rate, which is constrained to be equal to the labour and capital utilisation rates (and hence $y^P - y^{P*} = \log(\frac{Y_t^P}{Y_t^{P*}}) = \log(\frac{N_t^P}{N_t^{P*}}) = \log(\frac{K_t^P}{K_t^{P*}})$).

Private sector wage costs per worker can be represented as a convex function of the deviation of effective hours, h_t from normal hours, $\bar{h} = 1$:

$$W_t^P = \overline{W}_t^P \left[h_t + \frac{a_h}{2} \cdot (h_t - 1)^2 \right] \tag{13}$$

where a_h is the cost curvature parameter of the wage rate schedule.

Changes in private sector employment are coupled with adjustment costs. For estimation of the dynamic labour demand, we define the explicit form of the adjustment cost function $A_N(N_t, N_{t-1})$:

$$A_N(N_t^P, N_{t-1}^P) = \frac{a_N}{2} \cdot \Delta N_t^P \Delta n_t^P \tag{14}$$

where $n^P = \log(N^P)$. Now the dynamic system of first order conditions implies the following labour demand:

$$n_t^P = \frac{D_t}{(1 + D_t + a_h/a_N)} n_{t+1}^P + \frac{1}{(1 + D_t + a_h/a_N)} n_{t-1}^P + \frac{a_h/a_N}{(1 + D_t + a_h/a_N)} n_t^{P*} \tag{15}$$

where $n_t^{P*} = \log\left(N_t^{P*}\right)$ (inverted production function) and $D_t = \frac{\left(1+(w_{t+1}^P - w_t^P)\right)}{(1+r_t)_t} \cdot \frac{\left(1+\left(n_t^{P*}-n_t^P\right)+a_h\left(n_t^{P*}-n_t^P\right)^2\right)}{\left(1+\left(n_{t+1}^{P*}-n_{t+1}^P\right)+a_h\left(n_{t+1}^{P*}-n_{t+1}^P\right)^2\right)}$. = discounting factor ($\approx 1$)

The estimated parameters are presented in the table where we include both the forward and back roots of the equation along with the J-test statistic of overidentifying restrictions.

Employment

a_h/a_N	Root 1 (forward)	Root 2 (backward)	J-test (p-value)
0.0204 (0.0076)	1.1541	0.8677	0.5440

*Standard errors of estimates in brackets

4.1.2 Private non-housing capital formation

To capture the observed inertia in capital formation, the capital stock and its rate of change are coupled with adjustment costs which reflect time-to-build considerations. Also, an increase in the capital utilization rate causes convex costs that are assumed to approach infinity when the utilization rate approaches its technical maximum. Borrowing constraints affect the rate at which the future is discounted and in the estimated aggregate level equations we assume that for a certain percentage of firms these constraints are binding.

Similar to the adjustment cost function of employment, we define the adjustment cost function $A_K(K_t^P, K_{t-1}^P, K_{t-2}^P)$, as follows:

$$A_K(K_t^P, K_{t-1}^P, K_{t-2}^P) = \frac{a_K}{2} \cdot \Delta K_t^P \Delta k_t^P + \frac{a_K b_K^2}{2} \cdot \Delta K_{t-1}^P \Delta k_{t-1}^P - a_K b_K \cdot \Delta K_t^P \Delta k_{t-1}^P \tag{16}$$

where $k^P = \log K^P$ and $b_K \in [0, 1]$. Costs related to the time varying utilization rate are:

$$A_\kappa(\kappa_t, K_t^P) = \frac{utilmax \cdot \phi}{utilmax - \frac{Y_t^P}{Y_t^{P*}}} \cdot K_t^P \tag{17}$$

Now the dynamic system of first order conditions implies the following investment equation (see McAdam and Willman (2013b) and Dieppe et al (2012) for more details):

$$\frac{\left(1-\Lambda^B\right)^2 b_K}{\left(1+r_t\right)\left(1+r_{t+1}\right)}\Delta k_{t+2}^P - \left(\frac{\left(1-\Lambda^B\right)^2 b_K^2}{\left(1+r_t\right)\left(1+r_{t+1}\right)} + \frac{\left(1-\Lambda^B\right)\left(1+b_K\right)}{\left(1+r_t\right)}\right)\Delta k_{t+1}^P$$
$$+ \left(\frac{\left(1-\Lambda^B\right) b_K (1+b_K)}{\left(1+r_t\right)} + 1\right)\Delta k_t^P - b_K \Delta k_{t-1}^P$$
$$= \frac{1}{a_K}\left(MPK_t \frac{Y_t^P}{Y_t^{P*}} - \left\{UC_t + \frac{\phi}{(utilmax - \frac{Y_t}{Y_t^*})} - \frac{\phi}{(utilmax-1)}\right\}\right) \tag{18}$$

Λ^B = LG-multiplier related to the borrowing constraint; MPK = marginal product of capital with fully utilized input; UC = real user cost of capital; a_K and b_K are adjustment cost parameters, ϕ is the cost parameter of the utilization cost function and $utilmax$=1.035 is the technical maximum utilisation rate[7]. The estimated parameters are reported in the table.

Private non-housing investment

$1-\Lambda^B$	b_K	$1/a_K$	ϕ	utilmax	Root 1(backward)	Root 2	Root 3	J-test (p-value)
0.5323	0.9568	0.0132	0.00034	1.035	0.95683	1.90263	1.988	0.856
(0.057)	(0.007)	(0.0014)	(0.0001)					

*Standard errors of estimates in brackets

4.1.3 Price formation - private sector

Following McAdam and Willman (2010), we assume a staggered price setting of firms with a three-valued Calvo-signal, resulting in a conventional hybrid New Keynesian Phillips curve as in Gali and Gertler (1999). In this New Keynesian Phillips curve, part of the firms keep prices fixed, θ_p, another part change prices following a backward-looking rule ω_p and the rest set them optimally based on intertemporal optimization:

$$\left\{\theta_p + \omega_p\left[1-\theta_p\left(1-\beta\right)\right]\right\}\Delta p_t - \omega_p \Delta p_{t-1} - \beta\theta_p\Delta p_{t+1}$$
$$- \left(1-\omega_p\right)\left(1-\theta_p\right)\left(1-\beta\theta_p\right)\left(p_t^* - p_t + (a_h-1)(y_t - y_t^*)\right) = 0 \tag{19}$$

where p_t = log of private sector GDP deflator at factor costs; $p_t^* = w_t - mpn_t + \mu_t$ = log of the frictionless equilibrium price level; w_t = log of compensation per worker; mpn_t= log of the marginal product of labour (derived from the production function); $(y_t - y_t^*)$ is the capacity utilisation rate with y_t^* = optimal private sector output (log), y_t = actual private sector output (log); a_h is the cost curvature parameter of the wage rate schedule. θ_p is the probability that firms do not change their prices, and ω_p is the probability that prices are changed following a backward-looking rule.

In estimation we assumed a four per cent annual discount rate, which in quarterly data implies $\beta = 0.99$.

[7] utilmax was calibrated to exceed somewhat the highest estimated utilisation rate in the sample period.

New Keynesian Phillips Curve - Prices

θ_p	ω_p	a_h	Quarterly Duration	1/Root 1	Root 2	Root 3	J-test (p-value)
0.742358	0.347429	0.14847	3.88	0.7349	0.6057	0.5736	0.522
(0.014)	(0.021)	(0.003)					

*Standard errors of estimates in brackets

Our estimation results, reported in the table, imply that the duration, i.e. the average time firms keep prices fixed, is 3.9 quarters, which is broadly in line with the estimates we obtained for the euro area countries in the NMCM. This duration is somewhat shorter than the estimates of Gali and Gertler (1999) for the US and Gali, Gertler and Lopez-Salido (2001) for the euro area, whose estimated average duration was 5 - 6 quarters. But most studies undertaken using US micro data are consistent with an average duration of price changes of less than four quarters. Estimates range between 4 to 11 months, see Bils and Klenow (2004), Klenow and Krygstow (2008), Nakamura and Steinsson (2008), Dhyne et al (2006) and Eichenbaum, Jaimovich and Rebelo (2011). This is also confirmed by the median response of an interview study undertaken by Blinder et al (1998), covering around 200 US firms, which suggest that firms change their prices around 1.4 times per year. Furthermore, estimates from DSGE models of the US also suggest a duration of prices changes of less than 1 year (e.g. Smets and Wouters 2007). Given this broad evidence, we consider our estimates very plausible.

The GDP deflator at factor costs is an integral part of the supply side and the key determinant of other price variables in the model. In this regard, it is the central price in the model. However, adjustments in other prices also matter for the model's response. The consumer price indices (CPI) are determined by two equations. The first one is for the pre-tax non-energy CPI, in which we retain the Calvo staggered price setting framework with the same parameters reported above, and as with the Calvo price equation, we include the labour utilization rate variable. The other equation is for energy CPI. It is modelled as a mark-up of energy (or oil) prices and the GDP deflator.

The seasonally-adjusted post-tax CPI deflator excluding energy is defined as:

$$p_t^{HXS} = \frac{1 - tcir_0}{1 - tcir_t} p_t^{HXST} \tag{20}$$

where p_t^{HXST} is the pre-tax seasonally-adjusted CPI excluding energy, $tcir_t$ is the current implicit tax rate and $tcir_0$ is the tax rate in the base year of the price indices. As mentioned, we retain the Calvo price framework and parameters from above:

$$\begin{aligned} &\{\theta_p + \omega_p [1 - \theta_p (1 - \beta)]\} \Delta p_t^{HXST} - \omega_p \Delta p_{t-1}^{HXST} - \beta\theta_p \Delta p_{t+1}^{HXST} \\ &\qquad - (1 - \omega_p)(1 - \theta_p)(1 - \beta\theta_p)\left(p_t^{HXST*} - p_t^{HXST}\right) = 0 \end{aligned} \tag{21}$$

where p_t^{HXST*} is the long-run optimal non-energy CPI and is modelled as a weighted average of the optimal private sector GDP deflator p_t^* including indirect energy prices and imports deflator excluding energy, p_t^{MN} where the weights ϕ_1 are estimated by OLS and ϱ is set to 0.03 based on input output tables. In addition, as with the Calvo price equation, we include the labour (or equally the capacity) utilization rate:

$$p_t^{HXST*} = \phi_1 p_t^{MN} + (1-\phi_1)((1-\varrho)p_t^* + \varrho p_t^{EI}) + (a_h - 1)(n_t^* - n_t) \tag{22}$$

CPI energy (p_t^{HE}) is modelled as a mark-up of energy (or oil) prices and the (whole-economy) GDP deflator (p_t^T):

$$p_t^{HE} = \delta_1 p_t^{EI} + (1-\delta_1)\, p_t^T \tag{23}$$

CPI energy and CPI excluding energy and taxes

ϕ_1	δ_1
0.175	0.270
(0.009)	(0.015)

*Standard errors of estimates in brackets

The overall CPI, p_t^H, then becomes a weighted average of CPI non-energy (post-tax), p_t^{HXS} and CPI energy, p_t^{HE}, where w_{et} is the time-varying weight of CPI energy in the overall CPI.

$$p_t^H \;=\; w_{et} \cdot p_t^{HE} + \; (1-w_{et}) \cdot p_t^{HX} \tag{24}$$

The consumption deflator is linked via a simple bridge equation to seasonally adjusted CPI. All other domestic deflators (e.g. investment deflator) are specified as quasi-identities, i.e. modelled as weighted averages of domestic costs (measured by the value-added deflator defined above) and import prices (measured by the import deflator). This feature ensures static homogeneity in all price equations. For pre-tax deflators we assume that imports are 'cost, insurance and freight at the importer's border' (cif) and exports are 'free on board at the exporter's border' (fob). For this reason, indirect taxes are levied only on total consumption (private and public) and total investment. Since there is no distinction between indirect taxes on consumption goods and on investment goods, both tax rates will be equal in sample but will be treated separately for simulation purposes.

4.1.4 Wage Setting - private sector

As with the price setting, the specification of private sector wage setting is based on the staggered three-valued Calvo-signal mechanism, where part of unions keep wages fixed, θ_w, another part change wages following backward-looking rule, ω_w, and the rest set them optimally:

$$\begin{aligned}\{\theta_w + \omega_w[1-\theta_w(1-\beta)]\}\,\Delta w_t^P &= \omega_w \Delta w_{t-1}^P + \beta\theta_w E_t w_{t+1}^P \\ &+ \;(1-\omega_w)(1-\theta_w)(1-\beta\theta_w)\left\{w_t^{P*} - w_t^P\right\}\end{aligned} \tag{25}$$

where w_t^P = log of private sector compensation per worker, and β, the discount factor, = 0.99. The optimal frictionless wage rate, w_t^{P*} is based on two alternative behavioural assumptions. In a right-to-manage framework, where firms and unions negotiate to determine the optimal wage, the utilitarian unions maximize the member households' utility, allowing the employment rate to vary. The non-utilitarian unions target the employment rate via real wage demands. The share of utilitarian and non-utilitarian unions can be flexibly changed, depending for instance on the purpose of the simulation exercise. This is useful for model diagnosis because a non-utilitarian trade union alternative implies a constant long-run NAIRU, while the utilitarian alternative implies that the NAIRU reacts to permanent shocks, so that e.g. a permanent government expenditure shock affecting the GDP share of private consumption will also affect the long-run equilibrium unemployment rate. In addition, the utilitarian union alternative renders the model suitable for welfare analysis of economic policy. The two approaches are combined so that:

$$w_t^{P*} = a_{wu}\left[\left(p_t^C + c_t - n_t^F\right) - \log\left(\sigma - 1 + \frac{F_N^{CES}}{Y_t^P/N_t^P}\right) + \log\frac{\sigma\kappa}{\bar{h}}\right] + (1 - a_{wu})\left[p_t + \log\left(\frac{F_N^{CES}}{1+\mu}\right) + \chi\log\left(\frac{N_t}{N_t^F}\right)\right] \quad (26)$$

$$F_N^{CES} = (1 - \pi_0)\left(\frac{Y_0^P}{N_0^P}\Gamma_N(t, t_0)\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{Y_t^P}{N_t^P}\right)^{\frac{1}{\sigma}} \quad (27)$$

where c_t = consumption (log), p_t^C= consumption deflator (log) ; N_t^F = labour force. The gap between labour demand and supply accounts for the wage drift effect. F_N^{CES} is the marginal product of a fully utilized employee and $\sigma\kappa$ is taken from the supply-side.

The weight parameter a_{wu} is fixed to 0.2 in the context of the estimation of equation (25). Naturally, for scenario analysis its value can be imposed to unity or zero, depending if we want to assume all unions to be utilitarian or non-utilitarian[8]. As the size of χ plays a crucial role in the dynamics of price-wage-productivity dynamics, also its size was calibrated to safeguard reasonable model properties.

New Keynesian Phillips Curve - Wages

θ_w	ω_w	χ	a_{wu}	Quarterly Duration	1/Root 1	Root 2	Root 3	J-test (p-value)
0.7576	0.3619	0.05	0.2	4.126	0.7543	0.6583	0.5601	0.841
(0.009)	(0.015)							

*Standard errors of estimates in brackets

In the table we report the average duration between wage changes. The estimated wage duration in the US is around 1 year, again close to the estimates that we obtained for the euro area countries. Existing information on the frequency of wage changes is rather scarce, but recent finding of the Wage Dynamic Network (2010) based on European survey evidence suggested that the average duration of wages is about 5 quarters, see Druant et al. (2009), compared to an average duration of prices of slightly above 3 quarters. Estimated DSGE models, like Smets and Wouters (2007) typically find the average duration of wage contracts is around one year or even half a year in the case of Gali, Smets, and Wouters (2012) which includes unemployment as an observable. Hence, our estimates are broadly within the range of estimates and similar to the price duration effects, Taylor (1999).

4.1.5 Inventory investment

Following general practice and to neglect excessive complexity of the profit maximization framework, inventory formation is left outside that framework. Instead inventory formation is derived on the basis of a second stage optimization. *Here, we* assume that firms minimise a quadratic loss function specified in terms of the deviations

[8] For the simulations in the paper we assume all unions to be non-utilitarian.

of inventories from the optimal level, on *the* one hand, and the deviations of output (sales of storable goods) from the level corresponding to the optimal use of existing input*, on the other*.

The desired equilibrium inventory stock KII* is related to the normal level of production i.e. when hired capital and labor inputs are at full use:

$$KII^{*} = a + bF\left(K, N, t\right) \tag{28}$$

Short-run deviations from the desired equilibrium inventory stock are caused by fluctuations in current and expected future sales and production levels, leading to the following dynamic inventories equation:

$$\begin{aligned}(1 - r \cdot A)\Delta KII_t &= (1 - 2A)\,\Delta KII_t^{*} \\ &- A\left[\Delta S_t - \Delta KII_t - \Delta F\left(\cdot\right)\right] + (1 - r)\,A\left[\Delta S_{t+1} - \Delta KII_{t+1} - \Delta F\left(\cdot, t+1\right)\right]\end{aligned} \tag{29}$$

where S= Sales (Private consumption + exports)

Estimation results indicate that inventories adjust quickly to their optimal level.

Inventory

A	1/Root 1	Root 2	J-test (p-value)
0.3087	0.343	0.3465	0.877
(0.009)			

*Standard errors of estimates in brackets

4.1.6 Private residential investment and house prices

The profits of private construction firms which produces residential investment IH_t, depends on the price of housing minus the cost of construction and is defined as:

$$profit_t = p_t^{H} IH_t - p_t^{IH}\left[C\left(IH_t, r_t^{l}\right) + A\left(IH_t, IH_{t-1}\right)\right] \tag{30}$$

where p_t^H, is the nominal market price of housing, p_t^{IH} is the residential investment deflator. $C\left(IH_t, r_t^l\right)$ and $A\left(IH_t, IH_{t-1}\right)$ are the convex real production cost and real adjustment cost functions which are defined as:

$$C\left(IH_t, r_t^{l}\right) = C_0 + \left(c_0 + \mu_0 r_t^{l}\right) IH_t + \frac{\alpha}{2}\left(IH_t - IH_0\right)^2 \tag{31}$$

$$A\left(IH_t, IH_{t-1}\right) = \frac{a_0}{2}\left(IH_t - IH_{t-1}\right)^2 \tag{32}$$

In (31) the real interest rate r_t^l (deflated by the residential investment deflator) accounts for the cost of external financing and is a weighted average of short-term and long-term interest rates plus a spread. The weight on the short-term rate is calibrated at 0.148 which corresponds to the share of variable-rate mortgages in total single-family mortgages.

Now the intertemporal profit maximization of (30) results in the following relation for housing investment:

$$\left(\alpha - a_0 r_t^l\right) IH_t - a_0(1-r_t^l)IH_{t+1} - a_0 IH_{t-1} = Q_t - \mu_0 r_t^l + \kappa_0 \tag{33}$$

where $Q_t = \frac{p_t^H}{p_t^{IH}}$ is Tobin's Q and parameter $\kappa_0 = \alpha IH_0 - c_0$. The estimation of (33) with GMM (normalised with respect to IH_t) gave the following results:

Residential Investment

α	a_0	$\kappa 0$	$\mu 0$	1/Root1	Root2
0.001461	0.029226	0.06522102	-15.4987	0.6787	0.6817
(0.0001)	(0.00222)	(0.08307)	(3.46661)		

House price determination is based on the idea that the real market price of housing $p_t^{HR} = \frac{p_t^H}{p_t^C}$ equals the expected discounted real rental price of housing p_t^{HCR}. As regards the rental price we allow it to adjust via partial adjustment mechanism to the frictionless rental price p_t^{HCR*} that at each moment of time would equilibrate the demand for and the supply of housing services that will be defined later. Hence, neglecting for simplicity the expectation operator we can write:

$$p_t^{HCR} = (1-\lambda)\sum_{i=0}^{\infty} \lambda^i p_{t-i}^{HCR*} \tag{34}$$

$$p_t^{HR} = \sum_{i=0}^{\infty} p_{t+i}^{HCR}\left(\prod_{j=0}^{i} R_{t+j}\right) \tag{35}$$

where R_{t+j} is the discount factor. After inserting (34) into (35) we obtain

$$(1+\lambda R_t)\, p_t^{HR} - R_{t+1} p_{t+1}^{HR} - \lambda p_{t-1}^{HR} = (1-\lambda) p_{t+i}^{HCR*} \tag{36}$$

With the supply of housing services proportional to the existing housing stock that is given at each moment, the equilibrium of the demand for and the supply of housing services implies,

$$\log p_t^{HCR*} = c_0 + c_p \log C_t + k_p \log KH_{t-1} \tag{37}$$

where C_t is private consumption (or permanent income) and KH_{t-1} is the housing stock in the beginning of period t. As the frictionless rental price p_t^{HCR*} is not directly observable, we utilize the cointegration properties implied by (34) and (35) that $p_t^{HCR*} = \left(1-\overline{R}\right) p_t^{HR}$ in estimating the parameters of (37). In addition, by utilizing the approximation $\log\left(\frac{p_t^H}{p_t^C}\right) = p_t^{HR} - 1$, equations (36) and (37) imply,

$$\left[1+\lambda(1-r_{t-1}^l - 0.02)\right] p_t^{HR} - (1-r_t^l - 0.02)p_{t+1}^{HR} - \lambda p_{t-1}^{HR} = (\overline{r}^l + 0.02)(1-\lambda)(1+c_0+c_p \log C_t + k_p \log KH_{t-1}) \tag{38}$$

In (38) the discount factor is defined in terms of the real interest rate and the risk premium of 8 per cent per annum as $R_{t+1} = 1 - r_t^l - 0.02$ and $\overline{r}^l$ is the historical average of the real interest rate. Estimation gave the following results:

House Prices

λ	c_0	c_p	k_p	Root1	Root2
0.918511	-0.50325	1.03767011	-0.94947	0.91843	1.02429
(0.0229)	(0.470934)	(0.17667)	(0.2137)		

4.2 Households

Regarding households' utility maximisation problem, the indivisibility assumption simplifies the analysis, because the labour supply adjusts to the demand for labour, conditional on the wage contract set by unions either maximising the utility of member households or targeting the warranted wage rate consistent with a desired employment rate. The basic framework in household's utility maximisation is Blanchard's (1985) overlapping generation framework of perpetual youth that implies that the discount rate of the optimizing households is modified to include the probability of dying. This framework is especially useful in a small-open economy environment as it suffices to ensure that agents do not borrow indefinitely at the fixed external rate of interest. Hence, the long run stability of foreign assets is attained without resorting to the somewhat ad-hoc assumption of the foreign debt elastic interest rate or the endogenous discount factor, as is typically postulated in the infinite horizon DSGE models, see more closely Schmitt-Grohé and Uribe (2003). We extend Blanchard's overlapping generations framework in many ways. Firstly we incorporate income uncertainty into the utility maximization framework. To do it in a tractable way we apply a two-stage approach, see Willman (2007). In the first stage, the consumer evaluates her risk-adjusted non-human and human wealth conditional on uncertain lifespan and labour income. Thereafter, in the second stage, conditional on the risk adjusted life-time resource constraint, the consumer is assumed to determine her optimal planned path of consumption. This approach gives a closed form consumption function with precautionary saving depending positively on income risk and death probability. The explicit treatment of wealth allows us also to account for asset price (i.e. stock and house price) effects on the perceived wealth relevant for consumption decisions. Further, the assumption of imperfect front-loaded information on future income realisations changes also the weight structure in defining the present value of the expected income stream to be more front-loaded and increases the dependency of consumption on current income. Finally, the assumption of habit persistence introduces the dependency of current consumption on lagged consumption and is able to generate a hump-shaped response profile of consumption to shocks.

This results in a forward looking aggregate consumption function with strong backward-looking frictional elements. In the estimated specification we assume that the subjective discount rate *is* $\rho = \bar{r} = 0.01$ (and $\bar{R} = (1-\bar{r})^{-1}$), which corresponds to an annual rate of 4 percent. Further, to eliminate heteroscedasticity we divide both sides of equations by current period labour income. Hence, in estimating the equation with the method of instrumental variables, it can be written as follows:

$$E_t \left\{ \begin{array}{c} \left\{1 + \frac{\gamma}{1.01}\left[(1-\pi)^2 aR_t - (0.01+\pi)\left(1 - \frac{(1-\pi)a}{1.01}\right)\right]\right\} \\ \frac{C_t}{Y_t} - (1-\pi)\gamma \frac{R_t C_{t+1}}{Y_t} - \frac{a(1-\pi)}{1.01}\frac{C_{t-1}}{Y_t} - \left(\frac{1.01-(1-\pi)a}{1.01}\right)\left(\frac{0.01+\pi}{1.01}\right) \\ \left\{\left(\frac{1}{1-\pi} - \gamma\right)\left(\frac{V_{t-1}}{Y_t} + 1\right) + \left[\theta_1 + \theta_{10}(u_t - u_{t-4}) + \theta_{11}(u_{t-3} - 6.4)\right]\left(\frac{(1.01-(1-\pi)\gamma)}{0.01+\pi} - \frac{1}{1-\pi}\right)\right\} z_t \end{array} \right\} = 0 \tag{39}$$

where C_t is consumption, Y_t labour income net of payroll taxes minus transfers, V_{t-1} total net household wealth in the beginning of period. Parameter π= death probability; γ= forward information parameter; a= habit persistence parameter; and z_t refers to the set of instruments. Unlike in the euro area economies, income risk is assumed to be time-varying and dependent on the unemployment rate u_t : $[\theta_1 + \theta_{10}(u_t - u_{t-4}) + \theta_{11}(u_{t-3} - 6.4)]$.

As changes in real asset prices may affect the "perceived" wealth relevant for consumption decisions, total household net wealth at the beginning of period, V_{t-1}, is modelled as follows:

$$V_{t-1} = \frac{1}{(0.25a_1 + 0.75a_2)} \left[a1 \frac{fwon_{t-1}}{p_t^C} + a2 \left(\frac{p_t^S}{p_t^C} \right)^{b1} \frac{eqwn_{t-1}}{p_t^S} + a3 \left(\frac{p_t^H}{p_t^C} \right)^{b2} \frac{hwn_{t-1}}{p_t^H} \right] \tag{40}$$

where fwon_{t-1} is other financial net wealth including consumer durables, p_t^S is a stock price index, eqwn_{t-1} is equity wealth, p_t^H is a house price index and hwn_{t-1} is housing wealth, while $b1$ and $b2$ are the elasticity parameters. Now, if elasticity parameters $b1$ and $b2$ equal zero, then asset prices have no effects on consumption and "perceived" wealth equals total household net wealth at repurchasing prices. In the opposite polar case of b1 and b2 equalling one, changes in the stock and housing prices are fully transmitted into the perceived wealth and, further to consumption. The components of equity and housing wealth follow the evolution of the corresponding capital stock measures, where K_t is the private non-housing capital stock and KH_t is the private housing capital stock.

$$eqwn_t = c_{00} p_t^S \left(K_t + c_0 \right) \tag{41}$$

$$hwn_t = h_{00} p_t^H \left(KH_t + h_0 \right) \tag{42}$$

Other financial net wealth is determined by its past values, as well as by the changes in the current account CA_t and government deficit D_t.

$$fwon_t = fwon_{t-1} + CA_t + D_t \tag{43}$$

The parameters for wealth were estimated jointly with other parameters of the dynamic consumption function. According to the estimates, in the long-run about 64% of changes in stock prices and 71% of changes in house prices are transmitted to the respective wealth components and further to consumption. While the wealth effects on consumption estimated for the US and the euro area countries in the NMCM are not directly comparable as better data availability allows for a more detailed modelling of household wealth effects on consumption in the US than in the euro area countries, we find that the marginal propensity to consume out of wealth (with wealth being defined by (40)) is 7 cents to the dollar for the US, which is only slightly larger than for the euro area countries (where the corresponding estimate is around 6 cents for most countries).

Consumption

π	γ	a	θ_1	θ_{10}	θ_{11}	1/Root 1	Root 2	J-test (p-value)
0.0075	0.9532	0.79355	0.84355	-0.03	-0.03	0.9501	0.7909	0.79
	(0.026)	(0.123)	(0.012)	(0.00)	(0.00)			

Wealth

a1	a2	a3	b1	b2
0.040808	0.08433	0.08433	0.6421	0.7067
(0.031)	(0.011)	(0.000)	(0.069)	(0.215)

*Standard errors of estimates in brackets

4.3 External Sector Behaviour

4.3.1 Export formation

As the US is an open economy, in our two domestic good framework, with a government sector and a privately produced good, a part of the private output is exported. However, firms face separate demand functions in domestic and export markets, leading to pricing-to-market behaviour. This effectively separates the optimal price setting of exports from the rest of the firm's optimisation problem. We assume that the volume of exports is determined by the almost ideal demand system (AIDS) function (Deaton and Muellbauer 1980). The advantage of this functional form, compared to the conventional iso-elastic form, is that with the AIDS function the foreign competitors' price affects optimal export-price setting, in line with empirical evidence. The export demand function and the optimal price setting relation result in a two-equation system with cross-equation parameter constraints. This allows a model consistent way to estimate the price elasticity of export demand.

The two-equation system for export volumes and the export price is specified as follows:

$$\left(\frac{P^X X}{P^{CX} WD}\right) = a + b \cdot f(time) - (\phi - 1)\left(p^X - p^{CX}\right) \tag{44}$$

$$p^X = a + \frac{1 + (a + b \cdot f(time))/(\phi - 1)}{2 + (a + b \cdot f(time))/(\phi - 1)} \left((1 - a_x)(w - mpn) + a_x p^M\right) + \frac{1}{2 + (a + b \cdot f(time))/(\phi - 1)} p_{CX} \tag{45}$$

Where P^X = Export deflator (lower case refers to log); X = Export volume; P^{CX} = the external competitor export prices (lower case refers to log); WD = the world demand for exports; w = compensation per worker (log); mpn = marginal product of labour (log); p^M = import deflator (log), a = point market share (with indexed data close to unity); $\phi > 1$ is the representative point price elasticity of exports; b if different from zero measures the deviation of the income elasticity of export demand from unity; and a_X = import content of exports (input-output estimate). In the estimation, an additional free time trend was allowed in the price equation.

Exports

a	ø	a$_X$	Log-det	ADF volume-eq.	ADF price-eq.
1.04193	1.30642	0.123	-14.15	-3.227	-3.735
(0.009)	(0.015)				

*Standard errors of estimates in brackets

The dynamic export volume and export price equations follow a conventional error correction form for the log change in exports or export prices.

Dynamic Exports

Δx*	Δxt-1	EC-term	R^2	Durbin-Watson
0.45512	0.10866	-0.1199	0.729	1.788
(0.05)	(0.09)	(0.04)		

Dynamic export price

ΔP_x^*	ΔP_{xt-1}	EC-term	R^2	Durbin-Watson
0.59927	0.34435	-0.2092	0.479	1.862
(0.124)	(0.1139)	(0.779)		

*Standard errors of estimates in brackets

4.3.2 Import formation

Total import volumes are broken down into oil and non-oil imports. The modelling approach of non-oil imports is conventional, where the import supply curve is assumed to be horizontal and, hence, import volume is demand determined. Accordingly, the two key driving variables are domestic demand and the relative price of domestic production and imports. Domestic demand is broken down into the main expenditure components, which are weighted by their respective import content. The long-run equilibrium aggregate demand for non-oil import volumes is specified as follows:

$$mno^* = m_0 + \log \left[\begin{array}{c} {e_{MR}}^{c} \left(\frac{P^C}{TX_1 P^{MN}}\right)^{\sigma m} C + {e_{MR}}^{G} \left(\frac{P^G}{P^{MN}}\right)^{\sigma m} G + {e_{MR}}^{I} \left(\frac{P^I}{TX_1 P^{MN}}\right)^{\sigma m} I \\ + {e_{MR}}^{X} \left(\frac{P^X}{P^{MN}}\right)^{\sigma m} X + {e_{MR}}^{KII} \left(\frac{P^P}{P^{MN}}\right)^{\sigma m} \Delta KII \end{array} \right] + f(time) \quad (46)$$

where e_{MR}^i are the import content weights of the demand indicator, which were taken from Bussiere et al (2011) except for e_{MR}^{KII}, which is derived from input output tables; $P^{MN}$is import prices excluding energy; P^C is the consumer expenditure deflator, P^G the government consumption deflator, P^I the total investment deflator, P^P is the private sector GDP deflator at factor cost and TX_1 is the indirect tax rate (with the assumption that the indirect tax content of investment and consumption is equal). The dynamic equation follows a standard EC specification.

Non-oil imports

σm	Log(det)	ADF	${e_{MR}}^{c}$	${e_{MR}}^{I}$	${e_{MR}}^{G}$	${e_{MR}}^{X}$	${e_{MR}}^{KII}$
1.0564	-6.381	-4.539	0.119	0.173	0.062	0.123	0.206
(0.047)							

Dynamic non-oil imports

Δmno*+ Δ(pmn-p)	Δmno^*_{t-1}	$\Delta(\text{pmn-p})_{t-1}$	EC-term	Durbin-Watson
1.066754179	0.425404699	0.310769561	-0.134955	1.98
(0.367)	(0.151)	(0.186)	(0.05)	

Turning to real imports of petroleum products, these are assumed to have a zero price elasticity and a unit output elasticity in the long run. The resulting dynamic equation is

$$\Delta m_{Oil,t} = \theta_0 + \theta_1 \Delta m_{Oil,t-1} + \theta_2(m_{Oil,t-1} - y_t) \tag{47}$$

Dynamic oil imports

$\theta 1$	$\theta 2$	Constant $\theta 0$	R^2	Durbin-Watson
-0.13974481	-0.26366393	-0.996658213	0.196	2.06
(0.08)	(0.06)	(0.215)		

*Standard errors of estimates in brackets

The equation for the import deflator excluding energy is also quite traditional. The import deflator excluding energy p^{MN} depends on the private sector GDP deflator at factor cost (p^P) to capture possible pricing to market effects, and the competitors' import price (p^{CM}), all in logarithms, with static homogeneity condition imposed:

$$p^{MN} = \phi_1 p^P + (1 - \phi_1) p^{CM} \tag{48}$$

In addition a trend term was included. The equation is estimated via non-linear estimation methods. The dynamic equation follows a standard error correction specification.

Import prices (non-oil)

ϕ_1	1-ϕ_1	D-W
0.381	0.619	0.58
(0.05)		

Dynamic Import prices

Δp*MN	Δp*MN t-1	EC-term	R^2	D-W
0.46729	0.24472	-0.2227	0.413	2.00
(0.07)	(0.08)	(0.05)		

*Standard errors of estimates in brackets

Finally, the sum of the trade balance and net factor income equal the current account balance, which in turn is cumulated to give the stock of net foreign assets.

4.4 Government Sector and Accounts

Government output is assumed to be fully consumed by the government (i.e. implicity, government output capacity is continuously at full use) and government employment is directly linked to *the* output of the government sector, such that:

$$\Delta y_t^G = \Delta g_t \tag{49}$$

$$n_t^G = y_t^G - \varrho f(time) \quad (50)$$

where y^G is real government output, g is real government consumption and n^G is government employment (all in logs).

Government sector wages follow those of the private sector, and are also dependent on the size of the government sector. Meanwhile, government prices are assumed to be mainly driven by government wages, along with a proportional part driven by private sector prices:

$$w_t^G = w_t^P + \varrho_1(w_{t-1}^G - w_{t-1}^P) + \varrho_2(y_t^G - y_t^P) + \varrho f(time) \quad (51)$$

$$\Delta p_t^G = 0.7 * \Delta w_t^G + 0.3 * \Delta p_t^P \quad (52)$$

where p_t^G= log of government sector prices, w_t^G = log of compensation per government worker.

The rest of the fiscal block of the model comprises a set of identities in expenditure and revenue categories. Government receipts are split into a number of components: (i) direct tax receipts on income earned by households (T_F), which includes social security contributions by employers and employees; (ii) direct tax receipts from firms (T_o); (iii) indirect tax receipts (T_I), which include VAT and excise duties; and (iv) other government revenue (OR_G), which includes the gross operating surplus. The income tax rates that correspond to the revenue T_F is endogenously determined via a fiscal rule (see below), while both the indirect tax rate that determines T_I and direct tax receipts from firms (T_o), as well as other government revenue are set exogenous as a share of GDP. On the expenditure side, the fiscal authority makes (net) transfers (TR_F), which include pensions and countercyclical unemployment payments. To account for this countercyclical effect in model simulation, the otherwise constant GDP share of net transfers is adjusted to react negatively to the deviation of simulated employment from the predetermined baseline employment, $(N - N_{bas})$, mainly reflecting the dependency of unemployment compensation to the unemployed persons:

$$TR_F = tr.PY - \varkappa W(N - N_{bas}) \quad (53)$$

where $\varkappa$ is calibrated to 0.7. In addition, the fiscal authority makes net interest payments on government debt (IN_G) and different types of primary expenditure categories, namely, nominal government consumption (G_C) and nominal government investment (G_I) which are exogenous in real terms but can be shocked as part of a fiscal policy expansion, and other government expenditure (OX_G). Finally, the government consumption deflator follows both the price of domestically produced goods with a weight of δ^G and of imported goods with a weight of (1-δ^G) .

The public deficit (D) in each period is the difference between receipts and expenditures:

$$D = T_F + T_o + T_I + OR_G - TR_F - IN_G - G_C - G_I - OX_G \quad (54)$$

The fiscal authority's is faced by a budget constraint which says that public debt (B_t) is the cumulative sum of past public deficits (D_t) i.e.

$$B_t = B_{t-1} + D_t \tag{55}$$

As households are non-Ricardian, the path of government debt and taxes matter for the evolution of the economy. Therefore, governments aim to insure stability of the public debt stock. This is modelled via fiscal policy rules based on a reaction of both personal income taxes and corporate taxes to the deviation of the government's debt-to-GDP ratio from its predetermined target. This contributes to the adjustment towards the stock-flow equilibrium in the long-run.[9]

The fiscal rule is defined as follows:

$$\Delta\tau_t = \varphi_1(b_{t-1} - \bar{b}) + \varphi_2 \Delta b_{t-1} \tag{56}$$

where τ_t is the personal income tax rate (T_F/PY), b_t is the government debt to GDP ratio (B/PY), and $\bar{b}$ is the target. The parameters $\varphi_1$and φ_2 are set to 0.003 and 0.03 respectively.

For reporting purposes and comparison to published figures, we separate US *federal* government debt, deficit, revenues and expenditures which are linked to those of the general government sector via simple bridge equations.

4.5 Monetary Authority and Financial Markets

We close the model with the inclusion of the Central Bank, which sets monetary policy, and financial markets, which are forward-looking and determine exchange rates and long-term interest rates. Although the activation of a monetary policy rule and an Uncovered Interest Partiy condition for the exchange rate are required for long-run stability, there are scenarios where the model can be simulated when these aspects are exogenised (e.g. under a scenario of monetary policy accommodation).

Households and firms adjust their plans by taking into account the expected response by monetary authorities. The endogenous monetary policy rule provides the nominal anchor to the model and incorporates a smooth interest rate reaction to shocks in the short-run. The choice of the interest rate rule is an important determinant of model stability, see Clarida, Gali and Gertler (2000). In our standard model simulations, monetary policy follows a simple Taylor rule specification based on Christiano et al (2005), although we explore alternative rules in section 6.4. The short term interest rate is determined by expected inflation, and the output gap $q_t = (y_t - \tilde{y}_t)$ along with the lagged interest effect. This monetary policy rule means that the interest rate converges towards its long term target as expected inflation converges to the inflation target and the output gap closes. For the purpose of the simulations, the target level of inflation ($\Delta\hat{\bar{p}}_t$) is set to 2 per cent per annum, but it could, in principle, be set to any other *reasonable* level. The specification of the rule is:

$$i_t = 0.8 i_{t-1} + 0.3(E_t \Delta p_{t+1} - \Delta\hat{\bar{p}}_t) + 0.08 q_t + \varepsilon_t \tag{57}$$

[9] As the focus in the govenment sector stability target is more in the long than short-run and to strengthen the short run effects of fiscal policy, the fiscal policy reaction function could have longer lags from the debt ratio to the change of the income tax rate, but this would also increase the cyclicality of the model.

where i_t is the nominal interest rate and ε_t is a shock that is serially uncorrelated with the interest rate. In particular, the parameter governing the speed of reaction to the interest rate gap is set to 0.8 and inflation and output gap parameters are set to 0.3 and 0.08 respectively reflecting the dual mandate of the Federal Reserve.[10] As macroeconomic model reactions are sensitive to the parametrisation of the Taylor rule, as shown for example by Wieland et al (2012), we also experiment with alternative specifications (see section 6).

Financial markets are assumed to be forward-looking. The specification of the long-term interest rate equation is:

$$l_t = 0.9 l_{t+1} + 0.1 i_t \tag{58}$$

The exchange rate follows a standard real UIP equation:

$$e_t = e_{t+1} + (r_t^f - r_t)/400 \tag{59}$$

where e_t is the (log of) the real exchange rate and r_t and r_t^f are the domestic and foreign real interest rates respectively.

5 Learning setup - updating beliefs under limited information

The dominant way to model expectations has been via model-consistent rational expectations (strong rationality). While rational expectations (RE) can be taken as a theoretically well founded polar case, resorting only *to* them is not unproblematic. Rational expectations have been criticised by assuming too much information on the part of agents and it is known that rational expectations can give rise to a multiplicity of solutions. An alternative is the learning approach. Here, the literature (Evans 1986, Woodford 1990) has pointed out that often a particular learning specification will produce a unique solution, and when an expectations-stable equilibrium (E-equilibrium) is attained, the model has also reached a rational expectations equilibria (REE), as noted by Marcet and Sargent (1988, 1989). This implies that a particular REE is being chosen without recourse to arbitrary transversality conditions. However it is important to note that if the specific form of the learning process produces different solutions then the choice between these solutions (and implicitly the corresponding REE) is still being made on the basis of a largely arbitrary decision - i.e. the choice of the form of the learning rule itself can be crucially important.

In the adaptive learning literature a standard assumption is that agents form expectations by using the correct model of the economy, but do not know the parameters (Evans and Honkapohja 2001), i.e. agents have perfect knowledge about the structure of the economy and hence know the correct specification of the REE minimal state variable (MSV) solution, see McCallum (1983). However, in contrast to the RE solution, they have imperfect knowledge about the true values of the structural parameters and the implied parameter values of the true MSV solution. Hence, although correctly specified, the perceived law of motion (PLM) that agents use in updating their expectations deviates from the true MSV solution. Under these assumptions the actual law of motion (ALM) gradually converges to the model consistent RE solution. However, as discussed e.g. by

[10] In principle, a more refined approaches could be utilized here, for instance direct estimation.

Slobodyan and Wouters (2012), the short- and medium-run dynamics of the model may crucially depend on how much the initial estimates of the parameters deviate from those of the RE solution and, hence, ambiguity related to chosen initial values may introduce non-voluntary arbitrariness into the dynamics of the estimated model.

With large and medium-sized models, closed-form MSV solutions are difficult to attain given the large number of variables that could be included. Indeed, for non-linear models, closed-form MSV solutions do not necessarily exist. Therefore, an alternative strand in the recent learning literature has been that, instead of basing their PLM on the correctly specified MSV, agents base it on a misspecified MSV solution. This approach in effect drops the assumption of a common information set of rational agents that fully understand the world. Nevertheless, agents are still rational in the sense that they avoid systematic mistakes by being willing to learn from past errors and change their behaviour. A key question then becomes of how to select from the various PLM when an obvious choice is not available. There has been a number of approaches to this, including choosing the explanatory variables that minimise the standard error of the regression, ranking correlations according to their standard deviations, or identifying principle components and selecting the variables that most closely move with them (Beeby, Hall and Henry 2004).

Our approach deviates from all the aforementioned approaches. The structure of the whole model is assumed to be largely unknown to the different type of agents. Therefore, the agents are not assumed to know how shocks that hit the economy are transmitted to the expected developments of the variables which are key determinants of their optimized choices, which although being predetermined in their optimisation frameworks are endogenous in the whole model. This is the case even though shocks may quite often be observable. In line with these limitations in the information base, all forward looking equations of MUSEL are estimated by the single equation instrumental variable method of GMM that does not require a full specification of the model and is thus consistent with rationality under limited information. The basic principle of our learning approach is that it is compatible with the same limited information assumption that was used in deriving and estimating the behavioral equations of different optimizing agents. Hence, agents know the deep parameters of their own optimising frameworks, however, they are not assumed to know the structure nor the parameterisation of the rest of the economy. Neither are they assumed to know the stochastic processes generating shocks hitting the economy. Therefore, instead of basing their PLM on the correctly specified full model MSV, agents are assumed to base it on the single equation MSV where the fundament variable, although endogenous in the whole model, is treated as predetermined for the optimising agent. In addition, in line with the fact that most economic time series are I(1) variables, agents are assumed to know that the changes (or the growth rates) of fundament variables can be modelled as stationary ARMA(p,q) processes, the exact form of which is not, however, assumed to be known by agents. This suggests some form of heterogeneity of expectations, which could be due to cognitive limitations faced by agents in understanding the world, to the observability of economic variables being different across agents or to the costs of having full information being too large. This is also compatible with survey evidence showing clearly that expectations of aggregate economic variables differ across different sectors/agents, e.g. consumers and firms.

We now formalise our limited information expectations formation approach which follows the same approach as in Dieppe et al (2013), so only the key aspects are sketched out.

The following equation is an example of how the adaptive expectations approach is modelled:

$$y_t = \beta E_t y_{t+1} + \delta y_{t-1} + y_t^* + \nu_t \tag{60}$$

where ν_t is a white noise shock and y_t^* is the fundament variable (possibly a complicated function of several variables endogenous in the whole model), the development of which is outside the control of the optimising agent in question. Hence, although endogenous in the full model, the development of y_t^* is predetermined for the agent just like truly exogenous variables. As no uncertainty concerning the deep parameters of the underlying optimization framework (captured by GMM estimates of parameters β and δ) is assumed, the decision making agent also knows the correct parameterization of equation (60).

For our purposes it is useful to express the structural parameters of (60) in terms of its roots. The roots of the homogenous part (60) are $\lambda_1 = \frac{1}{2\beta}\left(1 - \sqrt{1-4\beta\delta}\right)$ and $\lambda_2 = \frac{1}{2\beta}\left(1 + \sqrt{1-4\beta\delta}\right)$. The saddle path stability requires $\beta + \delta < 1$ that implies that $|\lambda_1| < 1$ and $|\lambda_2| > 1$. Now (60) can be rewritten as,

$$(1 - \lambda_1 L) E_t y_{t+1} = -\frac{\lambda_1 + \lambda_2}{(1 - \lambda_2 L)} E_t (y_t^* + \nu_t) \tag{61}$$

where L with $Lx_t = x_{t-1}$ refers to lag operator. Now (61) implies the following current period solution (see Dieppe et al 2013),

$$y_t = \underbrace{\lambda_1 y_{t-1} + \left(\frac{\lambda_1 + \lambda_2}{\lambda_2 - 1}\right) y_t^*}_{f_t} + \frac{1}{\lambda_2} E_t \Delta pv_{t+1} + \left(\frac{\lambda_1 + \lambda_2}{\lambda_2}\right) \nu_t \tag{62}$$

where

$$E_t \Delta pv_{t+1} = \left(\frac{\lambda_1 + \lambda_2}{\lambda_2 - 1}\right) \sum_{i=0}^{\infty} \left(\frac{1}{\lambda_2}\right)^i E_t \Delta y_{t+1+i}^*. \tag{63}$$

This form of writing equation (60) depicts well the information available to the agent. In line with the information assumptions in the underlying optimization, the agent knows the f_t term but not how current and (possibly) lagged shocks hitting the economy are transmitted via the future changes of the fundament variable Δy_{t+i}^* to the expected present value term $E_t \Delta pv_{t+1}$. Econometricians know that most economic time series are I(1) variables, which implies that one can find a stationary ARMA(p,q) time series representation for their changes as, e.g.,

$$\Delta y_t^* = \mu + \phi(L)\left(\Delta y_{t-1}^* - \mu\right) + \psi(L) e_t \tag{64}$$

This gives the following Limited Information Minimal State Variable (LIMSV) presentation of equation (62),

$$\begin{aligned} y_t \quad = \quad & \underbrace{\lambda_1 y_{t-1} + \left(\frac{\lambda_1 + \lambda_2}{\lambda_2 - 1}\right) y_t^*}_{f_t} + \frac{(\lambda_1 + \lambda_2)}{(\lambda_2 - 1)^2} \mu + \Phi(L)\left(\Delta y_{t-1}^* - \mu\right) + \Psi(L) e_t \\ & + \left(\frac{\lambda_1 + \lambda_2}{\lambda_2}\right) \nu_t \end{aligned} \tag{65}$$

If the forecasting agent knew the exact parameterisation of (65), then she should use it for forecasting $E_t y_{t+1}$. However, we do not assume that the agent knows the correctly specified ARMA(p,q) process and, hence, polynomials $\Phi(L)$ and $\Psi(L)$. The only thing that is asumed to be known is that ARMA(p,q) is stationary.[11]

Hence, under assumption that the component f_t is known, then equation (65) gives an exact parameterisation of the forecasting equation only, if the true ARMA(p,q) were $\Delta y_t^* = \mu + e_t$. In this case polynomials $\Phi(L)$ and $\Psi(L)$ would vanish and the exact parameterisation of (65) would be known. However, the stationarity of (64), implies that $E_t \Delta pv_{t+1}$ term converges to a constant μ and hence, the true LIMSV solution of (65) asymptotically converges towards $f_t + \mu$. Hence, utilising this asymptotic property we specify the following PLM relation with time varying parameters:

$$y_t = \alpha_{0t} + \alpha_{1t}\left[\lambda_1 y_{t-1} + \left(\frac{\lambda_1+\lambda_2}{\lambda_2-1}\right) y_t^*\right] + u_t \tag{66}$$

$$\alpha_{0t} = \alpha_{0t-1} + \varepsilon_{0t} \tag{67}$$

$$\alpha_{1t} = \kappa + (1-\kappa)\cdot\alpha_{1t-1} + \varepsilon_{1t} \tag{68}$$

with $u_t \sim iid\left(0,\sigma_u^2\right)$ and $\varepsilon_{it} \sim iid\left(0,\sigma_i^2\right)$ and $\alpha_{it} \geq 0$. Parameter α_{0t} accounts for the drift and the adjustment dynamics of the unknown ARMA. The deviations of parameter α_{1t} from unity are related to the fact the f_{t-1} does not coincide exactly with correctly specified LIMSV. However, in line with the asymptotic properties of (64) parameter α_{1t} in equation (68) is specified to converge to unity with the speed determined by the size of parameter κ which was selected to be 0.03.

Now the PLM equation (66) implies the following forecasting equation

$$E_t y_{t+1} = \alpha_{0t} + \alpha_{1t}\left[\lambda_1 E_t y_t + \left(\frac{\lambda_1+\lambda_2}{\lambda_2-1}\right) y_t^*\right] \tag{69}$$

where the most straight-forward would be to assume that $E_t y_t = y_t$. This causes, however, simultaneity that is worsened by the fact that parameters α_{0t} and α_{1t} are also solved simultaneously with y_t. However, as discussed by Evans and Honkapohja (2001, Chapter 8.6) this problem can be circumvented by assuming that y_t is not included in the information set when forming expectations. This assumption is frequently used in the literature on indeterminacy (e.g. Milani 2007, 2011). Now, on the basis of equations (61) and (62) we re-define the square bracket term of (69) and end up with the following forecast equation for updating learning expectations,

$$E_t y_{t+1} = \alpha_{0t} + \alpha_{1t}\left[\underbrace{\lambda_1^2}_{\delta_1} y_{t-1} + \underbrace{\left(\frac{\lambda_1+\lambda_2}{\lambda_2-1}\right)(1+\lambda_1)}_{\delta_2} y_t^*\right] \tag{70}$$

Equation (70) together with (60) defines the actual law of motion (ALM).

[11] In addition, also the drift term μ may be regime dependent, for instance, with respect to the growth of technical progress and/or population as well as the inflation target of the central bank.

5.1 Kalman filter estimation of learning equations

The updating of the parameters is done via the Kalman filter approach. This has the advantage of being very general, and can capture alternative forms of learning. Equations (66)-(68) form a state-space model where (66) is the measurement and (67)-(68) are the transition equations and can be estimated by a Kalman filter recursion. In matrix form it can be presented as follows,

$$y_t = X_t \boldsymbol{\alpha}_t + u_t \tag{71}$$

$$\boldsymbol{\alpha}_t = T\boldsymbol{\alpha}_{t-1} + \boldsymbol{\kappa} + R\boldsymbol{\varepsilon}_t \tag{72}$$

where $X_t = [1 \;\; (\delta_1 y_{t-1} + \delta_2 y_t^*)]$, $\boldsymbol{\alpha}_t = [\alpha_{0t} \;\; \alpha_{1t}]'$, $T = \begin{pmatrix} 1 & 0 \\ 0 & 1-\kappa \end{pmatrix}$, $\boldsymbol{\kappa} = [0 \;\; \kappa]'$ $R = I$ and $\boldsymbol{\varepsilon}_t \sim N(0, Q_t)$. Following Harvey (1992) and Rockinger and Urga (2000), the variance-covariance matrix associated to $\hat{\alpha}_t$ is: $P_t = E_t[(\alpha_t - \hat{\alpha}_t)(\alpha_t - \hat{\alpha}_t)']$. The best estimates of P_t conditional on information at $t-1$ is

$$P_{t|t-1} = TP_{t-1}T' + RQ_tR^{'} \tag{73}$$

Denote the variance of the residual u_t of the measurement equation by $H = \sigma_u^2$. Now the Kalman updating equations become,

$$\hat{\alpha}_t = \hat{\alpha}_{t|t-1} + \frac{P_{t|t-1}X_t'(y_{t-1} - X_t'\hat{\alpha}_{t|t-1})}{X_tP_{t|t-1}X_t' + H} \tag{74}$$

$$P_t = (I_m - \frac{P_{t|t-1}X_t'X_t}{X_tP_{t|t-1}X_t' + H})P_{t|t-1} \tag{75}$$

Equations (74)-(75) are the standard updating equations of the Kalman filter. In (74) $(y_{t-1} - X_t'\hat{\alpha}_{t|t-1})$ is called innovation and $\frac{P_{t|t-1}X_t'}{X_tP_{t|t-1}X_t'+H}$ the Kalman gain.

One key aspect when setting-up the Kalman filter updating mechanism concerns the initial values (priors) for $\hat{\alpha}$. In some cases it has been found that the dynamics of learning models are sensitive to these choices, which adds a sense of arbitrariness that is *no*t present in the rational expectation solution. In our case, we use *as* starting values the expected long-run values i.e. α_0 =0 and α_1=1,[12] which is consistent with our framework and it thus is a reasonable starting point for the estimation. We nevertheless assign to these initial beliefs a large uncertainty by setting P_0 equal to a diagonal infinity matrix.

Another key aspect of the Kalman filter is the signal to noise ratio, Q_t, which is a measure of the rate at which agents are willing to update their forecasts. A higher Q means agents are more willing to learn and it also reflects their sensitivity to new information. One extreme is recursive least squares estimation which implies that agents have infinite memory, with each observation being given equal weight. As Q_t rises agents effectively discount past observations more rapidly and the Kalman filter becomes equivalent to the constant

[12] Instead of estimating learning forecast equations for the expected levels of dependent variables we estimate them for the expected changes. When the level form specification would imply the convergence of α_{0t} towards a small constant, the difference form specification implies convergence towards zero. We also set P_0 equal to a diagonal infinity matrix, i.e. we assign a large uncertainty to such initial beliefs estimates.

gain algorithm used in the literature where agents give more weight to more recent observations. However, there is a trade-off from discounting past observations in that there is a larger variance in the learning parameters, α_t. Contrary to other applications which simply use a calibrated Q_t, we optimise the degree of learning for each sector by calculating the mean square forecast errors:

$$MSE(y_i) = \frac{1}{T}\sum_{t=0}^{T}(y_{i,t} - \hat{y}_{i,t})^2 \tag{76}$$

where $\hat{y}_{i,t}$ is the forecast of the i-th component based on information at time t-1 and then computing the Q matrix that minimises the in-sample MSE by doing a grid search which is restricted to the range 0.0 (recursive least squares) to 1.9 (which implies a half life of around 5 quarters). This approach results in different degrees of discounting of information across agents.

Our learning approach is applied to the seven main forward looking stochastic equations which are: consumption, price (GDP deflator) and wage inflation, investment, employment, inventories and housing investment[13]. The uncovered interest rate parity condition for the real exchange rate also contains the expected forward exchange rate, however to retain the intrinsically forward looking nature of financial markets, this expectation is treated as rational.

The table below presents the key parameters for the learning equations estimated since 1991Q1. For the time-varying parameters, α_{0t} and α_{1t} the table shows the end point of the parameter estimate (i.e. 2011Q4). We also report the hyper-parameter Q.

In general, the α_{1t} coefficient at 2011Q4 are close to 1. The main exceptions are the price and wage equations where the α_{1t} parameters are lower. In general there is only small changes in the α_{1t} parameters over time with larger changes in the α_{ot} parameter. However, the small variation in the parameters has significant impact on the simulation results.

US Learning Estimates 2011q4

	α_0	α_1	Q	R^2
Private Sector Employment	0.0025	0.8128	1.9	0.757
Consumption	0.0726	0.9998	0.01	0.999
Investment	0.0008	0.9513	1.9	0.946
Prices	0.0008	0.7434	0.005	0.771
Wages	0.0007	0.7316	0.02	0.579
Inventories	11.004	1.0761	0.0	0.348
Housing Investment	-0.0428	1.0006	0.0	0.845

[13] A number of expected variables are derived from these key estimated expectations - e.g. expected HICP excluding energy is formed by a weighted average of the GDP deflator, the import deflator excluding energy, p_t^{MN}, and of energy prices (capturing the indirect effect of energy prices on core inflation).

6 Model evaluation

In this section we undertake a range of simulations to highlight the properties of the model for scenario and policy analysis. These simulations are not intended to provide a comprehensive account of the model's multipliers but are only indicative. Indeed, the model could be used to analyse many types of structural shocks, or even direct shocks to expectations. In undertaking scenario analysis it is important to consider exactly what underlying assumptions are required. Namely, it is important to consider what information set agents have, i.e. do households and firms have the same information set as Central Banks and Governments, or are there information asymmetries; is the shock anticipated or unanticipated, and finally it is important to distinguish between transitory and permanent shocks.

We consider two alternative approaches to expectation formation. The first approach is the case of perfect foresight (model consistent) rationality where all agents have the same information set and know the duration of the shock and how the Central Bank and indeed the economy will react to the shock. In this approach, agents adjust their behaviour as soon as a change is anticipated, we call this 'announced and credible' shock. In this case where expectations are assumed to be fully rational with perfect foresight, the future expected value is simply replaced by the model future realisations and the model solved iteratively, such that the expectations are fully model consistent, i.e. expected outcomes are replaced with model outcomes:

$$E_t y_{t+1} = y_{t+1} \tag{77}$$

To solve the model, given that the model has significant non-linearities, we use a stacked-time Newton-Raphson algorithm.We can use a partially solved-out version of the model, where the whole model is essentially written in terms of observables. In order to minimise the effects of the terminal condition on the solution at the beginning (Roeger and In't Veld, 1997), the model is solved over a horizon of 250 years, although we focus on the results over the first 10 years.[14]

Although this assumption of perfect foresight is extreme, these simulations provide a benchmark for our second approach, which is the bounded rationality approach outlined above, where agents learn about the shocks and how the economy responds to those shocks. Under this approach, we relax the assumption that all agents (households, firms, unions, central banks and governments) have the same information set. We assume the Central Bank follows a Taylor rule, where it adopts the private agents' expectations for inflation rather than assuming the Central Bank has more information than private sector agents, or even full information. However, we assume the real exchange rate follows a forward-looking, model-consistent UIP.[15] This second approach will enable us to isolate the implication of relaxing the assumption of perfect foresight.

The trajectories of economic variables in response to shocks depend on the parameter estimates (reflecting structural or economic factors), the initial states of the economy, and on the roots of the equations (i.e. the

[14] Under rational expectations long simulation horizons ensure that the early part of the simulation path is (at most) marginally affected by the choice of terminal conditions. See Dieppe et al (2012) for more details.

[15] The learning model can currently be simulated using a learning based UIP rule, but this clearly has different properties to the rational, model-consistent UIP. Using the latter could be interpreted as assuming rationality in the asset market.

degree of forward/backward lookingness), which determine the adjustments of the economy to fundamentals. For example, if a sector is more forward-looking, then typically this implies quicker adjustment, whereas backward-looking behaviour reflecting frictions implies that shock effects are more persistent, i.e. distributed over a longer period and potentially leads to hump shaped adjustment paths. In addition the speed of learning also affects the dynamics of the economy. All these factors provide a framework to interpret the simulations.

The scenarios are conducted under the assumption that the rest of the world is exogenous, namely: world demand and prices, world interest rates, and oil prices.[16]

The fiscal rule (56) used assumes that personal income taxes change to stabilise the debt-to-GDP ratio, while government consumption and investment are assumed to follow an exogenously determined path in real terms, and other fiscal variables remain constant as a share of GDP. The scenario results are sensitive to these fiscal assumptions and given the detailed nature of the fiscal block of the model, other specifications could be considered (for example also linking government expenditures to changes in the debt level).

We consider a variety of shocks, all occurring at time t. The first is an interest rate shock which informs us about the transmission of monetary policy within the MUSEL. We then consider a permanent demand side shock, namely a permanent increase in government spending. We also consider a permanent supply side shock – a technology shock – either labour or capital augmenting. Thereafter, we conduct two external shocks, the first being a permanent appreciation of the USD nominal exchange rate against all foreign currencies and the second a permanent world demand shock. Specifically we consider:

1. *Short-term interest rate shock* - defined as a 50 basis point increase in the short-term interest rate for 1 period followed by the interest rate rule.

2. *Permanent Government expenditure shock* – specified as a 0.5% of GDP increase in public consumption over the entire simulation horizon.

3. *Permanent Technology (TFP) shock* such that potential output is up by 0.1% over the entire horizon.

4. *Permanent appreciation of the dollar nominal exchange rate* by 5% against all foreign currencies and

5. *Permanent world demand shock* of 1 per cent increase in world import demand.

The simulations are presented in the figures at the end of the paper, where the results are presented as deviations from baseline (real variables and employment are expressed as percent deviations; prices as *percentage point* differences in year-on-year inflation rates and for interest rates, savings ratio, fiscal deficit, and unemployment, they are expressed as absolute deviations, either in percentage points or percent of GDP).

6.1 Short-term interest rate shock

We start by considering the reaction of the economy to a 1 period shock to the short-term interest rate of 50 bp followed by a interest rate rule. Figure 1 shows the response of the economy to this shock.

[16] Endogenisation of these variables would require a global model.

The transmission of monetary policy operates through a number of channels. First, via a change in real exchanges rates - due to the real UIP, which impacts both import prices and competitiveness and therefore, the trade balance. Secondly, monetary policy affects the user-cost of capital which raises the user cost of productive and residential capital, thereby acting as a depressant on firm and housing investment. Thirdly, households are directly affected due to a higher discount rate, and by intertemporal substitution effects where savings and future consumption become more attractive. Another channel is that net interest payments for firms, households and governments are also affected. Finally, expectations play a key role in all of this; for example households are already in the first period affected by expected lower wage growth which immediately decreases their consumption expenditure.

The combined effect of all this is a reduction in final demand as firms reduce output and thus, the demand for factors of production. This depresses prices, and combined with lower employment creation, causes lower wages and lower household income. The lower demand decreases imports, alleviating somewhat the downward pressures on GDP. Interest rates only gradually adjust down*wards* following a one period shock due to interest rate smoothing. Therefore, the price effect is relatively large as agents realise that the shock to interest rates takes time to unwind. However, lower prices start to lead to gains in competitiveness, which combined with improvements in productivity results in output growth returning to its previous level, halting the decrease in inflation. Overall, a temporary shock to short-term interest rate leads to a temporary fall in real GDP but a more gradual decrease in the price level. After 5 years, interest rates are nearly back to base.

The rational expectations case, where agents know that it is only a one period shock followed by a Taylor rule reaction is compared with the model simulated under learning. In both cases, both demand and prices initially fall, but then start to return back to base as increased competitiveness aided by lower interest rates boost the economy, as discussed above. Under the rational / model consistent expectations scenario, although demand reacts quicker, the responses are qualitatively similar. However, the initial price responses are much larger than under the assumption of learning and less persistent. This is because the learning model, to some extent, adds more frictions dampening the initial effects, so the model become*s* more like a backward-looking model. However, while the initial effects are different, the effect converge to each other in the longer run. In line with Slobodyan and Wouters (2012), differences in the adjustment paths under learning and rational expectations are larger on the price than real side of the model.

6.2 Permanent Government expenditure shock

As noted by Van Brusselen (2009), there is no consensus on the response of the economy to a fiscal policy expansion. Indeed estimates in the literature vary widely depending on the model used. As the fiscal block is quite extensive we can consider expansionary fiscal policy through alternative instruments, but in this section we focus purely on an expansion in government expenditure, more specifically on an unanticipated but credible announcement of an immediate and permanent increase in government spending of 0.5% of GDP. Figure 2 shows the impact of this shock under learning and under perfect foresight.

The increase in demand leads directly to an increase in employment and output. In the learning case, GDP initially rises by around 0.4%. While this increase is similar to the rational expectations it is more persistent. The increase in government spending leads to a less favourable government financial position which needs to be financed. While a reduction in government transfers to households resulting from lower unemployment achieve some reduction in spending, the fiscal rule employed in the model implies that households will also be affected by higher tax rates in order to bring government debt as a share of GDP back to its previous levels and as a result, households will have less future disposable income. Consequently, in the perfect foresight version of the model, household reduce their current consumption in anticipation. Therefore, following the initial increase, real GDP subsequently declines as there is also strong crowding out due to higher interest rates, and hence higher cost of capital which reduces both investment and consumption as well as helping to reduce the surge in inflation. Furthermore, consistent with the uncovered interest rate parity condition, the shock triggers an initial appreciation of the domestic currency, thus reducing export demand, followed by a gradual depreciation back to the baseline.

On the nominal side, the stronger demand puts upward pressure on prices and wages. The main transmission from a positive demand shock to prices is via marginal costs directly affecting the hybrid new Keynesian price and wage Phillips curves. Unions respond to declining unemployment by increasing their wage demands and firms respond by increasing prices. Further pressure on wages is due to the increase in tax rates which aims to ensure sustainability of government finances. The short-term interest rate reacts to the increased marginal cost and inflation by reducing demand. Through the increase in inflation, domestic producers become less competitive, so exports tend to decrease and imports increase, leading to a worsening trade balance.

In comparing the learning and the rational expectations approach, we notice that the learning model is stable and converging to the rational expectations solution. However, as also previously shown, the model with the learning process exhibits very different properties from the model with rational expectations, particularly in terms of the adjustment path to equilibrium and in terms of household behaviour.

6.3 Other simulations

In addition to the two shocks above we also report the simulation results of an appreciation of the US dollar, a permanent world demand shock and a supply shock in the form of two technology shocks. As we have a CES production function, we consider a shock both to labour and capital augmenting technical progress, such that ceteris paribus potential output is permanently higher by 0.1% – i.e. the economy can produce more output for a given labour and capital. As with the previous simulations, after these shocks, there is an adjustment to a new equilibrium which is the same in the rational and the learning. However the key differences remain - more sluggish adjustment, particularly on the nominal side.

For the exchange rate shock, the empirical literature suggests that pass through from exchange rate changes into both import and export prices is rather low in the US in comparison with other countries including the euro area (see for example Goldberg and Tille, (2006) who refer to estimates of exchange-rate pass-through into US import prices ranging between 25 and 40 percent). A key reason for this is the widespread dollar invoicing of external transactions with the United States. In line with this analysis, our results suggest a around 40 percent of exchange-rate changes are passed through into both import and export prices, while the remaining trade prices are insulated from exchange rate fluctuations.

6.4 Sensitivity to alternative monetary policy rules

It is important to benchmark our model against alternative assumptions and models. In this section we consider the sensitivity of the results to alternative monetary policy rules. Wieland et al (2012) have shown that macroeconomic model reactions are highly sensitive to the parameterisation of the monetary policy rule, with the output gap reacting more strongly to interest rate shocks under monetary policy rules that place a smaller weight on stabilising fluctuations in output rather than inflation. We investigate the sensitivity of MUSEL by simulating a 100 basis point increase in short-term interest rates under four alternative US specifications. We use four of the rules reproduced in Wieland et al (2012) based on Taylor (1993), Levin et al (2003), Christiano et al. (2005) and Smets and Wouters (2007) :

Taylor (1993):

$$i_t = \sum_{j=0}^{3} 0.38(\Delta p_{t-j} - \Delta\hat{\bar{p}}_t) + 0.5q_t + \varepsilon_t \tag{78}$$

Levin et al (2003):

$$i_t = 0.76i_{t-1} + \sum_{j=0}^{3} 0.15(\Delta p_{t-j} - \Delta\hat{\bar{p}}_t) + 1.18q_t - 0.97q_{t-1} + \varepsilon_t \tag{79}$$

Christiano et al (2005):

$$i_t = 0.8i_{t-1} + 0.3(E_t\Delta p_{t+1} - \Delta\hat{\bar{p}}_t) + 0.08q_t + \varepsilon_t \tag{80}$$

Smets and Wouters (2007):

$$i_t = 0.81i_{t-1} + 0.39(\Delta p_t - \Delta\hat{\bar{p}}_t) + 0.97q_t - 0.90q_{t-1} + \varepsilon_t \tag{81}$$

Figures 7 and 8 show that inflation and output responses are quite sensitive with respect to a chosen Taylor rule variant, especially under rational expectations. As mentioned above, this result is not specific to our model, but confirmed by the responses of other models to a short-term interest rate shock of the same magnitude (see Figure 1 in Wieland et al, 2012). The Christiano and Smets and Wouters rules, in particular, lead to larger output and inflation responses than the other rules, both under rational and learning expectations. Under learning expectations, the short-run effects are less pronounced, but more protracted and the differences in the responses across rules are smaller under learning - i.e. the results are less sensitive to the chosen Taylor rule.

6.5 Comparison with other models

We also use the platform developed by Wieland et al (2012) to compare simulation results from our model with other models developed for the United States. Taking advantage of the explicit and flexible modeling of expectations adopted in MUSEL, we can compare simulations results with some of the more popular models existing in the literature under rational expectations and also with recent models using adaptive learning specifications.

This provides a good overview of how MUSEL's impulse response functions rank across a wide range of models of different size and across different specifications and assumptions. Given the sensitivity of the results to the monetary policy rule we use the Smets and Wouters 2007 formulation for all models, for comparability purposes.

The models used in the rational expectations comparison are:

- two medium-scale state of the art DSGE models, including Smets and Wouters (2007) and the version of the Christiano, Eichenbaum and Evans model estimated by Altig et al (2011), which have been widely used in policy analysis (US_SW07 and US_ACELm)

- a DSGE model incorporating credit market imperfections developed by De Graeve (2008) (US_DG08)

- a larger-scale model usually employed to build forecast, given by the IMF projection model by Carabenciov et al. (2008) (US_PM08)

Under learning, MUSEL is compared with:

- the learning model developed by Milani (2007) to analyse macroeconomic persistence (US_M07)

- the medium-scale DSGE learning model proposed by Slobodyan and Wouters (2012), using the same parameter values as in Smets and Wouters (2007) (SW12)

- the learning model developed by Rychalovska (2013) to analyse the role of financial frinctions under learning (YR13)

- the Federal Reserve Board large-scale model (FRB/US, see Brayton and Tinsley, 2005), lineralized by Brayton and Laubach (2008) and incorporating mixed expectations (US_FRB08mx).

Simulating an increase in short-term interest rates yields results for our model that are close to other estimated models, see Figures 9 and 10. Assuming rational expectations, our model responses lie within the ranges of the other models for the output responses. Together with the IMFS's US_PM08 model, however, the inflation responses are on the high side and faster than in the other models for the US. Learning expectations turn the response effects more gradual and, in comparison to other models with learning expectations, the MUSEL responses are within the range of the respective response effects of other models, despite a somewhat more protracted reaction of real GDP growth.

6.6 Effects of government expenditure shocks over time

The impact of policy shocks may not be constant over time, but depend on the state of the economy as well as the expectations thereof. Against the background of the current debate regarding the size of fiscal multipliers[17], we use MUSEL to investigate whether the government consumption multiplier is larger during downturns. Using the bounded-rationality framework, the simulations in Figures 11 to 12 show the time-varying impact of a permanent shock to government expenditure of the size of 0.5% of GDP on real GDP and consumer price inflation under the assumption of no monetary accommodation (i.e. exogenous interest rates and exchange rates). The x-axis indicates the quarters in which the shock was implemented, focusing on the period 2000Q1 to 2011Q4, the y-axis denotes the time horizon (up to 12 quarters) and the z-axis denotes the size of the impact in percentage deviation from baseline. The simulations confirm that the government consumption multiplier in

[17] For a summary of the policy debate, see for example European Parliament (2013).

the US is larger during recessions than in expansionary periods, as indicated by the rise in the multiplier during both the 2001 and the 2007-2009 recessions[18]. By contrast, the impact on inflation is more stable over time. The time-variation in the response to the government consumption shock is due to the learning mechanism as agents adjust their expectations in response to the shocks hitting the economy and to the state of the economy as reflected by the fundamentals. Overall, our findings would support the effectiveness of expansionary fiscal policy to buffer economic downturns in the US.

6.7 Model fit

Finally, in order to assess how well the model as a whole fits the data, we derive the residuals from the key estimated dynamic equations by inverting the learning version of the model over the period 1998q1 to 2011q4. The advantage of our modelling approach, compared to e.g. the DSGE modelling approach, is that residuals of each estimated equation are easily produced revealing directly the data compatibility of specified equations. However, rather than on the whole model bases this assessment shows the data compatibility of the model on a single equation basis. An interesting but essentially more demanding approach would be to apply the stochastic simulation approach. This is, however, well beyond the scope of this paper, because that would require, inter alia, the definition of the exogenous variables as stationary stochastic processes for which there is no need in the normal use of the model.

The model residuals are reported in Figure 13 at end of the paper and complement the J-test results presented in section 4. It is worth noting that these residuals reflect also the goodness of the fit of the estimated longer-run relations (i.e. the medium-run supply-side system and the long-run relations estimated for foreign trade), because non-stationarity in the residuals of long-run relations would be transmitted also into the residuals of dynamic equations. We see, however, that the residuals of most equations appear well behaved without any persistent breaks around the recent financial crisis. The only exception is the increased volatility in the house price residuals in the period preceding the financial crises. Non-surprisingly that increased volatility corresponds to period of the US recent housing price bubble.[19] There are, though, also a notable deviation from white noise in the residual of CPI excluding energy. This reflects, however, that instead of genuinely estimating the equation for consumer prices they were calibrated to retain consistency with the estimated speed of adjustment for output prices and a priory information on the content of private consumption. Here, some extra work for improving the fit properties, while retaining internal consistency in price adjustment mechanism, would be desirable.

[18] This finding is consistent with earlier evidence by Dieppe et al. (2013) on the euro area as well as by Auerbach and Gorodnichenko (2012) for OECD economies.

[19] The estimation period of housing price equation covered the period 1953:q3-2011:q3. Over the whole period the residuals showed no autocorrelation. However, regarding the volatility of the residual we can separate three sub-periods. The first relatively volatile period extend to early 1980's.That was followed by the exceptionally nonvolatile sub-period extending to around 2003-2004. In the last , i.e. the bubble period the residual volatility is roughly double compared to that in the first sub-period.

7 Conclusions

In this paper, we have developed a model of the US economy specified to track both the short-run dynamics around the medium-run equilibrium and also the deviations of the medium-run developments from the balanced-growth-path. The model consists of three optimising private sector decision making units: firms, trade unions and households. Output is in the short-run demand determined and monopolistically competing firms set prices and factor demands. Labour is indivisible and monopoly-unions set wages and households make consumption/saving decisions. In addition the model has a separate government sector. In the medium-run the supply-side has cross-equation restrictions which allows for non-unitary elasticity of substitution between labour and capital and factor augmenting technical progress that is allowed to vary over time. The micro-founded theoretical coherence of the model addresses the Lucas critique, even though we assume that agents have only limited information, knowing only the parameters related to their optimization problem but not the rest of the model or the stochastic exogenous processes driving the model. The assumption of limited information boundedly rational expectations allow us to estimate the model with GMM.

In this paper, indicative simulations are undertaken with agents' expectations based on this learning approach or on rational, model consistent expectations. These simulations suggest that the impulse responses of the model to exogenous shocks are plausible. Under learning the effect of a monetary policy on output and inflation is more muted but persistent than under rational, but both formations are broadly comparable to other US macro models. Using the learning version, we find stronger expansionary effects of an increase in government expenditure during periods of downturns compared to booms.

The model is used to provide the underlying framework and interpretation of the US projections at the ECB. It is foreseen to link the model to the other country blocks of the NMCM so linkages between US and euro area can be explored.

8 References

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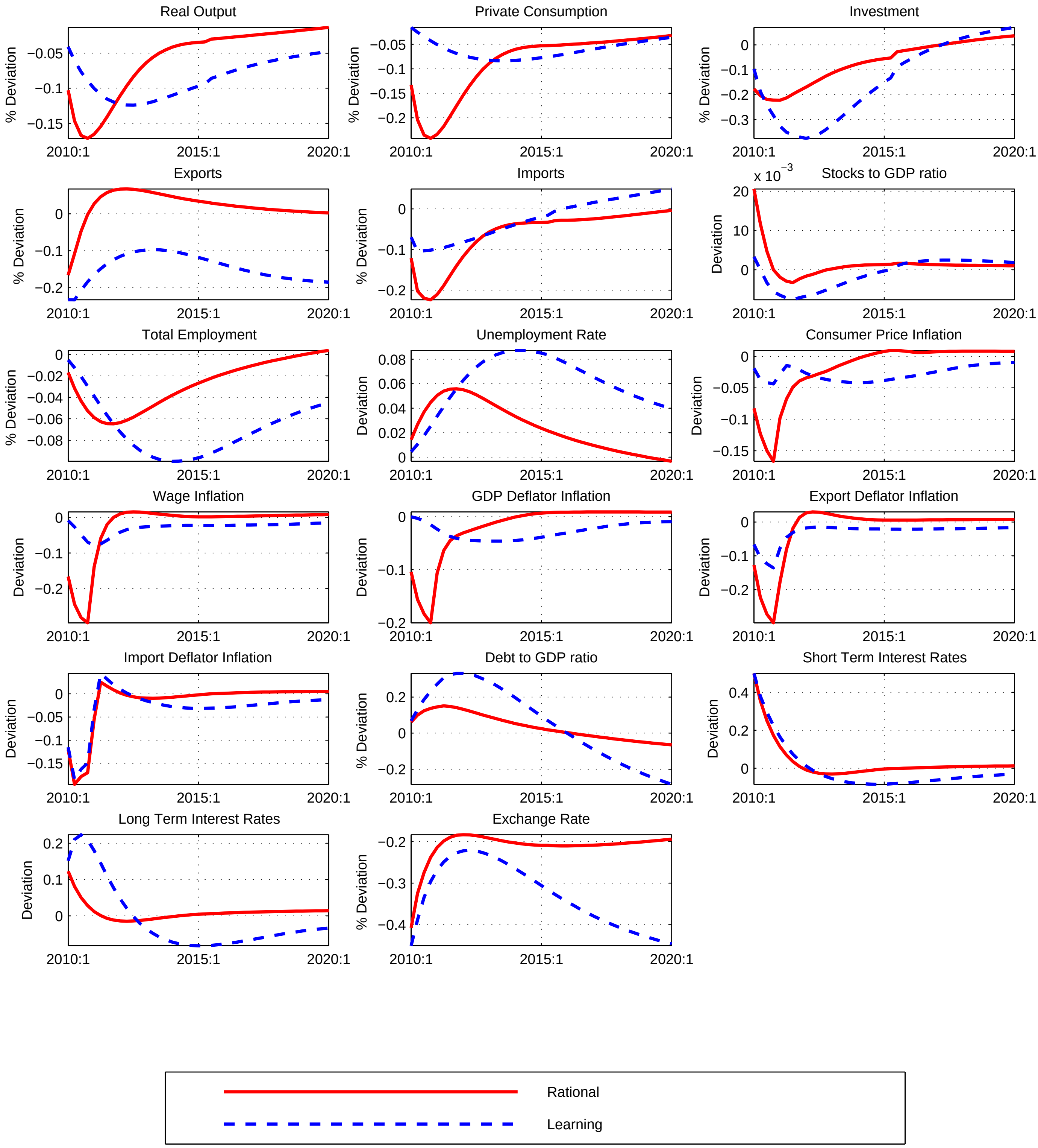
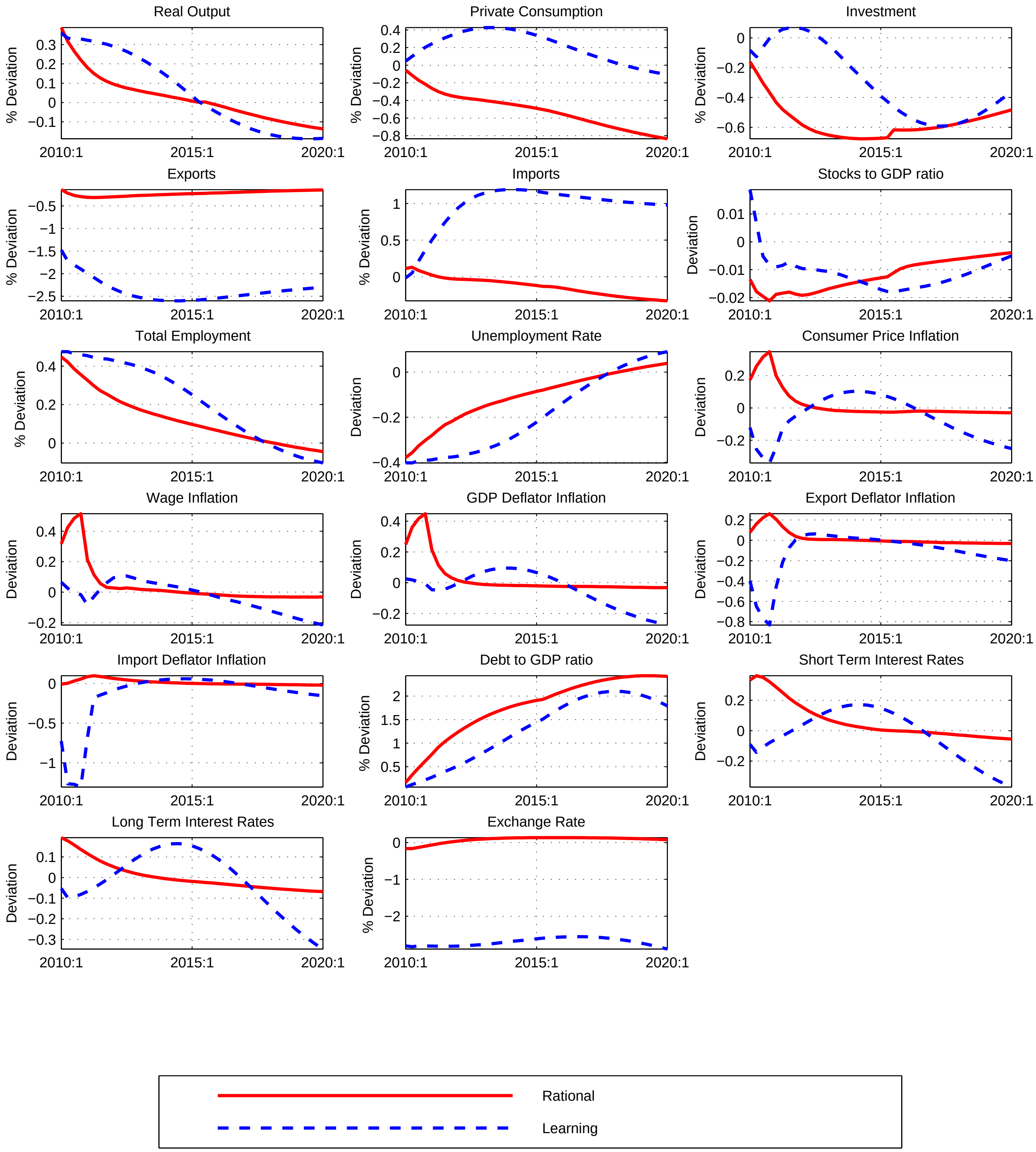
Figure 1: Shock to short−term interest Rates (50bp) − US



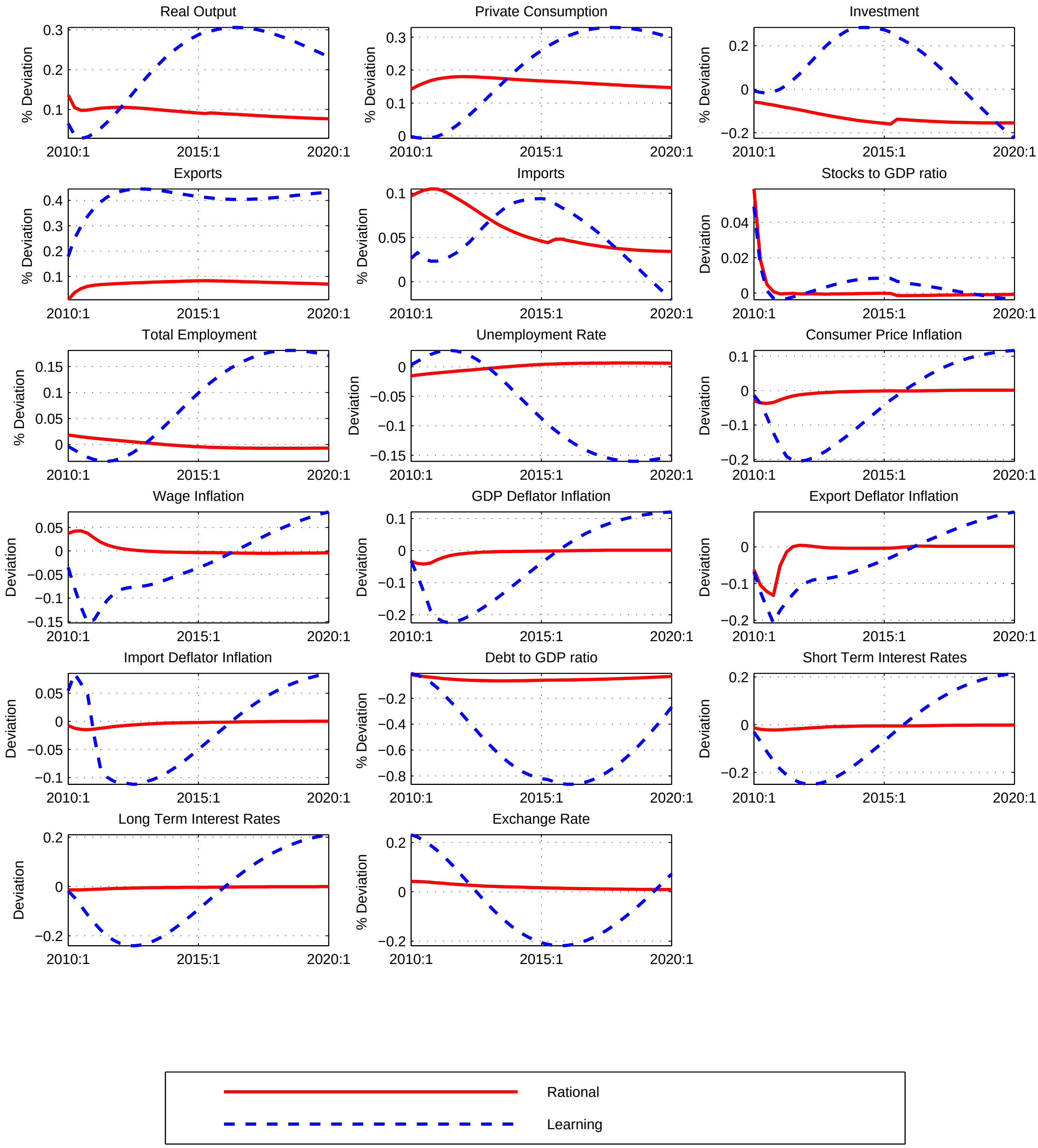
Real variables, employment and exchange rate are presented as percentage deviations from baseline, all other variables are in difference from baseline (either percentage points or percent of GDP). Inflation is computed as year-on-year rate of change.

Figure 2: Permanent Government consumption (0.5% of GDP) − US



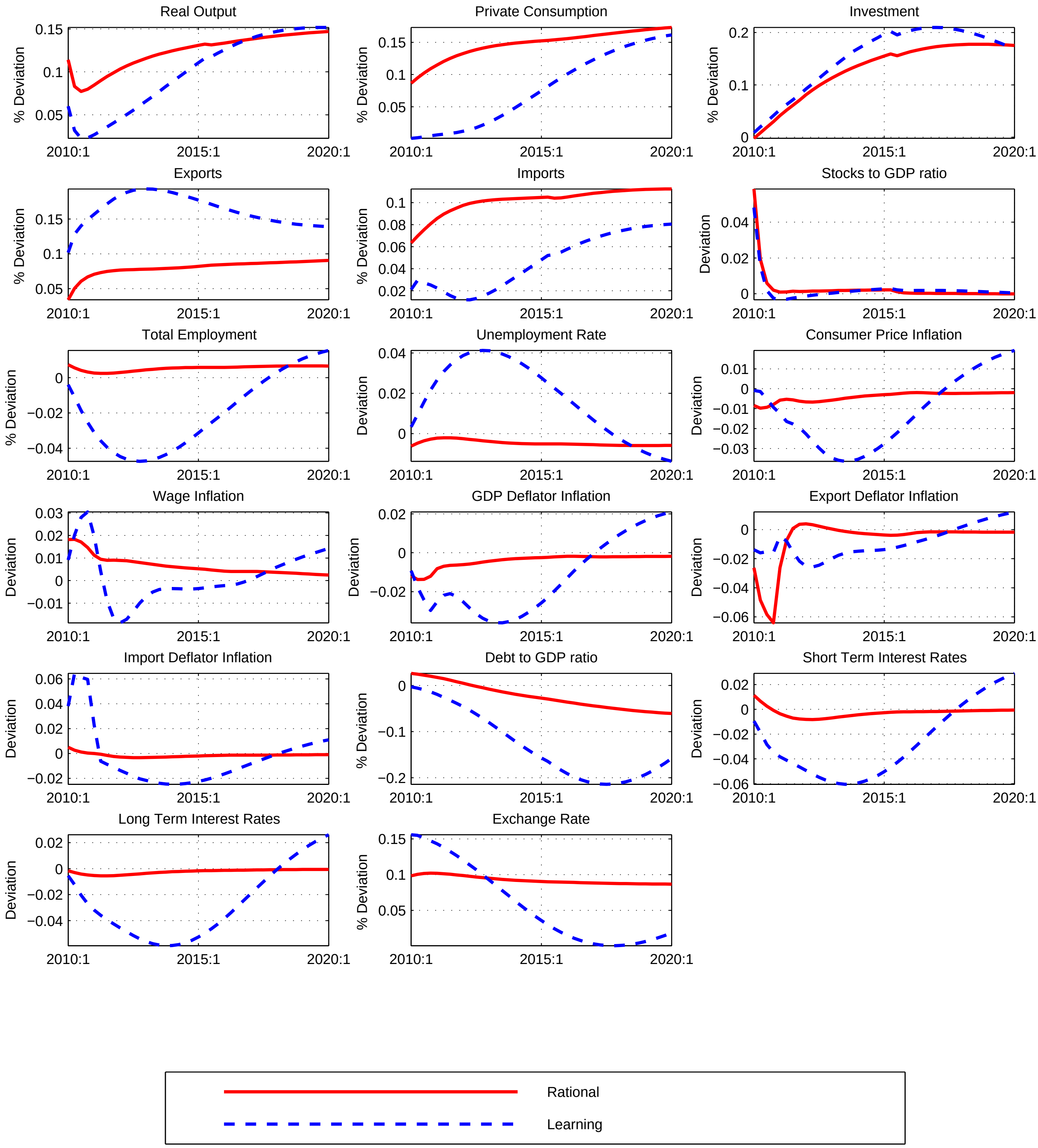
Real variables, employment and exchange rate are presented as percentage deviations from baseline, all other variables are in difference from baseline (either percentage points or percent of GDP). Inflation is computed as year-on-year rate of change.

Figure 3: Capital Augmenting Technology shock (0.1% GDP) − US



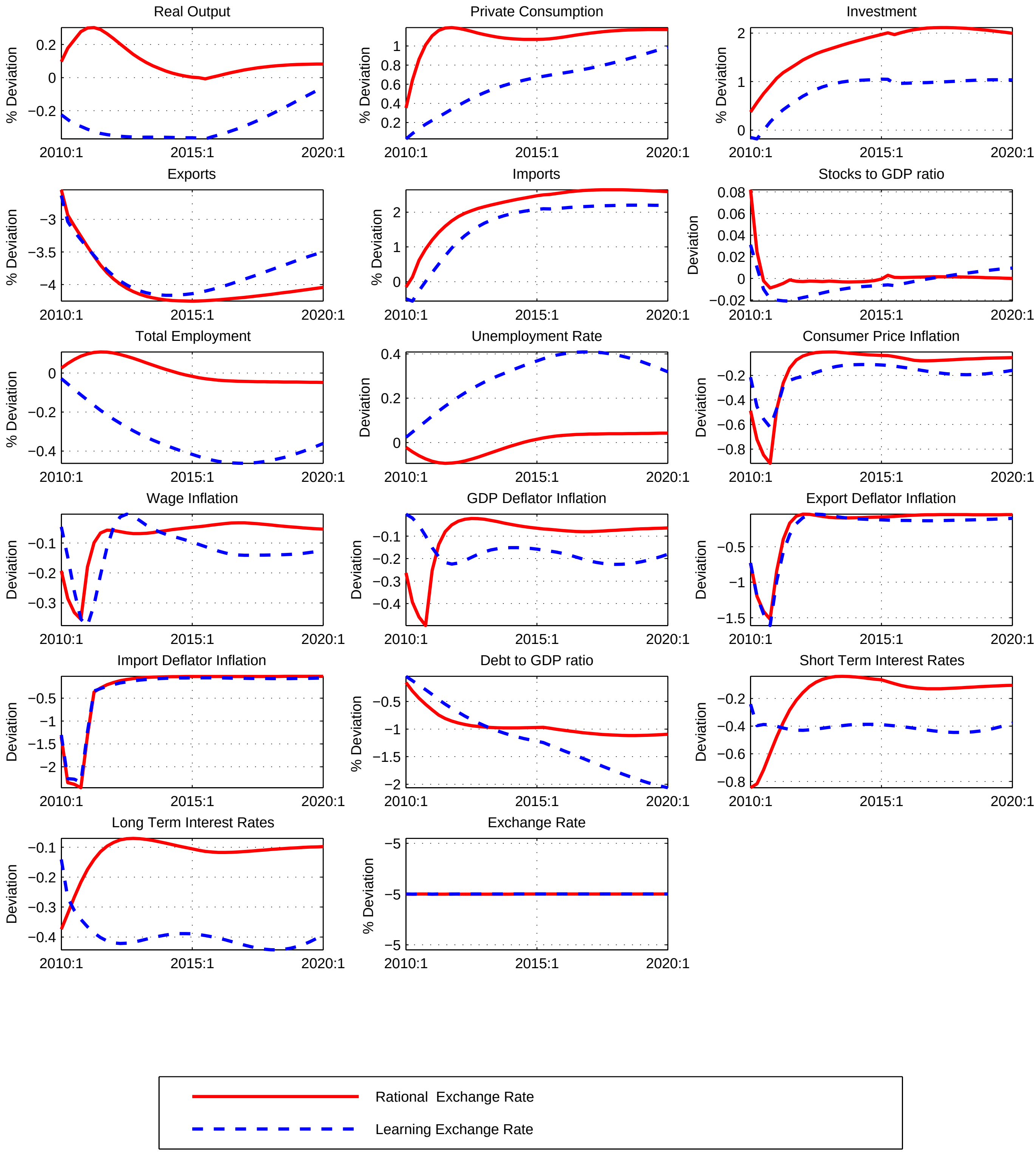
Real variables, employment and exchange rate are presented as percentage deviations from baseline, all other variables are in difference from baseline (either percentage points or percent of GDP). Inflation is computed as year-on-year rate of change.

Figure 4: Labour Augmenting Technology shock (0.1% GDP) − US



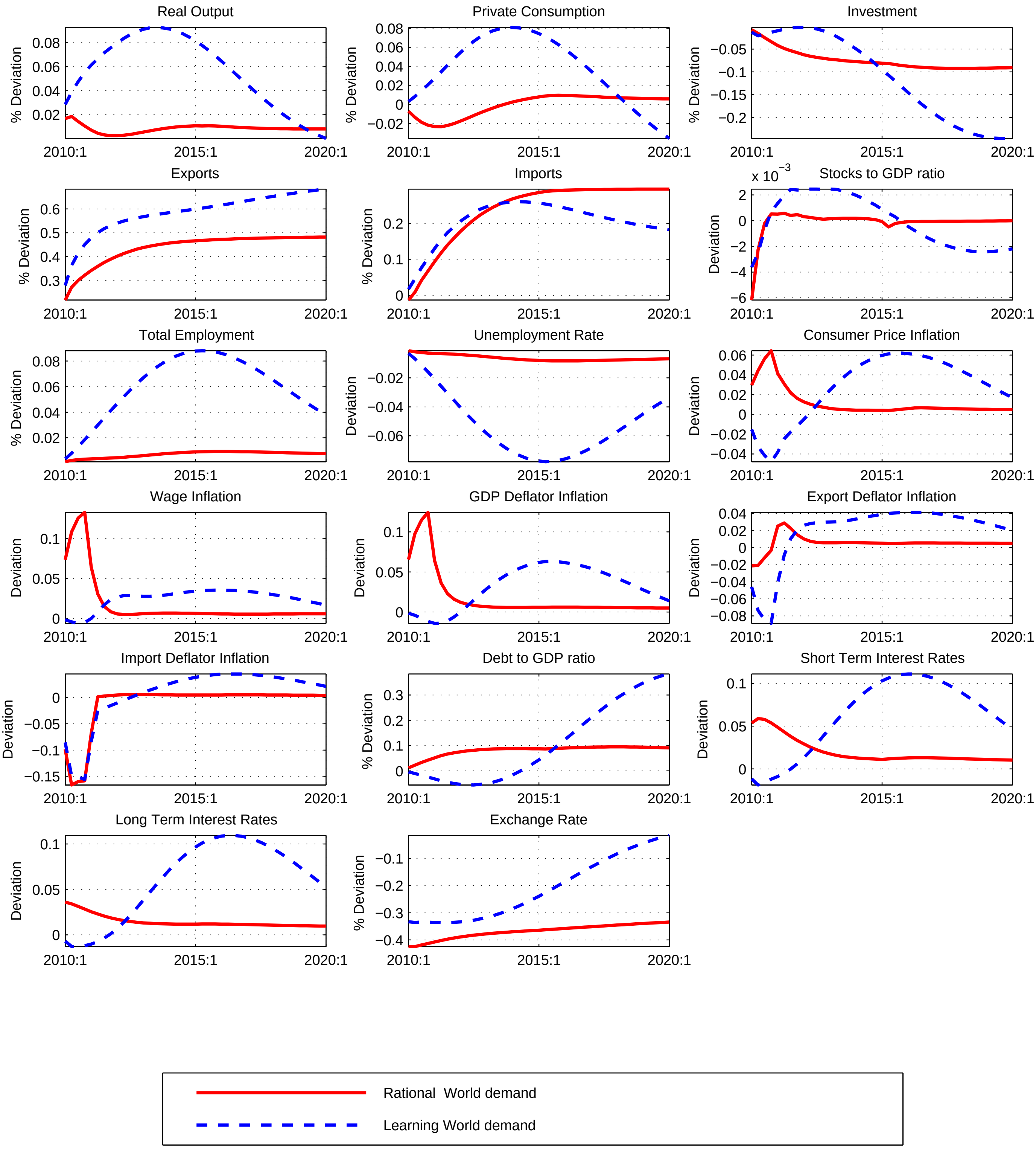
Real variables, employment and exchange rate are presented as percentage deviations from baseline, all other variables are in difference from baseline (either percentage points or percent of GDP). Inflation is computed as year-on-year rate of change.

Figure 5: 5% Permanent Exchange rate shock − US



Real variables, employment and exchange rate are presented as percentage deviations from baseline, all other variables are in difference from baseline (either percentage points or percent of GDP). Inflation is computed as year-on-year rate of change.

Figure 6: 1% World demand shock − US



Real variables, employment and exchange rate are presented as percentage deviations from baseline, all other variables are in difference from baseline (either percentage points or percent of GDP). Inflation is computed as year-on-year rate of change.

Figure 7: MUSEL with different Monetary Policy Rules - Rational expectations

Short-term interest rates *(difference from baseline, basis points)* | **CPI inflation** *(difference from baseline, pp year-on-year)* | **Real GDP growth** *(difference from baseline, pp quarter-on-quarter)*

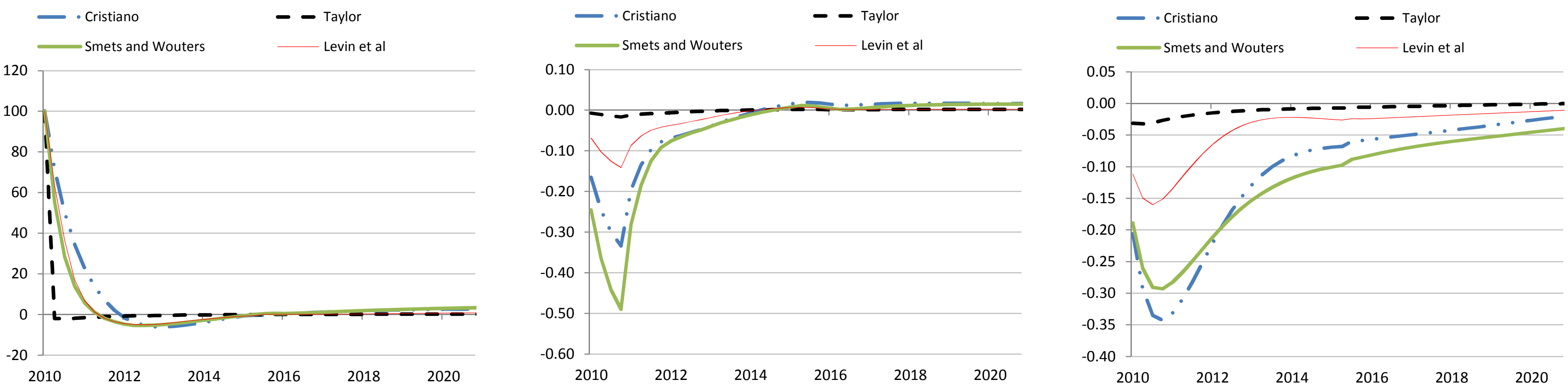


Figure 8: MUSEL with different Monetary Policy Rules - Learning expectations

Short-term interest rates *(difference from baseline, basis points)* | **CPI inflation** *(difference from baseline, pp year-on-year)* | **Real GDP growth** *(difference from baseline, pp quarter-on-quarter)*

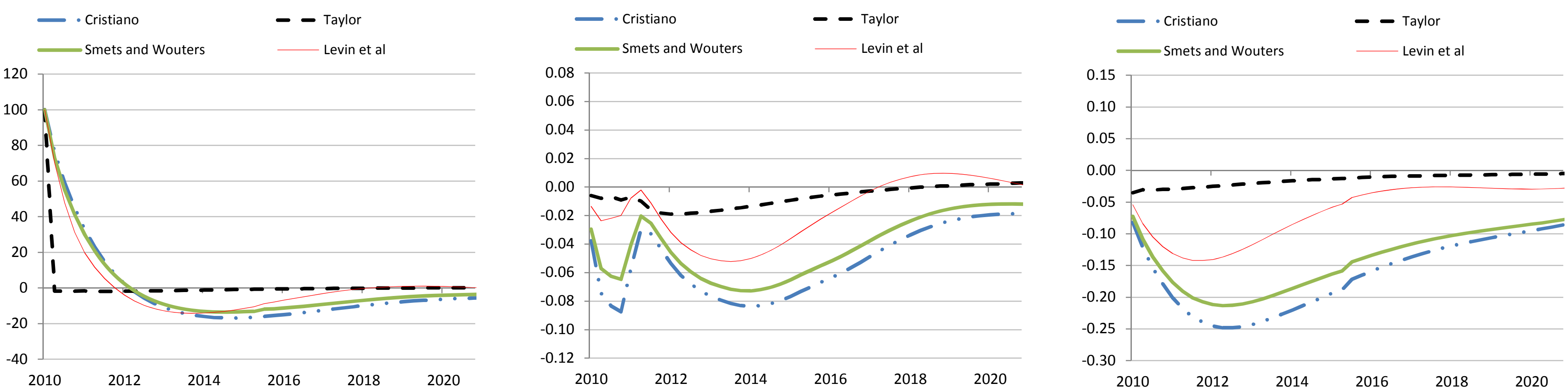


Figure 9: Different Models with the Smets and Wouters Monetary Policy Rule - Rational expectations

Short-term interest rates *(difference from baseline, basis points)* | **CPI inflation** *(difference from baseline, pp year-on-year)* | **Real GDP growth** *(difference from baseline, pp quarter-on-quarter)*

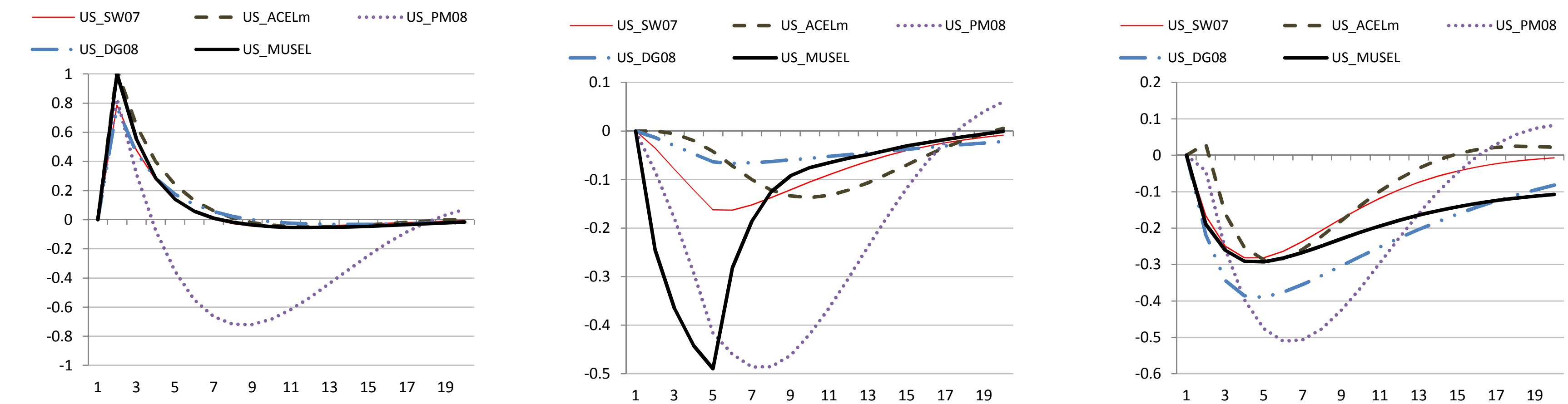
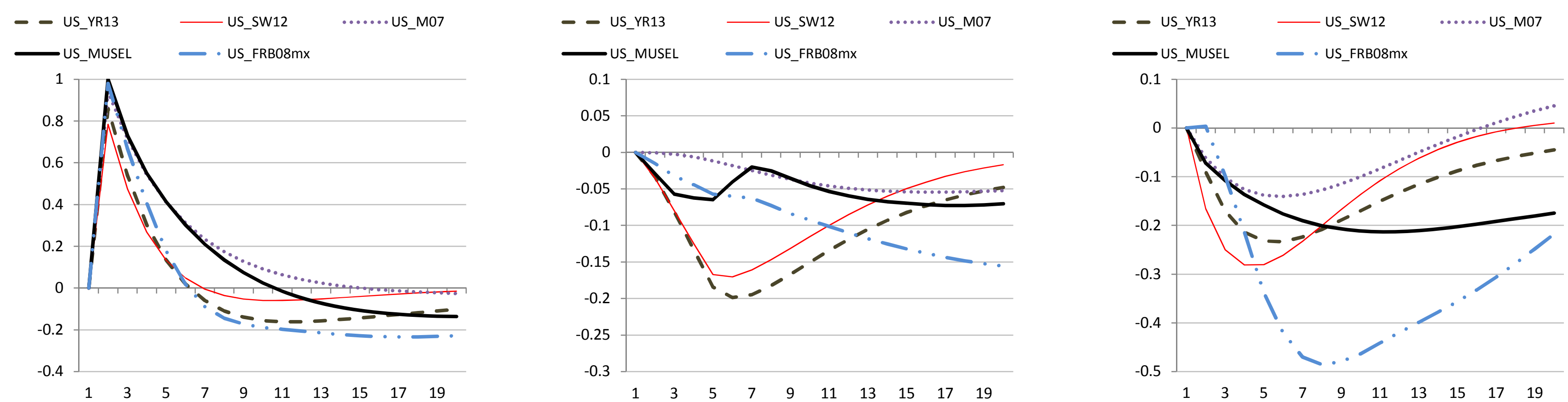


Figure 10: Different Models with the Smets and Wouters Moneteary Policy Rule - Learning / Mixed expectations

Short-term interest rates *(difference from baseline, basis points)* | **CPI inflation** *(difference from baseline, pp year-on-year)* | **Real GDP growth** *(difference from baseline, pp quarter-on-quarter)*



* For the US_M07 model, the line refers to the output gap

Figure 11: Permanent shock to government expenditure (0.5 % GDP):
Exogenous interest rates; Impact on Real Output

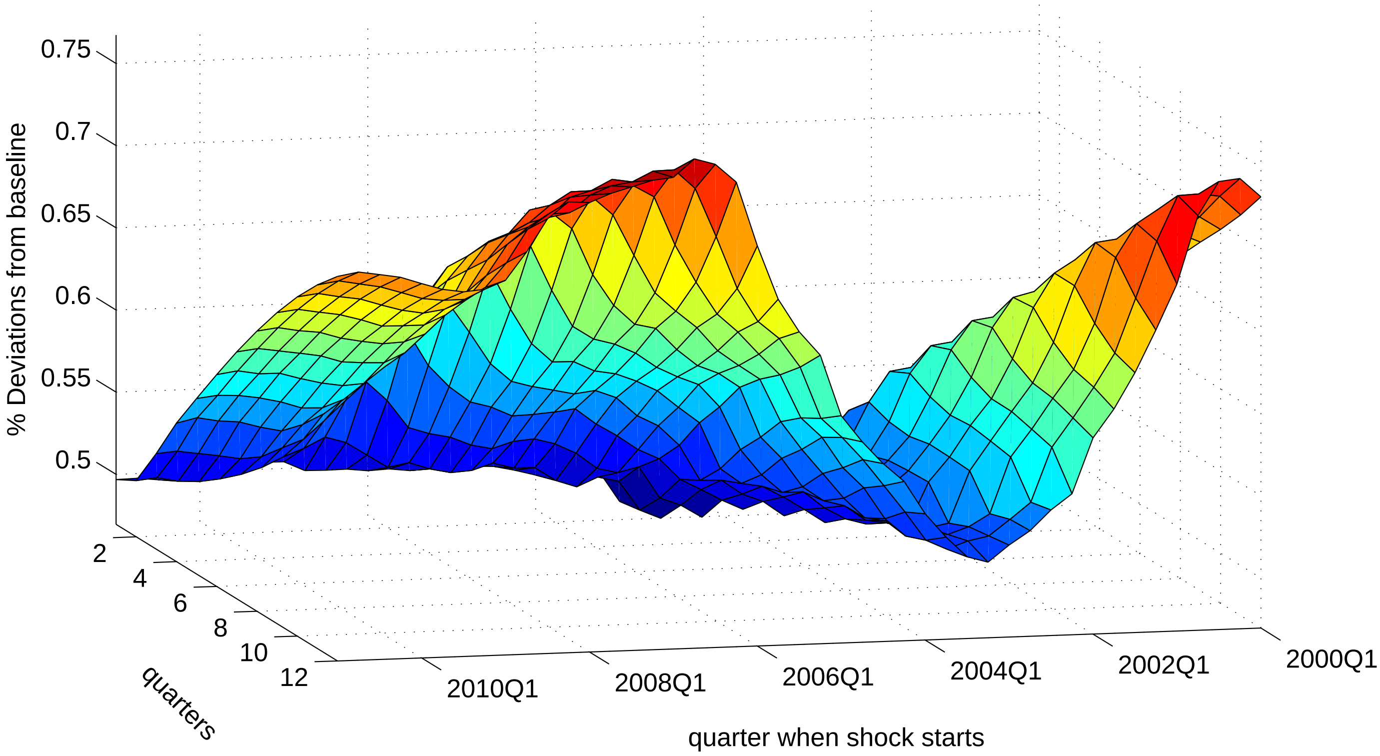


Figure 12: Permanent shock to government expenditure (0.5 % GDP): Exogenous interest rates; Impact on Consumer Price Inflation

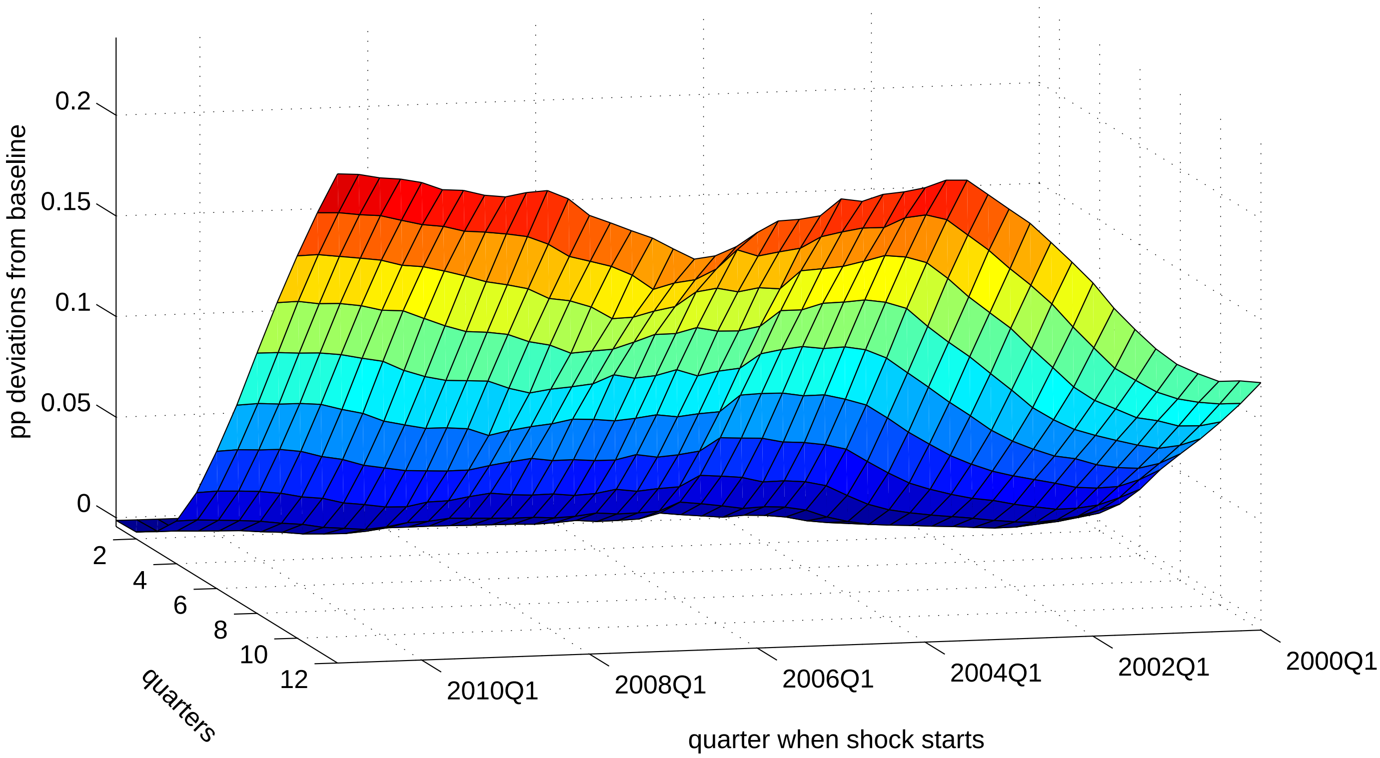


Figure 13: Residuals of the key dynamic model equations