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Liquidity, innovation, and endogenous growth

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Abstract

We study optimal liquidity management, innovation, and production decisions for a continuum of firms facing financing frictions and the threat of creative destruction. We show that financing constraints lead firms to decrease production but may spur investment in innovation (R&D). We characterize which firms should substitute production for innovation in the face of constraints and thus display a "gambling" type of behavior. We embed our firm dynamics into a model of endogenous growth and show that financing frictions have offsetting effects on economic growth.

Keywords: Innovation; Cash management; Financial constraints; Endogenous growth

JEL Classification Numbers: D21; G31; G32; G35; L11

Liquidity, Innovation, and Endogenous Growth

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One of the deepest concerns of European policymakers is the lack of growth and investment in the European economy after the last financial crisis. Many discussions of the causes of these problems have indicated that financing frictions might be an important driving force behind the slow recovery of the European economy. However, understanding the precise mechanisms through which financing frictions influence innovation and economic growth is difficult without a theoretical model that links firms' optimal investment behaviour to these frictions. The goal of the current paper is to develop such a theoretical model.

Our model belongs to the class of Schumpeterian models of endogenous growth that emphasize the importance of innovations and creative destruction for economic growth.[1] Importantly, incumbent entrepreneurs face the risk of creative destruction by new entrants: young, innovative firms that may create a new product or a product of a higher quality that may take away the market share of incumbent firms and force their exit. Thus, on the one hand, a high rate of creative destruction is beneficial for the economy because it leads to faster quality improvement and hence to stronger economic growth; on the other hand, it discourages innovation by incumbents because the expected rents that the incumbents will be able to extract from innovating decrease in the rate of creative destruction. An application of this model to economic policy would suggest that innovation subsidies for small and young firms might need to be balanced by subsidies for large incumbent firms.[2]

A key novelty to our model is the introduction of financing frictions into a Schumpeterian model of endogenous growth. Firms facing external financing costs hoard liquidity (cash) reserves to cover operating losses, production costs, and investment expenses.[3] Our model implies that financially constrained firms face an important trade-off between production and investment into innovation: due to constraints, firms are forced to substitute one activity for another. Therefore, firms need to solve the problem of the optimal cash management, dividend payout, investment into innovations (R&D), and production policies. This problem is complex because the firms need to optimally balance future expected costs and benefits jointly for all these policies. In this paper, we provide an explicit solution for firm's optimal policies. We show that, surprisingly, investment into innovations may be non-monotonic in cash reserves: Although investment in R&D is inherently uncertain, it is potentially highly

[1] Creative destruction is the innovation mechanism by which new production units replace outdated ones.

[2] An issue that for example emerges in this context is whether the Europe 2020 Flagship Initiative Innovation Union should take the size distribution of firms into consideration. Also see https://ec.europa.eu/research/innovation-union/pdf/innovation-union-communication_en.pdf

[3] In reality, external financing costs are often quite significant, and can be about 7% for seasoned equity offerings. Firms are well aware of this; liquidity hoarding has been a point of concern due to the secular growth of cash reserves over the last few decades: in 2014, European companies were holding EUR 963bn in cash reserves, which is EUR 250bn more than in 2007.

rewarding as it may lead to technological breakthroughs. When a firm achieves a breakthrough, it extracts monopoly rents from the brand-new technology and can raise funds on the capital market.[4]

In the second part of the paper, we embed the single firm optimal policies into an equilibrium setting where the rate of creative destruction, the rate of economic growth, and the interest rate are jointly and endogenously determined in equilibrium by the actions of all firms in the economy. A key new element of our model is the cross-sectional distribution of cash in the economy: It is through this distribution that firms' policies aggregate in equilibrium and determine the rate of economic growth. This could lead to a liquidity-redistributive channel of government policies: A new policy may not only change each individual firm's behaviour, it may also change the distribution of firm characteristics. In particular, the model suggests that stronger financing constraints may spur economic growth because they both reduce entry (creative destruction) and stimulate R&D by young and constrained firms. Applying this reasoning to the current lack of growth in Europe might indicate that firms hoard too much cash, and (counterintuitively) policies reducing cash holdings may stimulate growth. In terms of policy implications, our results also emphasize the major role of liquidity and its cross-sectional distribution for economic growth.

[4] These effects are particularly strong for young, constrained firms. In the real world, as in our theory, young firms substantially contribute to the advancement of the technological frontier despite their financial constraints.

1 Introduction

"Yes, I mean that. Less money raised leads to more success. That is the data I stare at all the time." (*Fred Wilson, co-founder of Union Square Ventures*)[1]

Innovation is pivotal to economic growth. The emergence of new goods and the improvements in the quality of existing goods shape the world we live in and drive the dynamics of the economy. Nonetheless, investment in innovation is costly and hard to finance. In fact, it requires a (possibly long) gestation period before becoming productive and its outcome is uncertain. To maintain financial flexibility, firms respond to these financing frictions by hoarding cash (or liquidity). Firms' production and innovation decisions depend on cash availability and affect the rate of economic growth. In turn, the growth rate affects firms' decisions through the market interest rate. The goal of this paper is to build a tractable framework to understand the links among cash hoarding, innovation, and economic growth.

To this end, we introduce financing frictions and corporate cash hoarding into a Schumpeterian model of endogenous growth. We study an economy in which the innovations of incumbents and entrants represent the key drivers of economic growth. Firms are financially constrained and have the incentive to hoard precautionary liquidity (cash reserves).[2] We first investigate how financing frictions affect the firms' production and innovation mix in relation to various firm characteristics. We then embed the firms' maximization problem into a general equilibrium setting and study the effects of financing frictions and liquidity hoarding on economic growth.

We start by solving the optimization problem of a continuum of incumbent firms facing the threat of creative destruction—that is, the threat of going out of business because new firms market better products and seize incumbents' market share. Firms are constrained in that external financing is costly. As in previous contributions, financing frictions make firms effectively risk-averse and generate the incentive to retain earnings in cash reserves.[3] In addition, in our model firms manage the risk-return trade-off

[1] http://avc.com/2013/09/maximizing-runway-can-minimize-success/

[2] Among others, Lyandres and Palazzo (2015), Ma, Mello, and Wu (2013), Falato, Kadyrzhanova, and Sim (2013), and Begenau and Palazzo (2015) suggests that investment in R&D is one major determinant of corporate cash reserves.

[3] See, e.g., Décamps, Mariotti, Rochet, and Villeneuve (2011), Bolton, Chen, and Wang (2011, 2013),

by adjusting production and investment in innovation. Cash flow volatility is thus endogenous and depends on corporate cash reserves.

We show that incumbent firms scale down production and increase markups in response to negative operating shocks. These shocks decrease a firm's cash reserves and weaken its financial strength. The firm then seeks to limit cash flow risk and operating costs in order to avert costly refinancing. As a result, production is cyclical and markups are countercyclical to firm-specific shocks.

By contrast, we highlight that firms may *increase* their investment in innovation in the face of tightening constraints. As we quote at the beginning, *"Less money [...] leads to more success."* Although investment in innovation is inherently uncertain, it is potentially highly rewarding as it may lead to technological breakthroughs. When firms attain such breakthroughs, they access monopoly rents from the brand-new technology and can raise fresh funds in light of a success (the achievement of technological improvements) rather than a failure (running out of funds after operating losses). Thus, constrained firms may substitute production for innovation in line with a *"gambling for resurrection"* behavior. While gambling behavior has been studied in relation to conflicts of interest between equityholders and debtholders, we show that it can also arise for all-equity firms.

We characterize which firms should display this gambling behavior. We illustrate that firms with low operating margins, volatile profits, or efficient innovation technologies (i.e., requiring a shorter gestation period) are willing to substitute production for innovation when constrained. These characteristics are typical of small and young firms. In the real world, small firms substantially contribute to the advancement of the technological frontier despite their financial constraints (see Akcigit and Kerr, 2015, or Acemoglu, Akcigit, Bloom, and Kerr, 2013). Our results also rationalize the observation that investment in innovation has not decreased uniformly during the recent financial crisis.[4]

We also study how endogenous cash flow volatility affects the dynamics of stock prices. In contrast with previous dynamic models with financing frictions (see, e.g.,

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and Hugonnier, Malamud, and Morellec (2015). In these models, as in ours, the marginal value of cash decreases with the level of cash reserves and reaches one at the target level.

[4]For example, Archibugi, Filippetti, and Frenz (2013) report that some firms have actually increased their R&D investment despite the toughening constraints.

Décamps, Mariotti, Rochet, and Villeneuve, 2011), we find that the relation between marginal value of cash and idiosyncratic stock volatility can be positive or negative. While we do observe the leverage effect (i.e., a decline in stock price leads to an increase in stock return volatility) for relatively unconstrained firms, the relation between changes in stock prices and volatility of stock returns can become positive when financing constraints are tight.

We then embed our firms' dynamics into a general equilibrium setting. We focus on a "balanced growth path" equilibrium in which all aggregate quantities grow at an endogenous, constant rate. In equilibrium, incumbents solve their maximization problem by taking the market interest rate as given. In each industry, incumbent's value and the rate of creative destruction (equivalently, the entry rate) are jointly determined by the free-entry condition. Incumbents' and entrants' innovations aggregate and determine the rate of economic growth, which affects the market interest rate through the representative household's Euler equation.

We find that financing frictions deter entry and so dampen creative destruction. As a result, incumbents invest more in innovation because they expect their profits to last for longer before new firms seize their market share.[5] Thus, financing frictions have offsetting effects on growth. On the one hand, they deter innovation by new firms (which decreases growth); on the other hand, they prompt innovation by incumbents (which increases growth) with respect to a benchmark, unconstrained economy. The latter (respectively, former) effect dominates when the entry cost is high (low). Even when the constrained economy features a smaller growth rate, consumption can be larger. By curbing the threat of creative destruction, financing frictions increase incumbents' expected monopoly rents. At the same time, the resulting lower growth rate decreases the interest rate that is used to discount future cash flows. These mechanisms all imply that incumbent firms in the constrained economy may be more valuable than in the unconstrained counterfactual. Thus, our analysis reveals that financing frictions may be *not* welfare-decreasing.

Related literature Our paper contributes to the literature that studies the determinants of corporate cash reserves, which has been spurred by the increase in cash-to-asset

[5] This effect is similar to Aghion and Howitt (1992).

ratios of U.S. firms over the last decades (see, e.g., Bates, Kahle, and Stulz, 2009). Notable contributions are, e.g., Riddick and Whited (2009), Décamps, Mariotti, Rochet, and Villeneuve (2011), Bolton, Chen, and Wang (2011, 2013), and Hugonnier, Malamud, and Morellec (2015). Within this strand, the papers that analyze the relation between cash reserves and investment do so by considering neoclassical models of investment and capital accumulation, either incremental (as in Bolton, Chen, and Wang 2011, 2013) or lumpy (as in Hugonnier, Malamud and Morellec, 2015). To the best of our knowledge, our paper is the first to study corporate cash management problem in a Schumpeterian framework.[6] This problem is economically important given the documented relations between cash reserves and R&D investment and between R&D and economic growth (see e.g. Caballero and Jaffe, 1993, Akcigit and Kerr, 2015, or Kogan, Papanikolaou, Seru, and Stoffman, 2014). Nonetheless, it is not trivial because it involves solving a problem with an infinite number of (sequentially arriving) growth options.

The paper also contributes to the literature studying the financing of innovation. Lyandres and Palazzo (2015) and Ma, Mello, and Wu (2014) investigate and test the relation among competition, R&D investment, and cash reserves. Falato, Kadyrzhanova, and Sim (2013) study and test the relation between investment in intangible capital and cash holdings in a neoclassical model with no growth. While we abstract from strategic competition within industries, we contribute to this strand by studying the joint dynamics of cash, production, and innovation as well as their impact on economic growth. In our paper, we make a clear distinction between target cash reserves and deviations from this target. Consistent with the evidence, we argue that R&D investment prompts firms to hoard cash reserves. In addition, we characterize how firms adjust their investment in innovation when operating shortfalls erode the reserves below the target level. In particular, we find that small firms might increase their innovation rate in the face of tighter financial constraints.

The link between R&D and cash hoarding is supported by vast empirical evidence. Among others, Hall (2005) and Hall and Lerner (2010) document that innovation is best financed through internal funds because it is subject to asymmetric information, it is not pledgeable, and has highly uncertain returns. Brown, Fazzari, and Petersen

[6]The key difference in neoclassical capital accumulation models and Schumpeterian innovation models is that, in the former models, investment implies an immediate increase in the output, whereas the payoff from investment in R&D arrives at uncertain (Poisson) times.

(2009) report a strong impact of financing constraints on innovation at both firm level and aggregate level and document that innovation decisions are related to the supply of internal and external equity finance.[7] Consistently, Hall (2005), Hall and Lerner (2010), Rajan (2012), and Acharya and Xu (2015) emphasize the relative importance of equity rather than debt for the financing of innovation, as we do in our theoretical model.

Schumpeter (1942) emphasized the importance of innovation for economic growth by introducing the concept of "creative destruction." The work of Schumpeter has led to the development of a subfield of macroeconomics, known as "Schumpeterian models of endogenous growth;" see Aghion, Akcigit, and Howitt (2014) for an overview. Our model belongs to this literature. In particular, we build on the endogenous technological change literature (see also Romer, 1990, Grossman and Helpman, 1991, or Klette and Kortum, 2004) and assume that investment in innovation is pursued by both incumbent firms and a continuum of potential entrants, as in the recent contributions of Acemoglu and Cao (2015), Akcigit and Kerr (2015), or Acemoglu, Akcigit, Bloom, and Kerr (2013).

Our paper also relates to the literature studying the effects of financial constraints on economic growth; see Levine (2005) and Beck (2012) for two surveys. Thus far, however, relatively little attention has been paid to the role of corporate liquidity in providing flexibility to constrained firms. Our model seeks to fill this gap.

The paper is organized as follows. Section 2 describes the model. Section 3 solves the model in a benchmark unconstrained economy, which serves as a counterfactual to single out the effects of financing frictions on innovation and growth. Section 4 solves the model in the economy featuring financing frictions. Section 5 provides a quantitative assessment of the model's predictions. Section 6 concludes. All proofs are in the Appendix.

[7] Brown, Fazzari, and Petersen (2009) report that, in the US, young publicly traded firms in high-tech industries finance R&D investment almost entirely with internal and external equity as debt financing is difficult due to non-pledgeability. Citing from their paper, *"While the large literature on finance and economic growth has good reasons to focus on debt and credit constraints, our results suggest that more attention should be given to equity finance [...] for models that emphasize innovation. [...] External equity is the more relevant substitute for internal cash flow for young high-tech firms."*

2 The Model

Throughout the paper, time is continuous and uncertainty is modeled by a probability space (Ω, F, P), equipped with a filtration $(F_t)_{t\geq 0}$ that represents the information available at time t. We consider an economy in which the representative household maximizes constant relative risk aversion (CRRA) preferences:

$$\int_0^\infty e^{-\rho t} \frac{\mathcal{C}_t^{1-\theta} - 1}{1-\theta} \, dt \, . \tag{1}$$

In this equation, $\mathcal{C}_t$ is the consumption rate at time t. The parameters ρ and θ represent the discount rate and the inverse of the elasticity of intertemporal substitution, respectively. Population is constant at L. Agents supply labor inelastically, for which they receive labor wage denoted by W_t.

Final good sector. There is one multipurpose final good $\mathcal{Y}_t$ serving as the numeraire of the economy. The final good is produced competitively using labor and a continuum of specialized intermediate goods indexed by $j \in [0,1]$, according to the production function

$$\mathcal{Y}_t = \frac{1}{1-\beta} \int_0^1 L^\beta \, \bar{X}_t(j)^{1-\beta} \, q_t(j)^\beta \, dj \; , \qquad \beta \in [0,1]. \tag{2}$$

In this equation, $\bar{X}_t(j)$ is the input quantity of the intermediate good j and $q_t(j)$ is the "quality" associated with it. The initial quality level is normalized to one; i.e., $q_0(j) = 1$ for all $j \in [0,1]$. We assume that only the highest quality version of each intermediate good j is used in the production of the final good. Improvements in the quality of each intermediate good j occur via two channels: innovation by incumbent firms and *creative destruction* by entrants.

Intermediate goods sector. As is common in Schumpeterian models, we assume that each intermediate good j is produced by the latest innovator in the industry j. The latest innovator enforces a patent on the highest quality version of the intermediate good j and becomes the incumbent monopolist in the production of that good. The patent is assumed to last forever but does not prevent other firms from improving further the quality of that good. Whenever a new firm launches a higher quality version of the intermediate good j, it enforces a new patent on it and becomes the new monopolist of the industry j.

Each incumbent firm invests in innovation to further improve the quality of the intermediate good it produces. We denote by z_t an incumbent's innovation intensity (or innovation rate). Innovation is costly, and its outcome is uncertain. Specifically, we assume that an incumbent firm paying the flow cost

$$\Phi(z_t, q_t) = \zeta \frac{z_t^2}{2} q_t, \quad \zeta > 0,$$

succeeds in increasing the quality of its intermediate good at a Poisson rate of ϕz_t. These Poisson events represent technological breakthroughs, whose occurrence is more likely when the innovation rate z_t is larger. When a technological breakthrough occurs in the product line j, the quality of the intermediate good j jumps from $q_{t^-}(j)$ to

$$q_t(j) = \lambda q_{t^-}(j) \,.$$

The parameter $\lambda > 1$ measures the incremental improvement in quality due to the incumbent's technological breakthrough.[9]

The operating revenues of each incumbent firm are primarily driven by the scale of production. In the following, we denote by $\bar{X}_t(j)$ the production quantity of the product line j and by $\Pi_t(j)$ the corresponding revenues. The dynamics of revenues are given by:

$$d\Pi_t(j) = \left[\bar{X}_t(j)(p_t(j) - m) - \Phi(z_t, q_t)\right] dt \;+\; \sigma \bar{X}_t(j) dZ_t(j) \,. \tag{3}$$

In this equation, p_t represents the price of the intermediate good j, whereas m represents the marginal cost of production. In the following, we normalized m to one. $Z_t(j)$ is a standard Brownian motion that represents random operating shocks.[10] Operating shocks are firm-specific and independent across firms. $\bar{X}_t(j)$ and $p_t(j)$ are endogenously determined in the following.

The cash flow process in equation (3) implies that incumbent firms are exposed to

[9] See also Acemoglu and Cao (2015) or Acemoglu et al. (2013). We do not allow incumbent firms to operate multiple product lines (as in Klette and Kortum, 2004); that is, each firm innovates only on one product line. As in Aghion, Howitt, and Mayer-Foulkes (2005), Acemoglu, Aghion, and Zilibotti (2006), or Acemoglu and Cao (2015), we do not consider skilled labor in the intermediate-good sector and assume that firms are run by entrepreneurs.

[10] The assumption of Brownian shocks is similar to the neoclassical AK-technology used in the model of Bolton, Chen, and Wang (2011, 2013) and DeMarzo, Fishman, He, and Wang (2012).

operating losses. If access to outside financing was frictionless, losses could be covered by raising fresh funds whenever needed. We depart from this assumption and assume that firms face financing costs, as in Bolton, Chen, and Wang (2011, 2013).[11] We assume that incumbents can refinance current operations at a cost $\epsilon > 0$ for any dollar raised. Incumbents can raise fresh financing whenever technological breakthroughs occur, by sharing the surplus created with the financiers via Nash bargaining. Denoting the bargaining power of financiers by $\alpha \in [0, 1]$ and the surplus created by S, the rents extracted by financiers are

$$\Gamma^* = \arg\max_{\Gamma \geq 0} \Gamma^\alpha \left[S - \Gamma\right]^{1-\alpha} = \alpha S,$$

whereas the firm retains $(1 - \alpha)S$. Notably, we model financing costs in a way that captures features that are typical among R&D-intensive firms (see, e.g., Lerner et al., 2012). This type of firms delay refinancing events until when they achieve a technological breakthrough or they are in the extreme need of funds.

To maintain financial flexibility, incumbent firms retain earnings in a cash reserve. We denote by $\bar{C}_t$ the firm's cash reserves at time t. Cash reserves earn a rate δ. The rate δ is lower than the market interest rate r, so holding cash entails an opportunity cost. The opportunity cost $r - \delta$ can be interpreted as an agency costs of free cash flows (as in Décamps, Mariotti, Rochet, and Villeneuve, 2011) or as tax disadvantages (as in Riddick and Whited, 2009). As we show in Appendix A.5, this opportunity cost may also arise when the supply of liquidity in the economy is scarce. Nothwithstanding its microfoundation, the wedge between r and δ leads to a dynamic trade-off between dividend payout and liquidity hoarding. Under the model assumptions, the dynamics of the cash holdings process $(\bar{C})_{t \geq 0}$ are given by

$$d\bar{C}_t = \left[\delta \bar{C}_t + \bar{X}_t(j)(p_t(j) - 1) - \Phi(z_t, q_t)\right] dt + \sigma \bar{X}_t(j) dZ_{jt} - dD_t + dF_t + dF_t^I. \quad (4)$$

The first term in the square brackets represents the return on cash reserves, whereas the second and third terms represent net operating profits. $dF_t \geq 0$ is the instantaneous net inflow from refinancing current operations, $dF_t^I \geq 0$ is the instantaneous net inflow from financing a technological breakthrough, and $dD_t \geq 0$ is the instantaneous flow of payouts.

[11] As in Bolton, Chen, and Wang (2011, 2013), financing costs are exogenous. Financing costs could be microfounded through limited enforcement (Albuquerque and Hopenhayn, 2002), asymmetric information (Clementi and Hopenhayn, 2002), or limited pledgeability (Holmstrom and Tirole, 2011).

Incumbent firms choose production $\{\bar{X}_t\}$, innovation $\{z_t\}$, financing $\{F_t\}, \{F_t^I\}$, and payout $\{D_t\}$ policies to maximize the present value of future dividends subject to the budget constraint (4) and the non-negativity constraint on cash reserves. We denote by $V(t, \bar{C}, q)$ the time-t value of an incumbent with cash reserves $\bar{C}$ and producing its intermediate good at quality level q, given by

$$V(t, \bar{C}, q) \;=\; \max_{\{\bar{X}_s\},\{z_s\},\{F_s\},\{F_s^I\},\{D_s\}} E\left[\int_t^{\mathcal{T}_d} e^{-r(s-t)}(dD_s - (1+\epsilon)dF_s - (1+\alpha S_s)dF_s^I)\right].$$

In this equation, $\mathcal{T}_d$ is the time of liquidation due to creative destruction. When creative destruction hits, the incumbent loses its market position, and shareholders receive a lumpy liquidation dividend equal to the firm's cash holdings; that is, $dD_{\mathcal{T}_d} = \bar{C}_{\mathcal{T}_d}$. As is common in Schumpeterian models, we assume that obsolescence drives the liquidation value of other assets to zero.

Entrants. Creative destruction hits when an entrepreneur markets a higher quality version of an intermediate good j. Specifically, we assume that there is a mass of entrepreneurs on the sideline, who develop blueprints to improve the quality of the intermediate goods. Once successfully developed, a blueprint has the potential to improve the current quality of a good j by a factor $\Lambda > 1$, i.e. quality may jump from $q_{t^-}(j)$ to

$$q_t(j) \;=\; \Lambda q_{t^-}(j).$$

We do not impose a priori restrictions on the relation between λ and Λ.[12]

We assume that entrepreneurs do not collaborate with incumbent firms due to frictions in the sale of ideas (see also Kondo and Papanikolaou, 2013 and Silveira and Wright, 2010). To exploit a successful blueprint, an entrepreneur needs to set up a firm. Creating a firm at a quality level q_t requires a technological cost equal to $\kappa_T q_t L$. To finance this cost, an entrepreneur contacts financiers, who charge a financing cost to the entrepreneur. We

[12] We allow for different sizes of quality jumps for incumbents and entrants; see Acemoglu and Cao (2015) or Akcigit and Kerr (2015). These contributions assume that $\Lambda > \lambda$ to model the difference between "incremental" innovations by incumbents and "radical" innovations by entrants. The difference between λ and Λ is not essential to our analysis. However, we set $\Lambda > \lambda$ in the numerical analysis.

denote this financing cost by $\kappa_I q_t L$, so the total entry cost is given by

$$K_t(j) = q_t(j)L(\kappa_T + \kappa_I) = \Lambda q_{t-}(j)L(\kappa_T + \kappa_I). \tag{5}$$

Notably, the ratio $\frac{\kappa_I}{\kappa_T+\kappa_I}$ is a measure of the financiers' ability to extract rents from entrant firms.[13]

Balanced growth path. We consider symmetric balanced growth path equilibria in which, for any industry j, the entry rate is given by x_d, and the equilibrium output $\mathcal{Y}_t$ in (2) grows at a constant rate g (endogenously determined in the following). We will frequently refer to the entry rate x_d as the rate of creative destruction, which is determined by the free-entry condition

$$V(t, \bar{C}_t^*, \Lambda q(j)) - \bar{C}_t^* = K_t(j). \tag{6}$$

In this equation, we assume that entrants start operations endowed with their target level of cash reserves, denoted by $\bar{C}_t^*$.[14] As we show below, this target level always exists and can be characterized explicitly.

To solve the model, we first derive the incumbents' optimal policies and then aggregate in general equilibrium. An equilibrium is an allocation characterized as follows: (i) Incumbent firms set production, innovation, payout, and financing policies to maximize the expected present value of net dividends; (ii) New firms enter at a rate that makes the free-entry condition binding; (iii) The final good producer maximizes profits; (iv) The representative household maximizes utility from consumption; (v) All the markets clear.

[13] Due to the free-entry condition (6), this is the fraction of firm value (net of the cash provided) that financiers extract from entrants. In the real world, adverse selection reduces entrants' bargaining power relative to that of an incumbent. To capture this effect, we assume that $\frac{\kappa_I}{\kappa_T+\kappa_I} > \alpha$.

[14] One can argue that financiers may be reluctant to provide the amount of liquidity that is optimal from the firms' perspective, due to adverse selection problems. We have solved a version of the model in which some of the entrants start with less cash than is optimal, and our main results do not change.

3 Benchmark unconstrained economy

We start by solving the model in a benchmark economy with no financing frictions, meaning that the parameters ϵ, α, and κ_I are zero.[15] In this setting, losses can be covered by raising external funds immediately and costlessly. Thus, idiosyncratic operating shocks do not matter in the definition of corporate policies. Under these assumptions, firms have no incentive to keep cash.

We begin the analysis by solving the optimization problem of the final-good producer,

$$\max_{\bar{X}(j)} \frac{1}{1-\beta} \int_0^1 L^\beta \, \bar{X}_t(j)^{1-\beta} \, q_t(j)^\beta dj \;-\; \int_0^1 p_t(j)\bar{X}_t(j)dj \,, \tag{7}$$

which delivers the standard demand curve for the highest quality version of any intermediate good $j \in [0,1]$, given by

$$\bar{X}_t(j) \;=\; L \left(\frac{q_t^\beta(j)}{p_t(j)} \right)^{\frac{1}{\beta}} . \tag{8}$$

Taking the demand schedule of the final-good producer as given, an incumbent firm maximizes its profits by setting the monopoly price[16]

$$p_t = p^* = \frac{1}{1-\beta}.$$

This price implies a constant markup above the marginal cost, equal to $\frac{\beta}{1-\beta}$. It follows that the optimal production quantity is $\bar{X}_t^*(j) = q_t(j) L X^*$, where

$$X^* = (1-\beta)^{\frac{1}{\beta}}$$

denotes the time-independent production quantity scaled by quality. As we explain below, we will often refer to scaled quantities. Substituting the expressions for X^* and p^* into

[15]In this unconstrained setting, our model follows Acemoglu and Cao (2015).

[16]As in other contributions, we rule out the case of limit pricing to keep the analysis tractable. For instance, Aghion and Howitt (1992) assume that innovations are always drastic so that a monopolist is unconstrained by potential competition from the previous patents. Akcigit and Kerr (2015) assume that the current incumbent and the former incumbents in a given product line enter a two-stage price-bidding game whereby each firm pays a fee to announce its price. Under this assumption, only the new leader pays the fee and announces its price. We adopt similar assumptions in our setup.

(2), we derive the aggregate output,

$$\mathcal{Y}_t^* \;=\; L(1-\beta)^{\frac{1}{\beta}-2}\int_0^1 q_t(j)dj\,, \tag{9}$$

and the competitive wage,

$$W_t^* \;=\; L\beta(1-\beta)^{\frac{1}{\beta}-2}\int_0^1 q_t(j)dj\,. \tag{10}$$

In this benchmark economy, we denote the value of an incumbent firm by $V(t,q)$ since it keeps no cash. An incumbent produces at the value-maximizing rate $\bar{X}_t^*$ and pays out any operating profit to shareholders. Following standard arguments, $V(t,q)$ satisfies the following Hamilton-Jacobi-Bellman (HJB) equation

$$rV(t,q) - V_t(t,q) = \max_z \left\{ \beta Lq\,(1-\beta)^{\frac{1}{\beta}-1} - \frac{z^2}{2}\zeta q + \phi z\,[V(t,\lambda q) - V(t,q)] - x_d\,V(t,q) \right\}\;. \tag{11}$$

The terms on the left-hand side represent, respectively, the return required by investors and the change in firm value as time elapses. The first two terms on the right-hand side represent operating profit net of production and innovation expenditures. The third term is the expected change in value when the incumbent markets a higher quality version of its production good following a technological breakthrough, weighted by its probability. The fourth term is the value discount due to creative destruction. To solve the incumbent's maximization problem, we conjecture that firm value is linear in the quality q_t,

$$V(t,q) = \;V(q_t) = \;q_t L v\,, \tag{12}$$

for some $v > 0$. In the following, we will refer to v as the scaled value of an incumbent. Substituting (12) into (11), we obtain

$$\max_z \left\{ \mu^* - \frac{z^2}{2}\,\frac{\zeta}{L} + \;\phi z(\lambda v - v)\; - \;x_d\,v\; \right\} \;=\; rv\,, \tag{13}$$

where we have defined the auxiliary quantity $\mu^* \equiv \;\beta\,(1-\beta)^{\frac{1}{\beta}-1}$. Differentiating (13), it follows that the value-maximizing innovation rate z^* is given by

$$z^* \;=\; \phi\,\frac{L}{\zeta}\,(\lambda-1)v\,. \tag{14}$$

The optimal innovation rate z^* is increasing in firm value v, in the rate of technological breakthroughs ϕ, as well as in the quality improvement λ. Substituting (14) into (13), we derive a closed-form expression for the value of an incumbent firm v^*, as reported in Proposition 1. Finally, the free-entry condition

$$v^* =\; \kappa_T\,. \tag{15}$$

determines the entry rate x_d^* (or the rate of creative destruction) for all the product lines j.

The expected increase in the quality of a given intermediate good j is given, per unit of time dt, by

$$E_{t-}[dq_t(j)] = q_{t-}(j)\,[(\lambda-1)\phi z^* \;+\; (\Lambda-1)x_d^*]\,dt.$$

In this expression, the first (respectively, second) term in the square brackets is the incumbents' (entrants') contribution to the improvement in quality of the intermediate good j. Innovations in different product lines $j \in [0,1]$ occur at independent Poisson times. By the law of large numbers, we obtain

$$\int_0^1 q_t(j)dj \;=\; e^{g^*t}, \tag{16}$$

where g^* denotes the rate of economic growth and is given by

$$g^* \;=\; (\lambda-1)\phi z^* \;+\; (\Lambda-1)x_d^*\,. \tag{17}$$

Notably, equation (16) implies that the aggregate output in (9) grows at the rate g^*. Finally, the Euler equation of the representative household determines the equilibrium interest rate r^*, which satisfies the following relation:

$$r^* = \rho + \theta g^*. \tag{18}$$

We summarize our findings in the following proposition.

Proposition 1 *In the unconstrained economy, the optimal innovation rate and the scaled value of an incumbent firm are respectively given by*

$$z^* = \phi \frac{L}{\zeta}(\lambda - 1)\ \kappa_T,$$
$$v^* = \frac{2\mu^*}{(x_d + r) + \sqrt{(x_d + r)^2 - 2\mu^* \phi^2 \frac{L}{\zeta}(\lambda - 1)^2}}.$$

Moreover, the equilibrium rate of creative destruction x_d^*, the rate of economic growth g^*, and the market interest rate r^* respectively satisfy

$$x_d^* = \frac{1}{1 + \theta\ (\Lambda - 1)} \left[\frac{\mu^*}{\kappa_T} - \rho + \kappa_T\ \phi^2\ \frac{L}{\zeta}\ (\lambda - 1)^2 \left(\frac{1}{2} - \theta\right)\right]$$
$$g^* = \frac{1}{1 + \theta\ (\Lambda - 1)} \left([\Lambda - 1)\left(\frac{\mu^*}{\kappa_T} - \rho\right) + \kappa_T\ \frac{\Lambda + 1}{2}\ \phi^2\ \frac{L}{\zeta}\ (\lambda - 1)^2\right]$$
$$r^* = \frac{1}{1 + \theta\ (\Lambda - 1)} \left[\rho + \ \theta\ (\Lambda - 1)\ \frac{\mu^*}{\kappa_T} + \kappa_T \theta \frac{\Lambda + 1}{2} \phi^2\ \frac{L}{\zeta}\ (\lambda - 1)^2\right].$$

Two remarks are worth a mention. First, the equilibrium growth rate g^* is non-monotonic in the entry cost. On the one hand, a low κ_T allows more entry; on the other hand, greater entry renders incumbent firms subject to more creative destruction and decreases their innovation rate. Second, it may appear surprising that the innovation rate z^* does not depend on the firm's monopoly rents, whose magnitude is determined by the parameter β. Indeed, the extant literature places a strong accent on the link between the incentives to innovate and the monopolistic rents a firm can extract (among others, see the seminal contributions of Arrow, 1962, Romer, 1990, or Aghion and Howitt, 1992). The reason is that, in the economy with no liquidity frictions, this link is fully offset by the free entry condition: an increase in monopoly rents stimulates entry and increases creative destruction, thereby exactly offsetting the positive effect of β on z^*. As we show in Section 4.2, liquidity frictions restore the link between rents and innovation decisions.

We denote aggregate consumption by $\mathcal{C}_t^*$ in the benchmark economy. By the budget constraint of the representative household, consumption at time t is the sum of labor wage (W_t^*) and dividends (d_t^*) net of financing funnelled to the corporate sector (f_t^*):

$$\mathcal{C}_t^* \;=\; W_t^* + d_t^* - f_t^* \,,$$

with

$$d_t^* \;=\; \int_0^1 \left(\beta L q_t(j)\,(1-\beta)^{\frac{1}{\beta}-1} - \frac{(z^*)^2}{2}\zeta q_t(j)\right) dj \;=\; \left(\mu^* - \frac{(z^*)^2}{2}\,\frac{\zeta}{L}\right) L e^{g^* t}\,, \qquad (19)$$

$$f_t^* \;=\; \int_0^1 x_d^* K_t(j) dj \;= x_d^* \Lambda L \kappa_T e^{g^* t}. \qquad (20)$$

As aggregate consumption grows at the balanced growth path rate, it follows that $\mathcal{C}_t^* = \mathcal{C}_0^* e^{g^* t}$, with

$$\mathcal{C}_0^* = W_0^* + \mu^* L - \frac{(z^*)^2}{2}\,\zeta - x_d^* \Lambda L \kappa_T\,.$$

Since equilibrium labor wage also grows at the rate g^*, the relation $W_t^* = W_0^* e^{g^* t}$ holds and it follows that

$$W_0^* = L\beta(1-\beta)^{\frac{1}{\beta}-2}$$

by (10). We analyze in detail the properties of these aggregate quantities in Section 4.4 and Section 5.2, where we compare the constrained and unconstrained economies.

4 The liquidity constrained economy

We now analyze the economy featuring financing frictions. In this setting, firms retain cash reserves as a precautionary buffer for the times of need. We start by deriving the optimal production, innovation, financing, and dividend decisions of incumbent firms. We then derive the stationary cross-sectional distribution of liquidity in the economy. Finally, we integrate single-firm quantities with respect to the stationary distribution to obtain the aggregate equilibrium quantities.

4.1 Deriving the value of incumbent firms

Recall that $V(t, \bar{C}, q)$ represents the time-t value of an incumbent with cash reserves $\bar{C}$ and producing an intermediate good at the quality level q. Since the benefit from an additional dollar saved is decreasing in the cash reserves and the marginal cost is constant, we conjecture that there exists some target level of cash reserves $\bar{C}^*(q)$ where the marginal cost and benefit of cash are equalized. Above this target, it is optimal to pay out excess cash as dividends to shareholders.

By assumption, an incumbent firm can always raise external funds by incurring the proportional cost ϵ. Raising the amount $dF > 0$ changes firm value from $V(t, \bar{C}, q)$ to $V(t, \bar{C} + dF, q) - (1 + \epsilon)dF$. The exercise of this financing option is optimal if and only if the marginal gain $V_c(t, \bar{C} + dF, q)$ is larger than the marginal cost $1 + \epsilon$ for some levels of cash reserves. We conjecture and verify that the marginal value of cash is decreasing in the level of cash reserves. As a result, an incumbent firm raises new funds only when the cash buffer is completely depleted, in which case the marginal gain from external financing equals the marginal cost; i.e., the condition $V_c(t, 0, q) = 1 + \epsilon$ holds.

In addition to this routine financing, an incumbent firm can raise funds when marketing a higher quality product (i.e., after a technological breakthrough). While this financing channel is not subject to the fee ϵ, financiers extract a fraction α of the surplus created. In this case, it is optimal for the firm to raise an amount that replenishes the cash reserves at the target level $\bar{C}^*(\lambda q)$. Thus, the total surplus generated by this transaction is given by $S(t, \bar{C}, q) = V(t, \bar{C}^*(\lambda q), \lambda q) - (\bar{C}^*(\lambda q) - \bar{C}) - V(t, \bar{C}, q)$. Because of bargaining with the financiers, firm value increases by $(1 - \alpha)S(t, \bar{C}, q)$ upon this transaction.[17]

As in the unconstrained economy, incumbents take the demand schedule of the final-good sector as given and choose their value-maximizing production rate. Differently, liquidity constraints may force firms to produce below the benchmark quantity X^* linked to the constant price p^* derived in Section 3. In the constrained economy, the optimal production decision $\bar{X}_t$ and the optimal innovation rate z_t depend on the level of the cash

[17]If an incumbent firm does not accept the financiers' surplus sharing offer, its value jumps from $V(t, \bar{C}, q)$ to $V(t, \bar{C}, \lambda q)$ after a successful innovation. Since firm value is concave in $\bar{C}$, marginal value of cash satisfies $V_c(t, \bar{C}, \lambda q) < V_c(t, 0, \lambda q) = 1 + \epsilon$. Thus, raising external funds through the "routine" procedure is never optimal, whereas moving from $V(t, \bar{C}, \lambda q)$ to $V(t, \bar{C}^*(\lambda q), \lambda q) - (\bar{C}^*(\lambda q) - \bar{C})$ is optimal, because the marginal value of cash is always above one. Hence, it is always optimal for the firm to take the financiers' offer to share the surplus $\widetilde{S}(t, \bar{C}, q) = V(t, \bar{C}^*(\lambda q), \lambda q) - (\bar{C}^*(\lambda q) - \bar{C}) - V(t, \bar{C}, \lambda q)$, independent of the fraction of the surplus they would like to extract. Note, however, that in our model we assume that financiers are extracting the fraction α of $S = \widetilde{S} + (V(t, \bar{C}, \lambda q) - V(t, \bar{C}, q))$. While this assumption is made for analytical tractability, one can interpret the term $\alpha(V(t, \bar{C}, \lambda q) - V(t, \bar{C}, q))$ either as an additional fixed cost of intermediation or as the cost of marketing the technological breakthrough.

buffer and can be characterized by solving the HJB equation for the firm value:

$$rV(t,\bar{C},q) - V_t(t,\bar{C},q) = \max_{z,\bar{X}} \left\{ \frac{\sigma^2}{2}\bar{X}^2 V_{cc}(t,\bar{C},q) + \left[\delta\,\bar{C} + (p-1)\bar{X} - \frac{z^2}{2}q\zeta\right] V_c(t,\bar{C},q) \right.$$
$$\left. +\ \phi z(1-\alpha)\left(V(t,\bar{C}^*(\lambda q),\lambda q) - (\bar{C}^*(\lambda q) - \bar{C}) - V(t,\bar{C},q)\right) \ +\ x_d\,(\bar{C} - V(t,\bar{C},q)) \right\}. \tag{21}$$

The left-hand side of this equation admits an interpretation analogous to (11). The right-hand side is the expected change in the value of the firm on a time interval. The first and the second terms capture the effect of cash flow volatility and cash savings, respectively. The third term captures the change in firm value when marketing a technological breakthrough. In these cases, financiers provide new funds to the firm to raise the cash buffer up to the target cash level $\bar{C}^*$ and extract a fraction α of the surplus created. Finally, the fourth term represents sudden exit due to creative destruction. When this happens, shareholders only recover the firm's cash reserves.

Similar to the benchmark economy, we conjecture and verify that the value of incumbents scales with q_t and with a non-linear function $v(c)$, i.e.

$$V(t,\bar{C},q) \ = \ q_t L\, v\left(\frac{\bar{C}_t}{q_t L}\right) \ \equiv \ Q_t\, v(c)\,, \qquad Q_t = q_t L. \tag{22}$$

As in the benchmark economy, $v(c)$ represents the value of an incumbent firm in the product line j scaled by the current quality of the intermediate good j. Differently, this value depends on the firm's cash reserves. We define scaled cash reserves c as

$$c \ \equiv \ \bar{C}_t/Q_t,$$

and we conjecture that the target cash level satisfies

$$\bar{C}^*(q_t) \ = \ Q_t C^*$$

for some constant $C^* > 0$. Scaled cash reserves represent the most important state variable in our model: optimal policies can be expressed as functions of c and are determined by solving the HJB equation for $v(c)$.

Defining the scaled production quantity as $X(c) \ \equiv \ \bar{X}_t/Q_t$ and substituting (22) into

(21), we obtain the following equation for $v(c)$,

$$\max_{z,X}\left\{\frac{\sigma^2}{2}X^2 v''(c) + v'(c)\left[\delta c \;+\; X^{1-\beta} - X - \frac{z^2}{2}\frac{\zeta}{L}\right]\right. \\ \left. +\;(1-\alpha)\phi z\left[\lambda(v(C^*) - C^*) - (v(c) - c)\right] \;+\; x_d(c - v(c)) \;-\; rv(c)\right\} \;=\; 0\,. \tag{23}$$

We define the curvature (i.e., the effective "risk aversion") of the value function as

$$\gamma(c) = -\sigma^2 \frac{v''(c)}{v'(c)} \tag{24}$$

and we let $F(\gamma)$ be the unique solution to the following equation

$$(1-\beta)F^{-\beta} \;=\; 1 + \gamma F\,, \tag{25}$$

see Appendix A.1 for further details. We conjecture that the marginal value of cash $v'(c)$ is monotone decreasing in c (that is, $v''(c) \leq 0$). Thus, the maximization problem in (23) has an interior solution for any c; specifically, the value-maximizing production rate $X(c)$ and innovation rate $z(c)$ are respectively given by

$$X(c) \;=\; F(\gamma(c)), \tag{26}$$

$$z(c) \;=\; \phi\,(1-\alpha)\,\frac{L}{\zeta}\,\frac{(\lambda w^* - v(c) + c)}{v'(c)}\,. \tag{27}$$

In this equation, the quantity

$$w^* \equiv w^*(C^*) = v(C^*) - C^*$$

represents the scaled firm value at the target cash level, net of cash reserves.

The intuition behind (26) is as follows. By (25), the optimal production policy $X(c)$ is determined by equalizing marginal gains $(1-\beta)F^{-\beta}$ from selling the with the effective marginal cost $1 + \gamma F$ of producing the good. The effective marginal cost accounts for the marginal dis-utility γF of the firm due to the firm's risk aversion γ. Similarly, the optimal innovation rate $z(c)$ is also determined by equalizing marginal gains and losses. Namely, the numerator $\lambda w^* - v(c) + c$ represents the change (jump) in the firm value in case of a successful innovation; the denominator $v'(c)$ is just the marginal utility loss from R&D

expenditures.

Substituting these expressions into (23), we obtain the following differential equation:

$$
\begin{aligned}
& -\frac{\sigma^2}{2} X(c)^2\, v''(c) + v'(c)\beta X(c)^{1-\beta} + v'(c)c\;\delta \\
& + \;\frac{\varphi^2}{2}\frac{(\lambda w^* - v(c) + c)^2}{v'(c)} + x_d\,(c - v(c)) \;-\; rv(c) \;=\; 0\,,
\end{aligned}
\tag{28}
$$

where the auxiliary quantity

$$
\varphi^2 \;\equiv\; \phi^2\,(1-\alpha)^2\left(\frac{L}{\zeta}\right) \tag{29}
$$

represents the effective rate at which technological breakthroughs occur.

In order to pin down the value of an incumbent firm, we solve (28) subject to the following boundary conditions:

$$
v'(0) \;= 1+\epsilon\;, \tag{30}
$$

$$
v'(C^*) = 1, \tag{31}
$$

$$
v''(C^*) = 0\,. \tag{32}
$$

The first boundary condition means that when the cash reserves are depleted, the marginal value of cash equals the marginal cost of raising funds (i.e., $1+\epsilon$). That is, an incumbent firm refinances current operations every time the cash reserves are depleted by issuing an infinitesimal amount that reflects the cash process back into the retention region. The second boundary condition means that the marginal value of cash at the target level C^* is equal to one. Above this target level, excess cash is paid out to shareholders and firm value is linear and given by

$$
v(c) = \;v(C^*)\; + c\; - C^*\,.
$$

The third boundary is the super-contact condition at C^*, which guarantees that the dividend threshold C^* is optimally chosen to maximize firm value. As c approaches the target level C^*, the firm's effective risk-aversion γ goes to zero, and the firm starts behaving as if it were unconstrained.

Substituting the boundary conditions $v'(C^*) = 1$ and $v''(C^*) = 0$ into (28), we derive the scaled value of an incumbent at the target cash level $v(C^*)$. By straightforward calculations, we obtain

$$w(C^*) \;=\; v(C^*) - C^* = \frac{2\left[\mu^* - (r-\delta)C^*\right]}{x_d + r + \left[(x_d+r)^2 - 2\varphi^2(\lambda-1)^2(\mu^* - (r-\delta)C^*)\right]^{1/2}}, \quad (33)$$

where $\mu^* \equiv \; \beta\,(1-\beta)^{\frac{1}{\beta}-1}$ as in Section 3. We summarize our findings in the following proposition.[18]

Proposition 2 *Suppose that ϵ is sufficiently small. Then, the value of an incumbent firm is given by (22), where $v(c)$ is the unique concave solution to (28) satisfying the boundary conditions (30). The scaled value of an incumbent firm holding its target level of cash reserves is given by $w(C^*) + C^*$ as from (33). By the free-entry condition, the following equality holds,*

$$w^*(C^*) = \kappa_T + \kappa_I.$$

Using these results, we analyze in detail the policies of incumbent firms.

4.2 Characterizing corporate policies

Firm value and liquidity hoarding. We start by investigating the value of an incumbent firm as a function of cash reserves. In the constrained economy, the concavity of the value function implies that $v(c) \le c + w^*(C^*) = c + \kappa_T + \kappa_I$ due to the free-entry condition. In order to gain a deeper understanding of the impact of liquidity frictions on firm value, we compute a Taylor approximation for $v(c)$ in a neighborhood of C^* in the next Proposition.

Proposition 3 *Suppose that ϵ is sufficiently small. For any c in a left neighbourhood of C^*, the value of an incumbent firm can be approximated as follows:*

$$v(c) \;\approx\; v(C^*) - (C^* - c) - \frac{1}{6}v_3(C^* - c)^3 + \frac{1}{24}v_4(C^*)(c - C^*)^4$$

[18] While our analytical proof only holds for small ϵ, numerical results indicate that firm value exists and is concave for all reasonable values of ϵ.

where

$$\begin{aligned} v_3 &= \frac{2\,(r-\delta)}{\sigma^2 X_0^2}, \\ v_4(C^*) &= -\frac{v_3}{X_0}\left[4\,\frac{r-\delta}{\beta} + \frac{2\,\beta}{\sigma^2(1-\beta)} + \frac{1}{\sigma^2 X_0}\,\left(2\delta C^* - \varphi^2(\lambda-1)^2(\kappa_T+\kappa_I)^2\right)\right]. \end{aligned}$$

In these expressions, $X_0 = X^* = (1-\beta)^{1/\beta}$ is the firm's production rate at C^*.

Proposition 3 illustrates that the main determinants of the loss in firm value in the constrained economy are (to the highest order) the opportunity cost of cash $r-\delta$ and the coefficient of cash flow volatility σ. Notably, the cost $r-\delta$ unambiguously has a negative effect on firm value. Differently, cash flow volatility σ increases firm value when the cash reserves are close to the target level C^*. Since the curvature of the value function tends to zero when σ does, it follows that firm value is non-monotonic in σ conditional on a value of C^*. This effect raises the question of the dependence of C^* on σ. The following proposition addresses this question by investigating the determinants of C^*.

Proposition 4 *Suppose that ϵ is sufficiently small. The target cash level C^* is given by*

$$\begin{aligned} C^* &\approx C_1^*\sqrt{\epsilon} + C_2^*\epsilon \\ C_1 = \left(\frac{2}{v_3}\right)^{\frac{1}{2}}, \; C_2 &= \frac{v_4(0)}{3\,v_3^2} \end{aligned}$$

The first crucial observation is that, to the highest order, the target level C^* is proportional to the square root of the refinancing cost, $\epsilon^{1/2}$. This means that a small financing friction ϵ can have a large effect on firm value. To the highest order, the target level of cash reserves is proportional to the quotient $\frac{\sigma X_0(\beta)}{(r-\delta)^{1/2}}$. Hence, it is largely driven by cash flow volatility and the opportunity cost of holding cash, as in previous cash holdings models. The target level also depends on the elasticity β. A larger β means that a firm sets a higher markup and produces less (X_0 is decreasing in β). Proposition 4 then suggests that firms with larger monopolistic rents should hold less cash.[19] Substituting the expression for $v_4(0)$ from Proposition 3 into the approximation of Proposition 4, we arrive at the following result.

[19]In this context, see also Morellec, Nikolov, and Zucchi (2014), who analyze the relation between corporate cash holdings and product market competition.

Corollary 5 (More innovative firms hold more cash) *Suppose that ϵ is sufficiently small. Then, the target cash level C^* is monotone increasing in λ and ϕ, and is monotone decreasing in ζ.*

Corollary 5 is very intuitive: More innovative firms naturally spend more on innovations. Since the outcome of R&D is inherently uncertain, they need to hold a large liquidity buffer in our to remain active in R&D even in bad, liquidity constrained times. The result of Corollary 5 is also consistent with the empirical evidence in Falato, Kadyrzhanova, and Sim (2013), Lyandres and Palazzo (2014), and Begenau and Palazzo (2015) suggesting that there is a strong positive link between cash holdings and innovations.

Production and markups. We next analyze the firm's optimal production rate $X(c)$. Equation (26) highlights that the dynamics of $X(c)$ are fully determined by the firm's effective risk aversion $\gamma(c)$ in (24) and the function F in equation (25). Since the function $F(\gamma)$ is monotone decreasing in γ, so is $X(c)$. The intuition is the following. When liquidity constraints tighten (the cash reserves decrease), effective risk aversion increases and the firm becomes reluctant to take on idiosyncratic risk. To limit operating volatility, the firm scales down production. Since liquidity constraints become negligible at the target cash level in that $v''(C^*) = 0$, the production rate at C^* equals the (constant) production rate $X_0 = X^* = (1-\beta)^{1/\beta}$ of the benchmark economy. As a result, $X(c) < X_0$ for all $c < C^*$. Thus, the model predicts that liquidity constraints lead the firm to decrease the production rate. This relation is monotonic: the tighter the firm's liquidity constraints are (and the smaller the firm's cash reserves), the smaller the optimal production rate is.

Recall that each incumbent firm j acts as the monopolist in the product line j. Given the demand schedule (8), selecting the production rate $X(c)$ is thus equivalent to setting the following price

$$p(c) = X(c)^{-\beta} \ \geq X_0^{-\beta} = \frac{1}{1-\beta}.$$

Interestingly, liquidity frictions lead the incumbent firm to deviate from the constant price p^* derived for the benchmark economy. In the constrained economy, the incumbent sets a markup equal to $p(c) - 1$, which exceeds the markup set by an identical firm operating in the benchmark economy. Following negative shocks that deplete the cash reserves, the financially-constrained incumbent decreases the production rate and increases markups.

Thus, liquidity constraints cause markups to be *countercyclical* to idiosyncratic shocks. Proposition 6 summarizes these results.

Proposition 6 (Liquidity and Markups) *Suppose that ϵ is sufficiently small. Then the optimal production rate $X(c)$ is monotone increasing in c whereas markups $p(c)-1$ are decreasing. Now, suppose that ϵ is sufficiently small. For c in a left neighborhood of C^*, the optimal scale of production can be approximated by*

$$X(c) \approx X_0 + X_1(c-C^*) + \frac{X_2}{2}(c-C^*)^2 \,,$$

where X_0 is defined as in Proposition 3, $X_1 = \frac{2(r-\delta)}{\beta}$ and

$$X_2 = \frac{X_1}{X_0}\left[X_1(\beta+5) + \frac{2\beta}{\sigma^2(1-\beta)} + \frac{1}{\sigma^2 X_0}\left(2\delta C^* - \varphi^2(\lambda-1)^2(\kappa_T+\kappa_I)^2\right)\right] .$$

This approximation implies that (1) A larger β leads to a lower sensitivity of markups to liquidity shocks; (2) Firms that face a lower rate of creative destruction (smaller x_d) or that have more efficient innovation technology (larger φ or larger λ) decrease their scale of production (lower $X(c)$) and set higher markups.

An important implication of Proposition 6 is that liquidity frictions create a link between markups and firm characteristics that is absent in the unconstrained economy. Ceteris paribus, if technological breakthroughs occur more often (larger φ) or are more path-breaking (larger λ), a firm invests more in innovation. As a result, such a firm depletes cash reserves faster, it has a higher effective risk-aversion, and it scales down production by a larger amount when cash reserves decrease. While previous endogenous growth models have stressed that monopoly rents generate firm's incentives to invest in innovation,[20] Proposition 6 warns that liquidity constraints may reverse the causality of this relation. Firms with more efficient innovation technologies are more R&D-intensive, which makes them more financially constrained and leads them to charge higher markups. Notably, the positive effect of innovation on markups is empirically supported by Cassiman and Vanormelingen (2013).

[20]See, e.g., Aghion and Howitt (1992) and Aghion, Akcigit, and Howitt (2014). Similarly, in Romer's (1990) variety model, lower rents for innovators lead to lower R&D incentives. Note, however, that Arrow (1962) argues that the incentive to invest in innovations is lower under monopolistic than under competitive conditions, due to "organizational inertia."

Investment in innovation. In this subsection we investigate the behaviour of the optimal innovation rate $z(c)$. Using (27) and a Taylor expansion of $z(c)$ around C^*, we arrive at the following result.

Proposition 7 *Suppose that ϵ is sufficiently small. The optimal innovation rate is*

$$z(c) \;\approx\; \tilde{z}(c) \;\equiv\; z_0 + \frac{1}{2}(C^* - c)^2\, z_2 - \frac{1}{6}(C^* - c)^3 z_3 \tag{34}$$

for c in a left neighborhood of C^*. In this expression, we have defined $z_0 = \phi\,(1-\alpha)\,\frac{L}{\zeta}(\lambda - 1)(\kappa_T + \kappa_I)$, $z_2 = -\phi\,(1-\alpha)\,\frac{L}{\zeta}\; v_3\;(\lambda - 1)(\kappa_T + \kappa_I)$, and $z_3 = -\phi\,(1-\alpha)\,\frac{L}{\zeta}\;\big(v_3\; + v_4(\lambda - 1)(\kappa_T + \kappa_I)\big)$, whereas v_3 and v_4 are defined as in Proposition 3.

The next corollary relies on Proposition 7 and on the monotonicity properties of the following auxiliary quantity:

$$\mathcal{Z} \equiv \frac{2\;(\lambda-1)(\kappa_T+\kappa_I)}{1\;-\frac{1}{X_0}\left[4\;\frac{r-\delta}{\beta}+\;\frac{2\;\beta}{\sigma^2(1-\beta)}+\frac{1}{\sigma^2 X_0}\;\;(2\delta C^* - \varphi^2(\lambda-1)^2(\kappa_T+\kappa_I)^2)\;\right](\lambda-1)(\kappa_T+\kappa_I)},$$

to characterize the optimal innovation rate $z(c)$.

Corollary 8 (Liquidity and Innovation) *Suppose that $\mathcal{Z} > 0$. Then,*

- *if $\mathcal{Z} > C^*$, $\tilde{z}(c)$ is monotone increasing;*
- *if $\mathcal{Z} < C^*$, $\tilde{z}(c)$ is decreasing for $c < C^* - \mathcal{Z}$. This pattern is more likely to arise for firms operating with: (1) larger cash flow volatility σ, (2) more severe financing constraints ϵ, (3) more frequent technological breakthroughs, i.e. larger φ, (4) smaller opportunity cost of cash $r - \delta$,*

Liquidity constraints may intuitively encourage firms to cut their investment in R&D in order to save financial resources for the times of need. Nonetheless, Corollary 8 illustrates that the optimal innovation rate might increase when cash reserves decrease. This pattern resembles a *gamble for resurrection* decision and is largely driven by financial constraints. Given two firms with the same fundamental characteristics but different cash reserves, gamble for resurrection implies that the firm with smaller cash reserves invests more in innovation. In so doing, the firm seeks to increase the probability of achieving a technological breakthrough. When a breakthrough occurs, the firm gains access to the monopoly rents related to the brand-new technology and can raise outside funds in light of a "success" rather than a "failure" (i.e., running out of funds due to operating losses). All else equal, less cash may lead to more success.

It is known (see, for example, Hugonnier, Malamud, and Morellec, 2015) that liquidity frictions may potentially make the firm value locally convex, which then naturally leads to risk-loving (gambling) behaviour. In contrast, in our model firm value is concave (see Proposition 2) and its gambling behaviour is driven by a different mechanism. Namely, by (27), the optimal innovation policy $z(c)$ is a constant multiple of the quotient $\frac{(\lambda w^* - v(c) + c)}{v'(c)}$. Both the numerator and the denominator are monotone decreasing in c : The numerator is decreasing because $v'(c) \geq 1$ and hence the gain $v(c) - c$ from injecting cash into the firm is decreasing over time; the denominator is decreasing in c because firm value is concave in c. Thus, $z(c)$ is decreasing if and only if the numerator decreases at a higher rate than the denominator. By direct calculation, this is equivalent to the inequality

$$\frac{v'(c) - 1}{\lambda w^* - v(c) + c} \geq \sigma^{-2}\gamma(c)$$

where $\gamma(c) = -\sigma^2 \frac{v''(c)}{v'(c)}$ is the effective risk aversion of the firm. Since $v'(0) = 1 + \epsilon$, $z(c)$ is decreasing for small values of c if and only if $\frac{\epsilon}{\lambda w^* - v(0)} \geq \sigma^{-2}\gamma(0)$. The effective risk aversion of the firm scales with the variance of cash flows, $\gamma \sim \sigma^2$, and hence the right-hand side is not very sensitive to volatility. At the same time, the gain $\lambda w^* - v(0)$ from replenishing cash reserves is lower when volatility is higher, which explains why $z(c)$ tend to be decreasing for high-σ firms. A similar logic applies to the opportunity cost $r - \delta$: Since v is increasing in δ, the gain $\lambda w^* - v(0) = \lambda(\kappa_T + \kappa_I) - v(0)$ is decreasing in δ. Furthermore, since larger financing costs ϵ make the firm more constrained, they make the firm more willing to engage in this behavior. Moreover, firms with more efficient R&D technologies (larger φ, meaning that technological breakthroughs occur more often) and more volatile profits are more likely to route resources from production to innovation, to increase the likelihood of a breakthrough while limiting operating volatility.

Our analysis also highlights that financing frictions make the optimal innovation rate $z(c)$ dependent on firm's characteristics that do not affect z^* in the benchmark economy (besides, obviously, the opportunity cost of cash). First, the optimal innovation rate *does* depend on β in the constrained economy whereas it *does not* in the unconstrained economy. Specifically, firms are more willing to gamble for resurrection when β is lower. In this case, the markup set by the firm is smaller and, hence, the firm has a greater incentive to decrease production and favor innovation. Second, the volatility coefficient σ has a major impact on a firm's innovation rate in the constrained economy. Specifically, gambling for resurrection arises in environments in which σ is sufficiently large. These two observations confirm our intuition that the gambling for resurrection effect should be particularly strong for small, contsrained firms with little market power and high operational risk. This is a key implication of our model: in agreement withe the empirical evidence, *young and financially constrained firms can be very R&D intensive.*

It is important to contrast the non-monotonic innovation pattern of Corollary 8 with the existing empirical evidence. Recent papers (for example Falato, Kadyrzhanova, and Sim (2013), Lyandres and Palazzo (2014), and Begenau and Palazzo (2015)) document a strong empirical link between R&D and cash holdings: essentially, R&D and innovative activities account for a major fraction of the cross-sectional variation in firms' cash holdings, and the cross-sectional relationship is very strong and positive. While this observation may seem to contradict the results of Corollary 8 at a first sight, this is actually not the case. Namely, one should carefully distinguish between target cash holdings (that is, C^*) and deviations from this target.

As follows from Corollary 5, more innovative firms indeed hold more cash. If the refinancing frequency is sufficiently high (which is the case when φ is large), such firms will most of the time stay close to the target level, and hence the positive link between cash and innovations will be observable empirically. At the same time, after a sequence large adverse liquidity shocks such *innovative firms may become even more innovative.* Thus, empirically testing the prediction of Corollary 8 presents a non-trivial challenge because one would have to be able to clearly distinguish between ex-ante heterogeneity (because firms differ in the parameters ϕ, λ, ζ) and ex-post heterogeneity whereby ex-ante identical firms differ in their cash holdings, which in turn influence their innovative activities.

We complete this discussion by discussing the behavior of firms during the last financial crisis. Anecdotal evidence suggests that most firms significantly increased their cash holdings during and after the last financial crisis. This is consistent with the result of

Corollary 4: After a crisis, volatility σ and financing costs ϵ go up, which in turn leads to an increase in C^*. Corollary 8 implies that such a global increase in cash holdings may actually be associated with a simultaneous drop in the *aggregate* R&D. At the same time, the behaviour on an *individual* firm level may be ambiguous. For example, Archibugi, Filippetti, and Frenz (2013) report that some firms have actually increased their R&D investment despite the toughening constraints. In a related study, Kanerva and Hollanders (2009) find no relation between firm size and decline in investment in 2008. This suggests that the actual link between R&D and innovations might be non-linear and non-monotonic, in agreement with the results of Corollary 8.[21]

Idiosyncratic volatility of returns and cash flows. The optimal production rate $X(c)$ affects the volatility of cash flows, given by $\sigma X(c)$. The monotonicity of $X(c)$ in c implies that cash flow volatility is monotone increasing in the level of cash reserves. That is, a firm scales down operating risk when liquidity constraints tighten. This result has interesting implications for the relation between cash flow volatility and cash reserves (e.g., Bates, Khale, and Stulz, 2009). Our model suggests not only that volatility affects the target level of cash reserves, but also that the level of cash reserves (i.e., a firm's financial stance) determines cash flow volatility via the optimal production rate. After positive operating shocks, cash reserves increase, the curvature of the value function decreases, and firms are willing to take on more risk. Conversely, negative operating shocks lead firms to reduce production, and, as a result, cash flow volatility decreases.

We relate endogenous cash flow volatility to the idiosyncratic volatility of stock returns. By (4), the realized volatility of stock returns $dR(c) = \frac{dv(c)}{v(c)}$ is given by

$$\sigma_R(c) \;=\; \sigma X(c)\,\frac{v'(c)}{v(c)}\,.$$

Since all cash flow shocks are idiosyncratic in our model, $\sigma_R(c)$ coincides with the idiosyncratic return volatility. A direct calculation implies that $\sigma_R(c)$ is locally monotone

[21] Filippetti and Archiburgi (2011) link the resilience of R&D spending to structural factors such as the quality of human resources, the specialization in the high-technology sector, or the development of the financial system.

increasing in c if and only if

$$\frac{X'(c)}{X(c)}\frac{v'(c)}{v(c)} \;>\; -\frac{d}{dc}\frac{v'(c)}{v(c)}\,.$$

Since $v(c)$ is concave and increasing in c, the ratio $v'(c)/v(c)$ is decreasing. Thus, if the rate of change in production is higher than the rate of change in the value function, idiosyncratic volatility will be locally increasing.

The monotonicity of $\sigma_R(c)$ is closely related to the leverage effect identified by Black (1976), according to which the volatility of stock returns increases after a negative shock to stock prices. In this context, Décamps et al. (2011) develop a cash management model with constant cash flow volatility to show that liquidity frictions may generate the leverage effect as $\sigma_R(c)$ is monotone decreasing (and stock price increasing) in c. Nevertheless, the empirical evidence on the co-movement between stock returns and idiosyncratic volatility appears ambiguous. Early studies report a positive relation (e.g. Duffee, 1995, or Malkiel and Xu, 2002), whereas more recent works (Ang, Hodrick, Xing, and Zhang, 2006, 2009) report a negative relation—which has been labeled as the idiosyncratic volatility puzzle. Our model is capable of capturing both the positive and the negative relation. In fact, the instantaneous covariation of returns and idiosyncratic volatility is given by

$$\langle v(c)^{-1}dv(c), d\sigma_R(c)\rangle \;=\; v(c)^{-1}\sigma_C(c)^2 v'(c)\sigma_R'(c)dt\,,$$

so the hump-shaped pattern for $\sigma_R(c)$ leads to a negative (positive) co-movement for firms with high (low) cash reserves. This result calls for a thorough empirical investigation of the role of corporate liquidity in the idiosyncratic volatility puzzle.

4.3 The stationary cross-sectional distribution of liquidity

Using the results in the previous sections, we now determine the cross-sectional stationary distribution of liquidity in the economy, which we denote by $\eta(c)$. For each individual incumbent (omitting the subscript j), the dynamics of scaled cash reserves satisfy

$$dC_t \;=\; \mu(C_t)dt \;+\; \sigma(C_t)dZ_t \;+\; dF_t \;+\; dF_t^I \;-\; dD_t \;+\; (\lambda C^* - C_t)dN_t\,. \qquad (35)$$

In this equation, N_t is a Cox process with stochastic intensity $\phi z(C_t)$, representing the occurrence of a technological breakthrough, while (as in Section 4.1), the quantities

$$\mu(c) \;=\; \delta c \;+\; X^{1-\beta}(c) - X(c) - \frac{z^2(c)}{2}\frac{\zeta}{L}, \quad \text{and} \quad \sigma(c) = \sigma X(c)\,,$$

denote, respectively, the operating profits and volatility. Since liquidity shocks are independent and identically distributed (i.i.d.) across firms, the cross-sectional distribution of liquidity satisfies the following Kolmogorov forward equation[22]

$$\frac{1}{2}\left(\sigma^2(c)\eta(c)\right)'' - (\mu(c)\eta(c))' \;-\; x_d\eta(c) \;-\; z(c)\phi\eta(c) \;=\; 0\,.$$

To solve for the cross-sectional density, we impose the following boundary conditions

$$\begin{aligned} 0.5(\sigma^2\eta)'(0) - (\mu(0)\eta(0)) \; &= 0 \\ 0.5(\sigma^2\eta)'(C^*) - (\mu(C^*)\eta(C^*)) \; &= \; \int_0^{C^*} \phi z(c)\eta(c)dc \;+\; x_d. \end{aligned} \tag{36}$$

For any level of cash reserves c, the quantity $0.5(\sigma^2\eta)'(c) - (\mu(c)\eta(c))$ represents the infinitesimal change in the mass of firms due to retained earnings and profitability (as captured by the drift $\mu(c)$) and due to idiosyncratic cash flow shocks (as captured by the volatility $\sigma(c)$). The first equation in (36) is the mass conservation condition at zero: it guarantees that there is no loss of mass for firms that run out of liquidity and use external financing to remain solvent. The second equation is the mass conservation at C^*: it ensures that the loss of mass on the left-hand side is offset by the inflow of successful innovators whose cash reserves jump to C^*. In the next sections, we use this stationary distribution to derive the equilibrium quantities of the model.

4.4 Equilibrium quantities

We embed the preceding analysis into a general equilibrium setting. Along a balanced growth path, output and aggregate consumption grow at the constant (endogenous) rate

[22] See the Appendix for a formal argument.

g, given by the following expression

$$g = (\lambda - 1)\phi \int_0^{C^*} z(c)\eta(c)dc + (\Lambda - 1)x_d\,. \tag{37}$$

Since innovation decisions are i.i.d. across firms, the law of large numbers implies that the contribution of incumbents to economic growth is given by the size of their quality improvements multiplied by the average innovation intensity in the population of incumbent firms (the first term in (37)). Similarly, the contribution of entrants is given by the size of their quality improvements times the entry rate (the second term). In equilibrium, the entry rate (i.e., the rate of creative destruction) is pinned down by the free-entry condition

$$v\left(C^*(x_d); x_d\right) - C^*(x_d) = \kappa_T + \kappa_I.$$

We solve for x_d by using (33) and the approximation for C^* in Proposition 4.

Proposition 9 *The equilibrium rate of creative destruction satisfies*

$$x_d \;=\; \frac{\mu^* - (r-\delta)C^*}{\kappa_T + \kappa_I} + \varphi^2(\lambda-1)^2\frac{\kappa_T+\kappa_I}{2} - r\,.$$

In particular, when ϵ is sufficiently small, we get

$$x_d \;\approx\; \frac{\mu^* - (r-\delta)^{1/2}\sigma X_0 \epsilon^{1/2}}{\kappa_T + \kappa_I} + \varphi^2(\lambda-1)^2\frac{\kappa_T+\kappa_I}{2} - r.$$

To single out the various effects at play, we begin by studying the predictions of Proposition 9 while holding r fixed (in equilibrium, the interest rate r is obviously endogenous and depends on all of the model parameters). This is akin a semi-partial equilibrium setting, for instance within a single industry whose impact on the interest rate is sufficiently small. Proposition 9 shows that the financial constraints of incumbents discourage fresh entry, and this effect is stronger when cash flow volatility σ, the opportunity cost of cash $r-\delta$, and the refinancing cost ϵ are larger. Moreover, the entry rate is monotone increasing in the elasticity β since X_0 is monotone decreasing in β.

Along the balanced growth path, consumption grows at the rate g. The equilibrium interest rate is pinned down by the Euler equation of the representative household and given by

$$r \;=\; \rho + \theta \left[(\lambda - 1)\phi \int_0^{C^*} z(c;r)\eta(c;r)dc + (\Lambda - 1)x_d(r) \right] . \tag{38}$$

This equation represents the fixed point equation for $r > \delta$, which we solve numerically.[23]

We now turn to the analysis of aggregate consumption and welfare. Similar to the unconstrained economy, consumption is the sum of labor income and aggregate dividends net of cash injections into the firms. In particular, the wage paid by the competitive final-good producer is given by

$$W_t \;=\; \frac{1}{1-\beta}\int_0^1 L^\beta\, \bar{X}_t(j)^{1-\beta}\, q_t(j)^\beta dj \;-\; \int_0^1 p_t(j)\bar{X}_t(j)dj\,.$$

In equilibrium, the wage grows at the rate g along the balanced growth path and satisfies $W_t \;=\; W_0 \int q_t(j)dj$ with

$$\int q_t(j)dj \;=\; e^{gt}.$$

By a direct calculation, it follows that

$$W_0 \;=\; \frac{\beta L}{1-\beta}\int \eta(c)X(c)^{1-\beta}dc\,.$$

In our continuous time model, calculating the dividend rate is not a trivial exercise. Even though the dividend process of every firm is singular,[24] aggregate dividends are smooth and grow at the rate g. At every instant, firms with cash reserves close to C^* may move to C^* according to the endogenous dynamics in (35), and eventually pay out dividends. Thus, computing the dividend rate requires to keep track of the whole cross-sectional distribution of liquidity in the economy. To address this issue, we proceed as follows.

We denote by d_t the aggregate dividend rate and by f_t the total rate of cash injections to the incumbent and entrant firms of the economy. In order to compute the rate $d_t - f_t$, we define $Y(q_t(j), c)$ to be the present value of a virtual "production unit" that we name *dynasty.* Namely, a dynasty represents the expected present value of dividends net of cash injections of all firms that will ever operate in the future in the product line j. As for

[23]Extensive numerical tests suggest that the solution is always unique.

[24]As in most other cash management models (see, for example, Décamps, Mariotti, Rochet, and Villeneuve, 2011; Bolton, Chen, and Wang, 2011, 2013; and Hugonnier, Malamud, and Morellec, 2015), dividends are non-smooth, and dividend payouts only happen when cash hits the threshold C^*.

the other quantities in a balanced growth path equilibrium, $Y(q_t(j), c)$ is homogeneous in the quality of the intermediate good j. Hence, the relation

$$Y(q_t(j), c) \;=\; Lq_t(j)y(c)$$

holds for some function $y(c)$. As shown in the Appendix, $y(c)$ solves the following equation:

$$\frac{1}{2}\sigma^2(c)y''(c) \;+\; \mu(c)y'(c) \;+\; \phi z(c)\left[\lambda(y(C^*) - C^*) - y(c) + c\right] \\ + x_d\left[\Lambda y(C^*) - y(c) + c - \Lambda\left(C^* + \kappa_T\right)\right] \;=\; ry(c)\,.$$

The first two terms on the left-hand side represent the effect of cash flow volatility and profitability. The third term represents the probability-weighted change in value after a technological breakthrough by the current incumbent of the product line j. In this case, investors provide an amount $\lambda C^* - c$ to the dynasty. The fourth term represents the probability-weighted change in value after a technological breakthrough by an entrant. Upon fresh entry, investors provide the amount $\Lambda\left(C^* + \kappa_T\right)$ to the entrant and collect the cash holdings of the outgoing incumbent. Notably, the financing fees ϵ, κ_I, and α have no direct impact on the cash flows to the representative household: by assumption, these costs are paid in equity shares and not in cash.[25] As a result, the marginal value of cash for the dynasty, $y'(c)$, is equal to one at $c = 0$ and $c = C^*$:

$$y'(C^*) = 1 = y'(0)\,.$$

Solving for $y(c)$ allows to determine the net dividend rate of the economy. Since net dividends grow at g on a balanced growth path, it follows that

$$d_0 - f_0 \;=\; (r - g)L\int_0^{C^*}\eta(c)y(c)dc\,.$$

Because aggregate consumption grows at g, we have $\mathcal{C}_t \;=\; \mathcal{C}_0 e^{gt}$ with $\mathcal{C}_0 \;=\; W_0 + d_0 - f_0$.

The analysis highlights that financial constraints unambiguously decrease the equilibrium wage. Since $X(c) < X_0$ for any $c < C^*$, it follows that $W_0 \leq W_0^*$. Moreover, W_0^*

[25]When a firm runs out of cash and raises an amount f, the cost to incumbent shareholders is ϵf. This cost is paid in shares of the company and has no impact on the wealth of the representative household.

only depends on β in the unconstrained economy, whereas liquidity frictions imply that W_0 depends on the other model parameters through their impact on production decisions $X(c)$ and the distribution of liquidity in the economy $\eta(c)$. The analytical comparison of the net dividend rate in the constrained and unconstrained economies is less trivial. On the one hand, the entry rate is lower in the constrained economy (as from Proposition 9), meaning that investors provide financing to new firms less often. On the other hand, the amount provided is larger in this economy, because investors finance not only the entry cost but also provide entrants with cash reserves. We provide a numerical analysis of consumption and welfare in the constrained/unconstrained economies in Section 5.2.

5 Model analysis

In this section, we provide a quantitative assessment of the model implications. Table 1 reports the baseline parametrization. Refinancing current operations entails a cost of $\epsilon = 8\%$ for any dollar raised, consistent with the estimation of Hennessy and Whited (2007). Financiers extract a share $\alpha = 6\%$ of the surplus when incumbents market a higher quality product. The financing component of the entry cost κ_I is 10% of the technological component, which means that financiers extract more rents from entrants than from incumbents. By setting $\lambda = 1.04$ and $\Lambda = 1.10$, we capture in a simple fashion that innovation by entrants tends to be path-breaking while innovation by incumbents tends to be incremental; see, e.g., Akcigit and Kerr (2015), and Acemoglu et al. (2013). We set $\sigma = 0.4$, which implies that cash flow volatility $\sigma X(c)$ varies between 9.9% and 12.7% under the baseline parametrization. The return on cash δ is equal to 4.9%, which implies an opportunity cost of cash $r - \delta$ around 0.5%.[26]

5.1 Corporate outcomes

We start our analysis by comparing the value-maximizing corporate policies in the constrained and unconstrained economies. Confirming the analytical results, Figure 1 shows that firms downsize production when cash reserves are low. As a result, the production

[26] We endogenize δ in Appendix A.5.

rate $X(c)$ in the constrained economy is always below that of the unconstrained economy. While intuition may suggest that financial constraints lead firms to reduce their investment in innovation, our analysis reveals that this may not be the case. Under the baseline parametrization, $z(c)$ is always higher than z^*, being roughly 7.14% higher for c close to zero. The innovation rate is decreasing when cash reserves are small and is almost flat when cash holdings are large, in line with a gambling behavior.

Figure 2 investigates further the patterns of the optimal innovation rate. An increase in the refinancing cost ϵ up to 14% makes the optimal innovation rate steeper around zero. That is, tighter financial constraints make firms more prone to gamble. Conversely, an increase in the cost of financing a technological breakthrough α up to 12% decreases the innovation rate very sharply. An increase in α erodes the incumbents' surplus from innovation; as a result, $z(c)$ lies below z^* when c is large enough. Note, however, that firms again find it optimal to increase their innovation rate when cash reserves are small. An increase in κ_I up to 0.1 (implying that the financing component of the entry cost is about 14.3% of the technological component) spurs innovation: e.g., $z(0)$ is 10.7% larger than in the unconstrained benchmark z^*. A larger κ_I deters entry and renders incumbents less threatened by creative destruction. This leads to an increase in their innovation rate.

We also explore the impact of cash flow volatility and profitability on the optimal innovation rate. Notably, the parameters σ and β *do not* affect the optimal innovation rate in the unconstrained economy, but they *do* in the constrained economy. Figure 2 shows that a decrease in β leads to a decrease in markups and prompts gamble for resurrection. When liquidity constraints are tight and profitability is low, it is relatively more convenient to cut core production and invest more in innovation. Moreover, a decrease in σ leads to a sharp drop in the optimal innovation rate and dampens gambling.

Our analysis reveals that $z(c)$ can be higher or lower than z^*, and it can be non-monotonic in c. These results suggest that financing frictions and liquidity constraints may boost investment in innovation and may help explain the innovation patterns of young entrepreneurial firms in comparison to mature firms. Young firms are more financially constrained (larger ϵ), they operate with lower margins (lower β), and their cash flows are volatile (larger σ). Despite these constraints, small firms can be very R&D intensive, as discussed in the Introduction. Our model can rationalize these patterns.

Moreover, Figure 1 shows that firms can be more valuable in the constrained economy than in the unconstrained economy. In the latter, entry is less costly as it does not involve any financing cost. The entry rate is then larger, which implies that incumbent firms are more likely to exit the industry. We investigate the general equilibrium effects of this result in the next section.

5.2 Aggregate quantities

The cross-sectional distribution of liquidity. Figure 3 displays the cross-sectional distribution of liquidity $\eta(c)$ on $c \in [0, C^*]$ under several parametrizations. The distribution is monotone increasing as in the model of Bolton, Chen, and Wang (2011). That is, incumbents' cash reserves are relatively large most of the time. Variations in the parameter values affect not only the shape of the distribution, but also its interval of definition. For instance, a decrease in the return on cash from 4.9% to 1% or in the coefficient of cash flow volatility from 0.4 to 0.2 leads to a decrease in the target level of cash reserves. In both cases, the distribution is defined over a tighter interval, and it becomes steeper. Conversely, a decrease in the elasticity β reinforces firms' precautionary policies. Firms enlarge their target level of cash holdings, and the distribution of liquidity becomes flatter around zero. That is, firms with smaller monopolistic rents hold more cash.

The shape of the cross-sectional distribution has important implications for the impact of firm-specific policies on aggregate outcomes. As an illustration, consider the effect of the potentially decreasing shape of $z(c)$ on the aggregate innovation $\int \eta(c)z(c)dc$. Since the stationary distribution is concentrated close to the target level C^*, the impact of the behaviour of highly constrained firms (i.e., those with c close to zero) is very small and largely negligible: Even if firms with c close to zero drastically increase their innovation rate, their mass in the economy and the impact of their policies on the aggregate innovation rate is small. This has interesting implications for government policies designed to subsidize innovations. As Acemoglu, Akcigit, Bloom, and Kerr (2013) notice, government interventions tend to benefit large incumbent firms that are usually more effective at obtaining subsidies. In our model, subsidizing large firms may also be efficient for two reasons. First of all, they tend to stay close their target cash holdings and hence, due to the nature of the distribution $\eta(c)$ the impact of their policies on the aggregate economy is the largest. Second, subsidizing small, constrained firms may actually reduce their innovative activities (Corollary 8).

Given that the government interventions often tend to be counter-cyclical, it is important to highlight the potential link between these interventions and nature of the cross-sectional distribution of cash holdings. After a major recession, volatility and financing costs spike, target cash holdings C^* increase, and the cross-sectional distribution gets more stretched out, implying that the contribution of low-cash firms to aggregate growth may change. Optimal subsidies should take such distributional effects into account.

Financing frictions, growth, and welfare. We next investigate the effects of financing frictions and corporate liquidity hoarding on economic growth and welfare. Under the baseline parametrization, economic growth is 1.916% in the constrained economy, whereas it is 2.013% in the identical but unconstrained economy.

Figure 4 shows creative destruction and growth as a function of the technological and financing components of the entry cost, κ_T and κ_I. It shows that creative destruction is decreasing in κ_T whereas growth displays a U-shaped pattern.[27] Departing from the baseline Schumpeterian paradigm whereby growth is spurred by creative destruction only, our model takes into account the incumbents' contribution to economic growth. It highlights that an increase in κ_T deters fresh entry but boosts the incumbents' innovation rate. When κ_T is low enough, an increase in κ_T leads to lower growth. In this case, the reduction in the entrants' contribution to growth overtakes the increase in the incumbents' innovation rate. When κ_T is large enough, conversely, a further increase in κ_T increases growth. In this case, the increase in the incumbents' contribution to growth more than offsets the decrease in creative destruction.

In our model, the entry cost includes a financing component on top of the technological component. By imposing barriers on entrants, financiers slow down creative destruction (i.e., the entry rate is lower). Indeed, Figure 4 shows that x_d decreases in κ_I. If creative destruction occurs less often, incumbents expect to enjoy monopoly rents for longer periods and thus increase their innovation rate (as illustrated in Section 5.1). Since the growth rate is non-monotonic in the entry cost, g can be larger than g^* *if* κ_T and κ_I are large enough. Notably, our analysis shows that financing frictions are not necessarily

[27]This may not be the case when risk aversion θ is less than (or equal to) 0.5. We do not consider these parametrizations, as they may bring along an interest rate greater than the rate of economic growth.

detrimental to economic growth.

An interesting question is how financing frictions affect aggregate consumption and welfare. To address this question, we make the following thought experiment. Suppose that the planner intervenes to relax the financing frictions in the economy.[28] After this intervention, the equilibrium quantities become those described in Section 3. Importantly, firms stop retaining earnings. Corporate cash reserves, amounting to

$$\tilde{C} \equiv L \int_0^{C^*} c\eta(c)dc,$$

are distributed to the representative household. The resulting intertemporal wealth of the household is the sum of consumption (in the unconstrained economy, $\mathcal{C}_0^*$) and corporate cash reserves. If the planner does not intervene, conversely, corporate cash reserves remain "trapped" in the corporate sector.

Recall that the dynasty represents the expected present value of dividends net of financing in a given product line j. In the unconstrained economy, the scaled value of a dynasty j equals the scaled value of the incumbent that currently produces the intermediate good j, which in turn is equal to κ_T due to the free-entry condition. In the constrained economy, in contrast, the scaled value of the dynasty is larger than the value of the corresponding incumbent; in particular, the following relations $y(C^*) - C^* > v(C^*) - C^* = \kappa_T + \kappa_I \geq \kappa_T = v^* = y^*$ hold (see the results of Section 4.4). This implies that if the planner does not intervene and the economy remains constrained, agents enjoy larger net dividends. If the planner does intervene, agents enjoy larger wages and receive cash holdings of the corporate sector.

Figure 5 shows consumption and welfare as a function of κ_T and of κ_I if the planner does or does not intervene. The figure shows that welfare is non-monotonic in κ_T and follows a U-shaped pattern as the rate of economic growth. Importantly, the figure shows that financing frictions may *be not* welfare-decreasing. When κ_I is large enough, a firm operating in the constrained economy is more valuable than an identical firm operating in the unconstrained economy.[29] As explained, the former is less threatened

[28] In our though experiment, we assume that the planner can remove the financing friction and switch the economy from constrained to unconstrained (i.e., as in Section 3).

[29] Recall that the financing cost κ_I is paid in shares of the company's stock and therefore has no direct impact on aggregate consumption. Of course, κ_I has an indirect effect through the rate of creative

by creative destruction. The constrained dynasty promises greater net dividends. In the constrained economy, aggregate consumption is larger than in the unconstrained economy if the increase in dividends more than offsets the decrease in wages.

6 Concluding remarks

Several empirical studies document strong relations between corporate cash reserves and investment in innovation, and between innovation and growth. In this paper, we develop a theoretical model that studies the micro and macro implications of financing frictions and liquidity hoarding for investment in innovation and economic growth. To this end, we embed liquidity frictions into a Schumpeterian model that features innovations by incumbents and entrants. Our modeling of financial frictions accounts for several major features that characterize innovation-intensive industries. In particular, firms prefer internal funds and delay external (equity) financing until technological milestones are attained or when in extreme need of funds. The model generates rich joint dynamics of cash holdings, production, and R&D investment decisions, which capture documented stylized facts and deliver new testable predictions. One of our main findings is that liquidity frictions can spur investment in innovation for young, constrained firms. We also find that, in the presence of financial constraints, firms can be more valuable and pay out more dividends, which can lead to an increase in equilibrium consumption and welfare.

We believe that the interactions between liquidity frictions, innovations, and growth are important for the dynamics of the global economy and policy-making. Our paper makes only the first step in this direction. First of all, we abstract from aggregate shocks. Nonetheless, it would be interesting to investigate the impact of government liquidity policies on production, innovation, and long-run growth over the business cycle (see also Aghion, Farhi, and Kharroubi, 2012). Second, we take the costs of external financing as exogenous and do not microfound them. Such microfoundations based on asymmetric information and limited pledgeability of assets could also allow one to investigate full-fledged dynamic capital structure choices and study their micro and macro implications. We leave these aspects for future research.

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destruction.

A Appendix

A.1 Deriving incumbents' policies and value

In the presence of liquidity frictions, the incumbents' HJB equation is

$$0 = V_t + \max_{z,\bar{X}} \Big\{ \frac{\sigma^2}{2} \bar{X}^2 V_{cc} + V_c \left(\delta \, \bar{C} + X(p-1) - \frac{z^2}{2} q\zeta \right) \\ + \phi(1-\alpha) z (\lambda(V - \bar{C}^*) - (V - \bar{C})) \; + \; x_d \, (\bar{c} - V) \; - \; rV \Big\}. \tag{39}$$

We conjecture

$$V(t, \bar{C}, q) \; = \; qL \, v\left(\frac{\bar{C}}{qL} \right) \; \equiv \; Q \, v(c) \quad Q = qL \, .$$

Having defined $X \; \equiv \; \bar{X}/Q$, $\; c \; \equiv \; \bar{C}/Q$so the expression for the price of input becomes

$$p = \; \left(\bar{X}^{-1} Q\right)^{\beta} = X^{-\beta} \, .$$

Substituting

$$V_t \; = 0 \; , \quad V_c \; = \; v'(c) \; , \quad V_{cc} \; = \; \frac{1}{Q} v''(c)$$

into (39), we get[30]

$$\max_{z,\bar{X}} \Big\{ \frac{\sigma^2}{2} X^2 v''(c) + v'(c) \left(\delta c \; + \; X^{1-\beta} - X - \frac{z^2}{2} \frac{\zeta}{L} \right) \\ + \; \phi(1-\alpha) z (\lambda(v(C^*) - C^*) - (v(c) - c)) \; + \; x_d (c - v(c)) \; - \; rv(c) \Big\} \; = \; 0 \, . \tag{40}$$

Let $A(c) \; = \; -\sigma^2 \frac{v''(c)}{v'(c)}$. The first order condition for X takes the form

$$-A(c) X + (1-\beta)(X)^{-\beta} \; = \; 1. \tag{41}$$

Define $F(a)$ to be the unique solution to

$$-aF(a) + (1-\beta)(F(a))^{-\beta} \; = \; 1 \, . \tag{42}$$

Then, $X(c) \; = \; F(A(c))$. Denote $w^* \; \equiv \; v(C^*) - C^*$. By the first order condition, we have that the optimal innovation rate solves

$$z \; = \; \phi(1-\alpha) \frac{L}{\zeta} \, \frac{(\lambda w^* - v(c) + c)}{v'(c)}$$

[30]The true surplus created upon financing a technological breakthrough occurs is given by $\tilde{S}(c) = \lambda \left(v(C^*) - C^* \right) + c - \lambda v(c/\lambda)$. To ease the analysis, we take a slightly modified version of this surplus, i.e. $S(c) = \lambda(v(C^*) - C^*) + c - v(c)$. This is without loss of generality. In fact, we can show that $0 < \tilde{S}(c) < S(c)$. We have $\lambda(v(C^*) - v(c/\lambda)) \; > \; \lambda(C^* - c/\lambda) = \lambda C^* - c$ since $v'(c) \geq 1$. Hence, it is always optimal for the firm to bargain with the financiers.

and the corresponding term in the HJB is

$$\frac{\phi^2(1-\alpha)^2}{2}\frac{L}{\zeta}\frac{(\lambda w^* - v(c) + c)^2}{v'(c)}$$

and we define $\varphi \equiv \phi(1-\alpha)\left(\frac{L}{\zeta}\right)^{1/2}$. Now, by (41), it follows that

$$F^{1-\beta} - F = \beta F^{1-\beta} - \sigma^2 \frac{v''(c)}{v'(c)} F^2$$

and hence we get

$$0.5\sigma^2 F(A(c))^2\, v''(c) \;+\; v'(c)(F^{1-\beta} - F) \;=\; -0.5\sigma^2 F(A(c))^2\, v''(c) \;+\; v'(c)\beta F^{1-\beta}\,.$$

Thus, the HJB equation can be written as follows:

$$\begin{aligned} &-\frac{\sigma^2}{2} F(A(c))^2\, v''(c) + v'(c)\,\beta\, F(A(c))^{1-\beta} + \; v'(c) c\,\delta \\ &+\; \frac{\varphi^2}{2}\frac{(\lambda w^* - v(c) + c)^2}{v'(c)} + x_d\,(c - v(c)) \;-\; rv(c) \;=\; 0\,, \end{aligned} \tag{43}$$

subject to the following set of boundary conditions

$$v'(0) \;= 1 + \epsilon\,, \qquad v'(C^*) \;= 1, \qquad v''(C^*) = 0\,.$$

It follows that $A(C^*) = 0$ and $X(C^*) = X_0$. Then, at C^*, the following equation

$$\mu + \delta C^* + 0.5(\varphi(\lambda - 1)w^*)^2 - x_d\, w^* \;-\; r(w^* + C^*) \;=\; 0 \tag{44}$$

holds, where we have defined $\mu = \mu^* \;=\; \beta(X_0)^{1-\beta}$ and $X_0 = \; X(C^*) = \; (1-\beta)^{\frac{1}{\beta}}$. This reveals that the solution to w^* is given by[31]

$$w^* = w(C^*) = \frac{x_d + r - \left((x_d + r)^2 - 2(\varphi(\lambda-1))^2(\mu - (r-\delta)C^*)\right)^{1/2}}{\varphi^2(\lambda-1)^2}.$$

[31]The choice of the solution to pick is motivated as follows. Consider the frictionless case $C^* = 0$. In the limit when $\phi \to 0$, $w^* = \frac{\mu}{r + x_d}$, where $\mu \;=\; \beta(X_0)^{1-\beta}$. Our solution can be rewritten as

$$w^* \;=\; \frac{2\mu}{x_d + r \mp \left((x_d + r)^2 - 2(\varphi(\lambda-1))^2(\mu - (r-\delta)C^*)\right)^{1/2}}.$$

Obviously, to get the continuous solution in the limit, we need to pick the plus sign, as the other solution blows up. Thus, by continuity, we pick that with the plus sign.

A.2 Approximations

In this section, we derive $F^i(c)$, $A^i(c)$, $v_i = v^{(i)}$, where F, A, v are the functions derived in the previous section. We start with the function $A(c)$. By simple calculations, it follows that

$$A'(c) \;=\; -\sigma^2\Big(\frac{v'''(c)}{v'(c)} - \frac{(v''(c))^2}{(v'(c))^2}\Big), \quad \Rightarrow\; A'(C^*) = -\sigma^2 v'''(C^*),$$

and

$$A''(c) \;=\; -\sigma^2\Big(\frac{v''''(c)}{v'(c)} - 3\frac{v''(c)v'''(c)}{(v'(c))^2} + 2\frac{(v''(c))^3}{(v'(c))^3}\Big), \quad \Rightarrow\; A''(C^*) = -\sigma^2 v''''(C^*)\,.$$

We turn to F. By differentiating (42), we have

$$-F - aF' - \beta(1-\beta)F^{-\beta-1}F' = 0$$

so

$$F'(A(c)) \;=\; -\frac{F(A(c))}{A(c) + \beta(1-\beta)F(A(c))^{-\beta-1}} \quad \Rightarrow\; F'(A(C^*)) = -\frac{X_0^{\beta+2}}{\beta(1-\beta)}. \tag{45}$$

To arrive at F'', we differentiate again the equation above

$$-2F' - aF'' - \beta(-\beta-1)(1-\beta)F^{-\beta-2}(F')^2 - \beta(1-\beta)F^{-\beta-1}F'' = 0 \tag{46}$$

$$\Rightarrow\; F''(A(C^*)) = \frac{X_0^{2\beta+3}}{\beta^2(1-\beta)^2}(\beta+3)$$

Summarizing, the following relations hold at C^*

$$\begin{aligned} A'(C^*) &= -\,\sigma^2\, v_3 \\ A''(C^*) &= -\,\sigma^2\, v_4 \end{aligned}$$

and

$$\begin{aligned} F'(A(C^*)) = -\,\frac{X_0^{\beta+2}}{\beta(1-\beta)} &= -\Big(\,(1-\beta)^{1/\beta}\,\Big)^{\beta/2} \\ F''(A(C^*)) &= (\beta+3)\,\frac{X_0^{2\beta+3}}{\beta^2(1-\beta)^2}. \end{aligned}$$

By simple calculations, we also obtain

$$\begin{aligned} X(C^*) = \; X_0 &= F(A(C^*)) = (1-\beta)^{\frac{1}{\beta}}, \\ X'(C^*) = \; X_1 &= F'(A(C^*))A'(C^*) = \frac{2(r-\delta)X_0^{\beta}}{\beta(1-\beta)} = \frac{2(r-\delta)}{\beta}, \\ X''(C^*) = \; X_2 &= F''(A(C^*))(A'(C^*))^2 + F'(A(C^*))A''(C^*) \\ &= \frac{X_0^{2\beta-1}(2(r-\delta))^2}{\beta^2(1-\beta)^2}(\beta+3) - \frac{X_0^{\beta+2}}{\beta(1-\beta)}(\sigma^2 v_4), \end{aligned}$$

where we omit the arguments of the functions to ease the notation. We will provide an expression for the term $v4 = v^{(4)}(c)$ in the following.

We now differentiate equation (43), obtaining

$$
\begin{aligned}
&-\frac{\sigma^2}{2}v'''(c)F(A(c))^2 - \sigma^2 v''(c)F(A(c))F'(A(c))A'(c) + v''(c)\beta(F(A(c)))^{1-\beta} \\
&+ v'(c)\beta(1-\beta)F^{-\beta}(A(c))F'(A(c))A'(c) + v''(c)c\delta + \; v'(c)\delta - \frac{\varphi^2}{2}\frac{(\lambda w^* - v(c) + c)^2 v''(c)}{(v'(c))^2} \\
&+ \varphi^2(\lambda w^* - v(c) + c)\left(\frac{1}{v'(c)} - 1\right) + x_d\,(1 - v'(c)) \; - \; r v'(c) \; = \; 0.
\end{aligned}
$$

Using the boundary conditions, we obtain an expression for the third derivative of v at the target cash level

$$
v'''(C^*) \equiv \; v_3 = \; \frac{2\,(r-\delta)}{\sigma^2}X_0^{-2}.
$$

Differentiating again the ODE, we obtain

$$
\begin{aligned}
&-\frac{\sigma^2}{2}v''''(c)F^2 - 2\sigma^2 v'''(c)FF'A' - \sigma^2 v''(c)(F')^2(A')^2 - \sigma^2 v''(c)FF''(A')^2 - \sigma^2 v''(c)FF'A'' \\
&+ v'''(c)\beta F^{1-\beta} + 2v''(c)\beta(1-\beta)F^{-\beta}F'A' - v'(c)\beta^2(1-\beta)F^{-\beta-1}(F')^2(A')^2 \\
&+ v'(c)\beta(1-\beta)F^{-\beta}F''(A')^2 + v'(c)\beta(1-\beta)F^{-\beta}F'A'' + v'''(c)c\delta \; + 2v''(c)\delta \\
&+ \varphi^2\frac{(-v'(c)+1)^2}{v'(c)} \; - \; \varphi^2\frac{(\lambda w^* - v(c) + c)v''(c)}{(v'(c))^2}(2 - v'(c)) \\
&+ \varphi^2 \; \frac{(\lambda w^* - v(c) + c)^2(v''(c))^2}{(v'(c))^3} - \frac{\varphi^2}{2}\;\frac{(\lambda w^* - v(c) + c)^2 v'''(c)}{(v'(c))^2} - \; (r + x_d)v''(c) \; = \; 0.
\end{aligned}
$$

At C^*, the above equation becomes:

$$
\begin{aligned}
&-\frac{\sigma^2}{2}v''''(C^*)X_0^2 \; - \; 2\sigma^2 v'''(C^*)XF'A' + \; v'''(C^*)\beta X^{1-\beta} \\
&- \beta^2(1-\beta)X^{-\beta-1}(F')^2(A')^2 \; + \; \beta(1-\beta)X^{-\beta}F''(A')^2 \; + \; \beta(1-\beta)X^{-\beta}F'A'' \\
&+ v'''(C^*)C^*\delta - \frac{\varphi^2}{2}\;(\lambda w^* - v(C^*) + C^*)^2 v'''(C^*) = \; 0.
\end{aligned}
$$

Therefore, we also obtain an expression for the fourth derivative of v at C^*,

$$
v^{(4)}(C^*) \equiv \; v_4 \; = \; \frac{2v_3}{\sigma^2 X_0^2}\left(\frac{\varphi^2}{2}\;(\lambda-1)^2(w^*)^2 - \beta X_0^{1-\beta} - \delta C^* - v_3(\sigma^2)^2\;\frac{X_0^{\beta+3}}{\beta(1-\beta)}\right).
$$

Suppose that ϵ is sufficiently small. For c close to C^*, firm value can be approximated by the following expression

$$
\begin{aligned}
v(c) &\approx v(C^*) + v'(C^*)(c - C^*) + \frac{v''(C^*)}{2}(c - C^*)^2 + \frac{v'''(C^*)}{6}(c - C^*)^3 \\
&+ \frac{v''''(C^*)}{24}(c - C^*)^4 + \; O((c - C^*)^5) \approx \\
& w^* \; + \; c \; + \; \frac{v_3}{6}(c - C^*)^3 \; + \; \frac{v_4}{24} \, (c - C^*)^4 \; + \; O((c - C^*)^5)
\end{aligned} \tag{47}
$$

while the first and the second derivative of v satisfy

$$
\begin{aligned}
v'(c) &\approx 1 + v''(C^*)(c - C^*) + \frac{v'''(C^*)}{2}(c - C^*)^2 + \frac{v''''(C^*)}{6}(c - C^*)^3 = \\
& 1 \; + \; \frac{v_3}{2}(c - C^*)^2 \; + \frac{v_4}{6}(c - C^*)^3
\end{aligned} \tag{48}
$$

$$
\begin{aligned}
v''(c) &\approx v''(C^*) + v'''(C^*)(c - C^*) + \frac{v''''(C^*)}{2}(c - C^*)^2 = \\
& v_3(c - C^*) \; + \frac{v_4}{2}(c - C^*)^2 \; .
\end{aligned} \tag{49}
$$

Using these approximations, it follows that

$$
\begin{aligned}
v'(0) &= 1 \; + \; \frac{v_3}{2}(C^*)^2 \; - \frac{v_4}{6}(C^*)^3, \\
v''(0) &= - \; v_3 \; C^* + \; \frac{v_4}{2}(C^*)^2.
\end{aligned} \tag{50}
$$

The threshold C^* is then obtained by exploiting the boundary condition at zero, i.e.

$$
C^* : \quad 1 \; + \; \frac{v_3}{2}(C^*)^2 \; - \frac{v_4}{6}(C^*)^3 \; = \; 1 + \; \epsilon.
$$

We use

$$
C^* \approx \left(\frac{2}{v_3} \right)^{\frac{1}{2}} \epsilon^{\frac{1}{2}} \; + \; a\epsilon
$$

where a can be found by solving the following equation

$$
\frac{v_3}{2}\epsilon \left(\left(\frac{2}{v_3} \right)^{\frac{1}{2}} + a\epsilon^{\frac{1}{2}} \right)^2 \; - \frac{v_4}{6}\epsilon^{\frac{3}{2}} \left(\left(\frac{2}{v_3} \right)^{\frac{1}{2}} + a\epsilon^{\frac{1}{2}} \right)^3 \; = \; \epsilon.
$$

Ignoring the terms of order higher than $\frac{1}{2}$, we obtain

$$
a \; = \; \frac{v_4}{3 \; v_3^2}.
$$

Then, the approximation for the target level of cash holdings is given by

$$
\begin{aligned}
C^* &\approx C_1^* \sqrt{\epsilon} + C_2^* \epsilon, \\
C_1 = &\left(\frac{2}{v_3}\right)^{\frac{1}{2}}, \\
C_2 = &\frac{v_4}{3\ v_3^2}.
\end{aligned}
$$

We also calculate the approximation for $z(c)$, as follows:

$$
z(c) \approx\ z(C^*)\ +\ z'(C^*)(c - C^*) +\ \frac{z''(C^*)}{2}(c - C^*)^2\ + \frac{z'''(C^*)}{6}(c - C^*)^3.
$$

To ease the notation, we define the auxiliary quantity $\phi^* \equiv \phi\,(1-\alpha)\,\frac{L}{\zeta}$. The derivatives of $z(c)$ at C^* are given by

$$
\begin{aligned}
z'(c) &= \phi^* \left(-1 + \frac{1}{v'(c)}\right) - \frac{\phi^*\ v''(c)\ (\lambda w^* - v(c) + c)}{(v'(c))^2}, \\
z''(c) &= \frac{\phi^* v''(c)}{v'(c)}\ - \frac{\phi^*\,(2v''(c) + v'''(c)\,(\lambda w^* - v(c) + c))}{(v'(c))^2}\ +\ \frac{2\phi^*\ (v''(c))^2\ (\lambda w^* - v(c) + c)}{(v'(c))^3}, \\
z'''(c) &= \frac{2\phi^* v'''(c)}{v'(c)}\ -\ \frac{\phi^*}{(v'(c))^2}\left(3v''(c)^2 + 3v'''(c) + v^{(4)}(c)(\lambda w^* - v(c) + c)\right)\ + \\
&\quad + \frac{6\phi^*}{(v'(c))^3}\left(v''(c)^2 + v''(c)v'''(c)(\lambda w^* - v(c) + c)\right)\ -\ \frac{6\phi^*}{(v'(c))^4} v''(c)^3(\lambda w^* - v(c) + c).
\end{aligned}
$$

So,

$$
\begin{aligned}
z(C^*) &=\ z_0 =\ \phi^*(\lambda - 1)w^*, & z'(C^*) &= z_1 =\ 0, \\
z''(C^*) &=\ z_2\ =\ -\phi^*\ v_3\ (\lambda - 1)w^*, & z'''(C^*) &=\ z_3\ =\ -\phi^*\ \left(v_3\ + v_4(\lambda - 1)w^*\right),
\end{aligned}
$$

and the resulting approximation for $z(c)$ is therefore

$$
z(c) \approx z_0 +\ \frac{z_2}{2}(c - C^*)^2\ + \frac{z_3}{6}(c - C^*)^3. \tag{51}
$$

Finally, the approximation for $X(c)$ is

$$
X(c) \approx\ X(C^*) +\ X'(C^*)(c - C^*)\ + \frac{X''(C^*)}{2}(c - C^*)^2,
$$

where $X^{(i)}(C^*)$ are calculated as above. Then, we have

$$
X(c) \approx\ X_0 +\ X_1(c - C^*)\ + \frac{X_2}{2}(c - C^*)^2.
$$

A.3 The cross-sectional distribution of liquidity

For any incumbent firm, the dynamics of scaled cash holdings satisfy

$$dC_t \;=\; \mu(C_t)dt \;+\; \sigma(C_t)dZ_t \;+\; dF_t \;+\; dF_t^I -\; dD_t \;+\; (\lambda C^* - c)dN_t$$

where N_t is a Cox process with stochastic intensity $\phi\, z(C_t)$. Since cash is i.i.d. across firms, the cross-sectional distribution of firms will satisfy the Kolmogorov Forward equation

$$\frac{1}{2}\left(\sigma^2(c)\eta(c)\right)'' - (\mu(c)\eta(c))' \;- x_d\eta(c) \;-\; z(c)\phi\eta(c) \;=\; 0\,.$$

By calculations, it follows:

$$\begin{aligned}(\mu(c)\eta(c))' &= \mu'(c)\eta(c) + \;\mu(c)\eta'(c)\\ \left(\sigma^2(c)\eta(c)\right)' &= 2\sigma(c)\sigma'(c)\eta(c) + \sigma^2(c)\eta'(c)\\ \left(\sigma^2(c)\eta(c)\right)'' &= 2(\sigma'(c))^2\eta(c) + 2\sigma(c)\sigma''(c)\eta(c) + 4\sigma(c)\sigma'(c)\eta'(c) + \sigma^2(c)\eta''(c).\end{aligned}$$

Now, we need to determine the boundary conditions. By assumption, firms never vanish and are only replaced by new entrants. So the mass always stays constant, equal to 1. At zero, the reflection boundary condition implies that the equality

$$0.5(\sigma^2(c)\eta(c))'(0) - (\mu(0)\eta(0)) \;=\; 0$$

holds. With reflection at C^*, and with innovating firms jumping to C^*, we will also have an additional term

$$\frac{1}{2}\left(\sigma^2(c)\eta(c)\right)'' - (\mu(c)\eta(c))' \;- x_d\eta(c) \;-\; z(c)\phi\eta(c) \;=\; 0\,.$$

By integrating, it follows that

$$\begin{aligned}0 \;&=\; \int_0^{C^*}\left(\frac{1}{2}\left(\sigma^2(c)\eta(c)\right)'' - (\mu(c)\eta(c))' \;-\; x_d\eta(c)\right)dc \;-\; \int_0^{C^*}\phi z(c)\eta(c)dc\\ &=\; \left(\frac{1}{2}(\sigma^2\eta)'(C^*) - (\mu(C^*)\eta(C^*))\right) - \left(\frac{1}{2}(\sigma^2\eta)'(0) - (\mu(0)\eta(0))\right)\\ &\quad - x_d\int_0^{C^*}\eta(c)dc \;-\; \int_0^{C^*}\phi z(c)\eta(c)dc\,.\end{aligned} \tag{52}$$

Thus, mass conservation

$$\int_0^{C^*}\eta(c)dc \;=\; 1$$

is equivalent to

$$\frac{1}{2}(\sigma^2\eta)'(C^*) - (\mu(C^*)\eta(C^*)) \;-\; \int_0^{C^*}\phi z(c)\eta(c)dc \;-\; x_d \;=\; 0\,,$$

or equivalently,

$$\Big(\sigma'(C^*)\sigma(C^*) - \mu(C^*)\Big)\eta(C^*) + 0.5\sigma^2(C^*)\eta'(C^*) \; = \; \Psi(\epsilon). \tag{53}$$

In this equation, we have defined

$$\Psi(\epsilon) \; \equiv \; \int_0^{C^*} \phi z(c;\epsilon)\eta(c;\epsilon)dc \; + \; x_d$$

Having derived the cross-sectional distribution of liquidity, which we solve numerically, we turn to analyze the general equilibrium properties of the model.

A.4 General equilibrium analysis

Using the notation introduced in Section 4.4, we derive the scaled value of the dynasty $y(c)$, the equilibrium net dividends, labor wage, and aggregate consumption. After a technological breakthrough at time t, outside investors inject the amount $q_{t-}(j)(\lambda C^* - c)$ of liquidity into the firm. As compensation, they receive a fraction of the surplus, $\alpha q_{t-}(j)(\lambda v(C^*) - v(c))$ in shares of the firm. Note that compensation cannot be in cash because this would mean they had to inject more than C^* and would then receive their own cash back immediately. Thus, after this transaction, the cash buffer rises to $q_{t-}(j)\lambda C^*$ whereas the value of the firm changes from $q_{t-}(j)v(c)$ to $q_{t-}(j)(v(c) + (1-\alpha)(\lambda v(C^*) - v(c)))$. However, the effect of bargaining is a pure share dilution, and has no effect on the outside liquidity in the economy.

The scaled value of the dynasty is given by the difference between the expected present value of the dividends paid (denoted by the function $D(c)$) and the expected present value of the financing received by all of the firms that will ever operate in the dynasty j. That is, $y(c) = D(c) - F(c)$. In particular, the function $D(c)$ satisfies

$$\begin{aligned}&\frac{1}{2}\sigma^2(c)D''(c) \; + \; \mu(c)D'(c) \; + \; \phi z(c)\Big(\lambda D(C^*) - D(c)\Big) + \\ &+ x_d\Bigg(c + \Lambda D(C^*) - D(c)\Bigg) \; = \; rD(c) \; , \; D'(0) = 0, \; D'(C^*) = 1 \, ,\end{aligned}$$

whereas the function $F(c)$ satisfies

$$\begin{aligned}&\frac{1}{2}\sigma^2(c)F''(c) \; + \; \mu(c)F'(c) \; + \; \phi z(c)\Big(\lambda F(C^*) - F(c) + (\lambda C^* - c)\Big) + \\ &+ x_d\Bigg(\Lambda F(C^*) - F(c) + \Lambda\left(C^* + \kappa_T\right)\Bigg) \; = \; rF(c) \; , \; F'(0) = -1, \; F'(C^*) = 0 \, .\end{aligned}$$

Taking the difference, it follows that $y(c)$ satisfies[32]

$$\frac{1}{2}\sigma^2(c)y''(c) \;+\; \mu(c)y'(c) \;+\; \phi z(c)\Big(\lambda(y(C^*)-C^*)-y(c)+c\Big)+$$
$$+\,x_d\left(\Lambda y(C^*)-y(c)+c-\Lambda\left(C^*+\kappa_T\right)\right) \;=\; ry(c)$$

with

$$y'(C^*)=1=y'(0)\,.$$

The present value of dividends net of financing is then given by

$$\int_0^\infty e^{-rt}(d_t-f_t)dt \;=\; \int_0^\infty e^{-rt}(d_0-f_0)e^{gt}dt$$

Since we normalize $q_0(j)=1$ for all j, it follows:

$$\frac{d_0-f_0}{r-g} \;=\; \int_0^\infty e^{-rt}(d_t-f_t)dt \;=\; \int Y(q_0(i),c(i))di \;=\; L\int \eta(c)y(c)dc\,.$$

Turning to the wage, straightforward calculations deliver

$$W_t \;=\; \frac{1}{1-\beta}\int_0^1 L^\beta\,\bar{X}_t(j)^{1-\beta}\,q_t(j)^\beta dj \;-\; \int_0^1 p_t(j)\bar{X}_t(j)dj\,.$$

By the results in Section 4, the term $\bar{X}_t(j)^{1-\beta}$ in the above equation is given by

$$\bar{X}_t(j)^{1-\beta} \;=\; q_t(j)\,(p(c(j))^{-1/\beta}L)^{1-\beta}\;,\;\bar{X}_t(j)\;=\;(p(c(j)))^{-1/\beta}\,q_t\,L.$$

Moreover, as discussed in the main text, the following relation

$$W_t \;=\; W_0\int q_t(j)dj$$

holds. In this equation, the expression for W_0 is given by

$$W_0 \;=\; \frac{L}{1-\beta}\int \eta(c)p(c)^{-(1-\beta)/\beta}dc - L\int \eta(c)p(c)^{1-1/\beta}dc\,,$$

[32]For completeness, we also define the dynasty in the frictionless benchmark, denoted by y^* and satisfying the following relation

$$\mu-\frac{(z^*)^2}{2}\frac{\zeta}{L}-x_d^*\kappa_T\Lambda=\Big(r^*-x_d^*(\Lambda-1)-\phi z^*\left(\lambda-1\right)\Big)y^*$$

Therefore, it follows that

$$d_0^*-f_0^*=L\left(\mu-\frac{(z^*)^2}{2}\frac{\zeta}{L}-x_d^*\kappa_T\Lambda\right)=L(r^*-g^*)y^*.$$

that is equivalent to

$$W_0 \;=\; \frac{\beta L}{1-\beta}\int \eta(c)X(c)^{1-\beta}dc\,.$$

Furthermore,

$$\int q_t(j)dj \;=\; e^{gt}.$$

Then, $\mathcal{C}_t \;=\; \mathcal{C}_0 e^{gt}$, with

$$\mathcal{C}_0 \;=\; W_0 + (d_0 - f_0)\;.$$

Finally, total welfare is given by the utility of the representative consumer, i.e.

$$\int_0^\infty e^{-\rho t}\,\frac{\mathcal{C}_t^{1-\theta}-1}{1-\theta}\,dt = \frac{1}{\theta-1}\left(\frac{1}{\rho}-\frac{\mathcal{C}_0^{1-\theta}}{\rho+g(\theta-1)}\right).$$

A.5 Government debt and the liquidity premium

The wedge between the market interest rate and the return on cash (i.e., the opportunity cost $r-\delta$) plays a major role in determining the impact of financing frictions on corporate decisions. It affects the target level of cash reserves and, thus, the firms' financial resilience. In the extant cash literature, this wedge is typically attributed to agency frictions (the free cash flow problem) or to tax disadvantages.[33] In this section, we propose a general equilibrium channel to endogenize the wedge $r-\delta$, which is based on the interplay between the demand (from the corporate sector) and the supply (from the government) of liquidity. We follow Woodford (1990) and assume that liquidity can only be stored by holding government bonds.[34] When the supply by the government is scarce, bonds trade at a premium that results in a positive wedge $r-\delta$.

As in previous contributions, we assume that the government faces an exogenous expenditure stream. We denote this expenditure stream by $\mathcal{G}_t$ and assume that it constitutes a fixed fraction $\mathcal{G} \in [0,1]$ of the final-good output, i.e. $\mathcal{G}_t = \mathcal{G}\mathcal{Y}_t$. To finance these expenditures, the government levies income taxes and issues public debt. We denote the amount of taxes collected at time t by T_t. We assume that taxes represent a fraction τ of wages, i.e. $T_t = \tau W_t L$. Moreover, we denote the public debt outstanding at time t by B_t. We assume that public debt promises a return of δ, which is endogenously determined in equilibrium. Under these assumptions, the budget constraint of the government is given by

$$\mathcal{G}_t + \delta B_{t-1} = \tau W_t + B_t \tag{54}$$

at any time t. The left-hand side of this equation denotes the government outflow, i.e., the expenditure stream plus the repayment of the maturing debt. The right-hand side denotes the government inflow, i.e., the tax inflow plus the newly issued debt.

We assume that public debt grows at the equilibrium rate g : $B_t = gB_{t-1}$, so the debt-to-output ratio is constant. Under this assumption, if the budget constraint of the government

[33] See also Zucchi (2015), who shows how this wedge may arise endogenously due to market illiquidity.

[34] See also Holmstrom and Tirole (1998), Bansal, Coleman, and Lundblad (2011), Krishnamurthy and Vissing-Jorgensen (2012, 2013), Greenwood, Hanson, and Stein (2015), and Nagel (2014).

holds at time zero,

$$\mathcal{G}\mathcal{Y}_0 + \delta B_0 = \tau W_0 + g B_0,$$

it does hold at any t. In this equation, $\mathcal{Y}_0$ denotes the final-good output at time zero, which is given by

$$\mathcal{Y}_0 = \frac{L}{1-\beta}\int_0^{C^*} X(c)^{1-\beta}\eta(c)dc\,.$$

The equilibrium return δ^* makes the government debt market clear, i.e.,

$$L\int_0^{C^*(\delta)} c\,\eta(\delta, c)\,dc = B_0\,.$$

That is, the demand for liquidity in the economy equals the supply of bonds issued by the government.[35] Fixing the supply B_0 amounts to determining the equilibrium tax rate $\tau(B_0)$ from the government budget constraint, which is then given by

$$\tau(B_0) \;=\; \frac{\mathcal{G}\mathcal{Y}_0(\delta^*) - B_0(g(\delta^*) - \delta^*)}{W_0(\delta^*)}\,.$$

In the absence of liquidity frictions, government bonds trade at the market rate r and have no impact on the dynamics of the economy. By contrast, the supply of government bonds has real effects in the presence of liquidity frictions. By affecting the liquidity premium, debt supply influences corporate policies. In particular, as B_0 increases, liquidity frictions vanish, and the economy converges with the frictionless one. We numerically analyze the interplay between liquidity demand and supply in the next section.

We give a quantitative assessment of the results. We use again the benchmark parametrization, and in addition we set $\mathcal{G} = 10\%$.[36] Consistent with Bansal, Coleman, and Lundblad (2011) and Krishnamurthy and Vissing-Jorgensen (2012), Figure 6 shows that an increase in government debt B_0 leads to an increase in δ. When the supply of liquidity is large, the liquidity premium $r-\delta$ narrows. At the same time, a larger δ makes debt more costly for the government, which results in a higher tax rate τ. As a result, higher levels of government debt imply higher taxes, as in Woodford (1990).

Figure 6 also shows that a larger B_0 is associated to a larger rate of economic growth (and to a larger rate of creative destruction, not displayed in the figure). That is, when firms are financially constrained, policies that increase liquidity in the economy might effectively enhance growth by stimulating the incumbents' and entrants' innovation rate. The mechanism is the following. An increase in B_0 decreases the cost of holding cash and thus leads to an increase in the target level of cash C^* (Figure 7, right panel). As cash is the resource used to finance R&D investment, the incumbents' innovation rate also increases in B_0 (see Figure 7, left panel). The value of incumbent firms then increases, which makes it more attractive for new firms to enter the industry. As a result, the rate of creative destruction increases. Although larger liquidity supply leads to a larger tax rate, in unreported results we find that an increase in B_0 can increase aggregate consumption and welfare via the positive effect on firms' innovation and on economic growth.

[35] Note that the representative agent does not hold government bonds because $\delta < r$.

[36] The equilibrium tax rate τ is uniquely pinned down by the government's budget constraint.

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Table 1: Baseline parametrization.

Symbol	Description	Value
ρ	Discount rate	1.5%
θ	Risk aversion	2.00
σ	Cash-flow volatility coefficient	0.40
ϕ	Innovation rate coefficient	3.00
ζ	Innovation cost coefficient	1.00
β	Elasticity of substitution	0.25
δ	Return on cash	4.9%
ϵ	Proportional refinancing cost	0.08
α	Surplus haircut	0.06
λ	Quality improvement (incumbents)	1.04
Λ	Quality improvement (entrants)	1.10
κ_T	Entry cost (technological component)	0.70
κ_I	Entry cost (financing component)	0.07
L	Labor supply	1.00

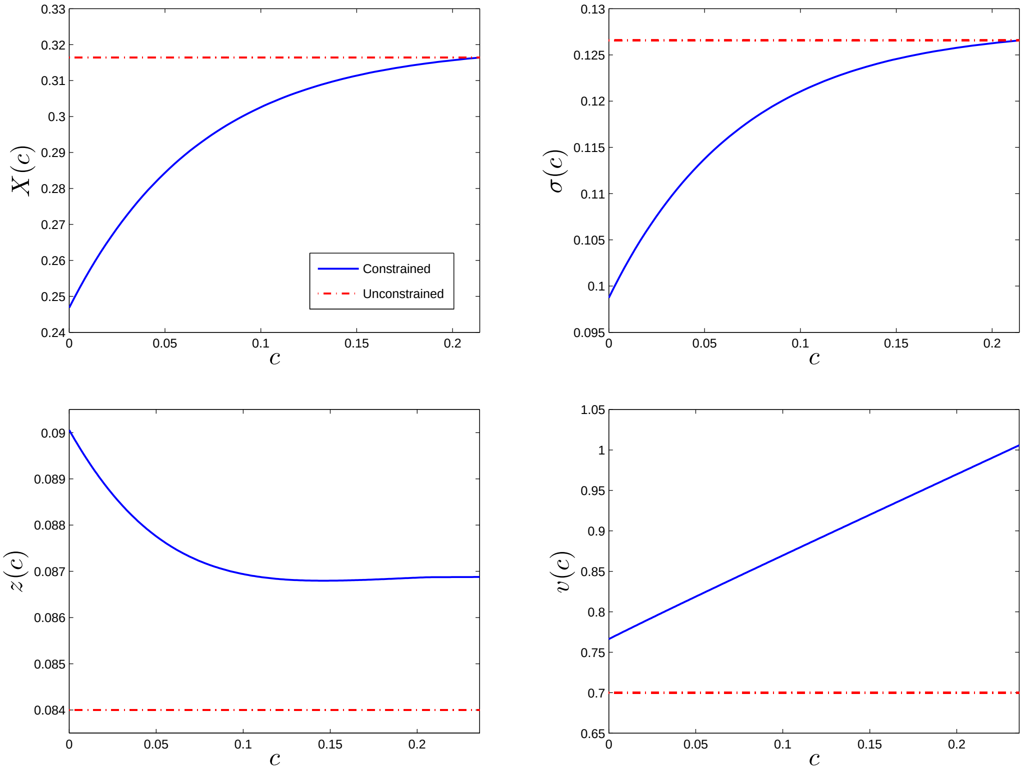

Figure 1: Corporate policies.

The figure illustrates the optimal production quantity $X(c)$, the volatility of cash flows $\sigma(c)$, the innovation rate $z(c)$, and the scaled value of an incumbent firm $v(c)$ as a function of the cash reserves $c \in [0, C^*]$, under the baseline parametrization. The red line represents the benchmark unconstrained economy, whereas the blue line represents the constrained economy.

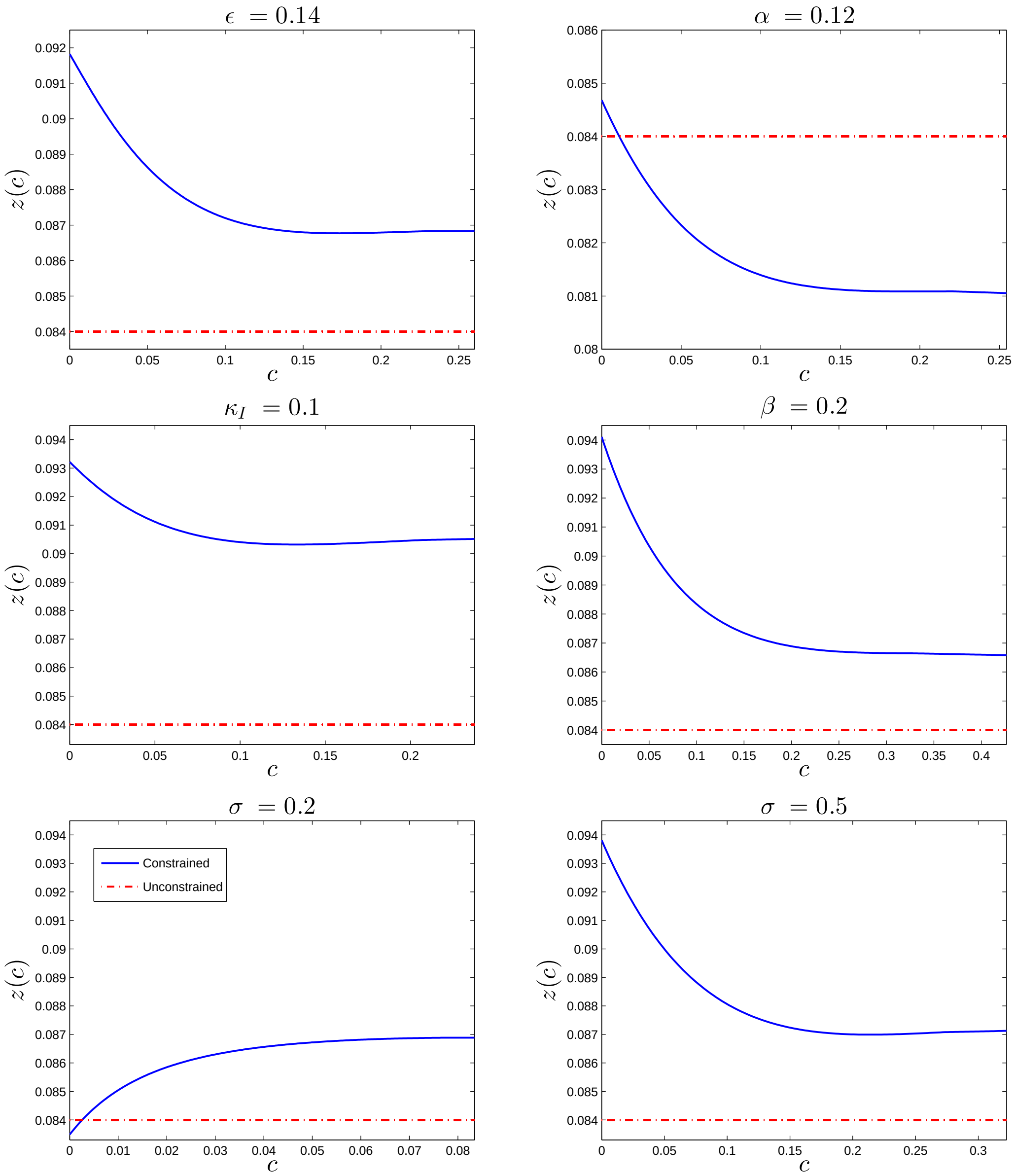

Figure 2: Innovation, Financing frictions, and Firm characteristics.

The figure illustrates the endogenous innovation rate $z(c)$ as a function of the cash reserves $c \in [0, C^*]$ when varying the financing costs ϵ, α, and κ_I, the elasticity β, and the coefficient of cash flow volatility σ. The red line represents the benchmark unconstrained economy, whereas the blue line represents the constrained economy.

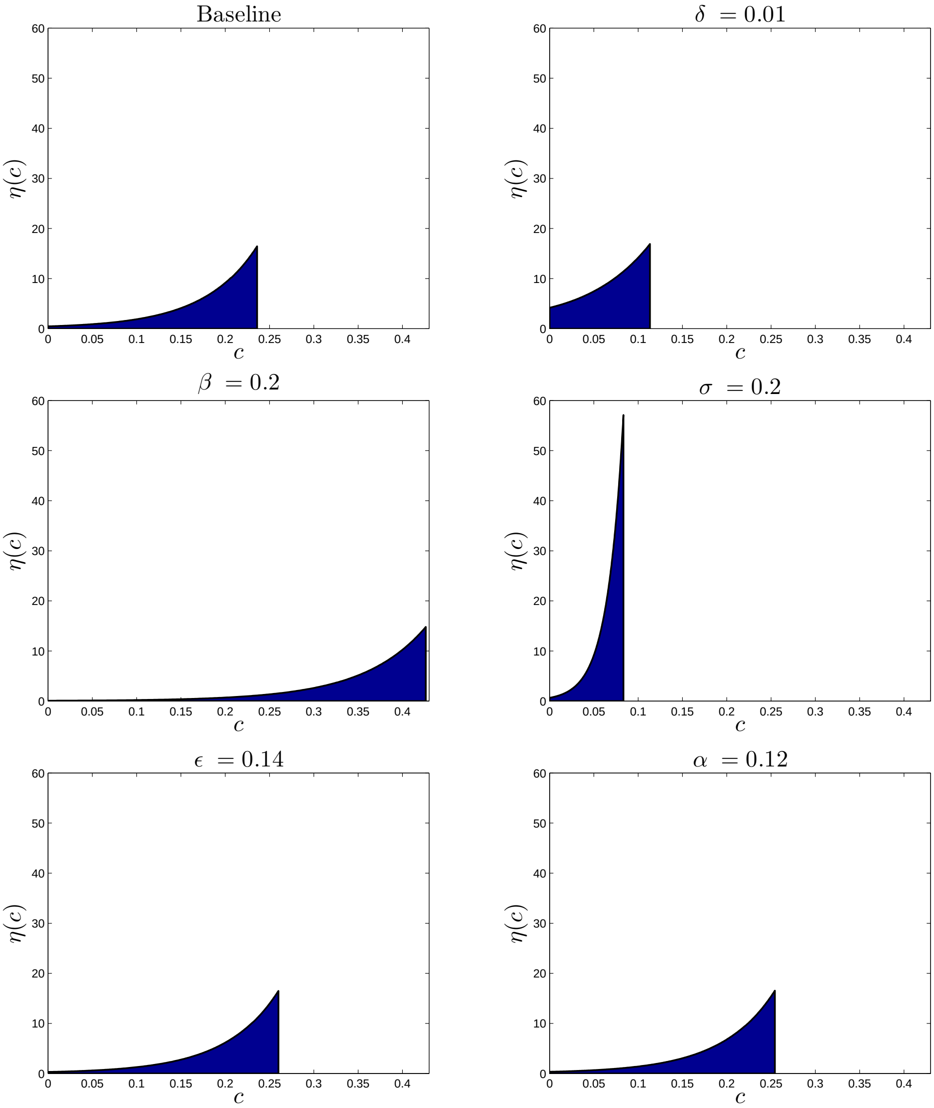

Figure 3: Stationary distribution of liquidity.

The figure illustrates the stationary cross-sectional distribution of liquidity $\eta(c)$ as a function of $c \in [0, C^*]$ in the baseline parametrization and when varying the return on cash δ, the elasticity β, the coefficient of cash flow volatility σ, and the financing cost ϵ and α.

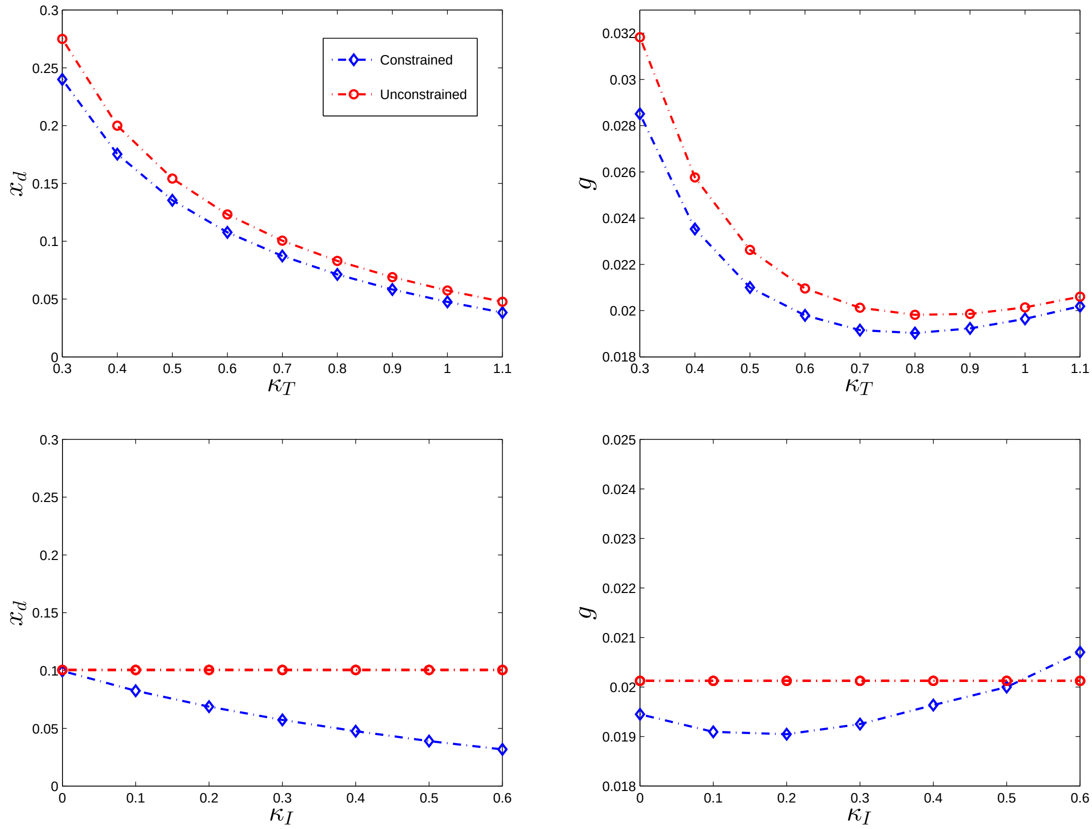

Figure 4: Creative destruction and Growth.

The figure shows the equilibrium rate of creative destruction x_d and the equilibrium growth rate g as a function of the technological κ_T (top panel) and financing κ_I (bottom panel) components of the entry cost. In the top panels, we vary κ_T while setting κ_I to be 10% of κ_T. In the bottom panels, we vary κ_I while keeping κ_T as in the baseline. The red line represents the benchmark unconstrained economy, while the blue line represents the constrained economy.

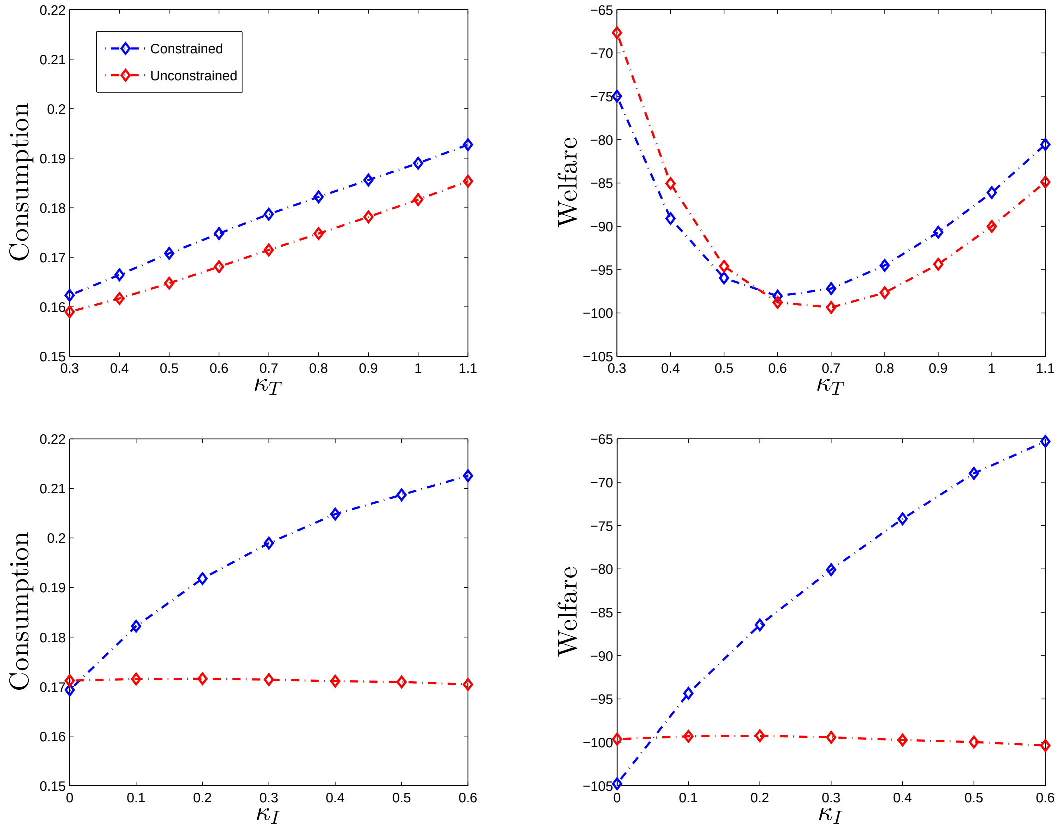

Figure 5: AGGREGATE CONSUMPTION AND WELFARE.

The figure shows aggregate consumption and welfare as a function of the technological κ_T (top panel) and financing κ_I (bottom panel) components of the entry cost. In the top panels, we vary κ_T while setting κ_I to be 10% of κ_T. In the bottom panels, we vary κ_I while keeping κ_T as in the baseline. The red line represents the benchmark unconstrained economy, while the blue line represents the constrained economy.

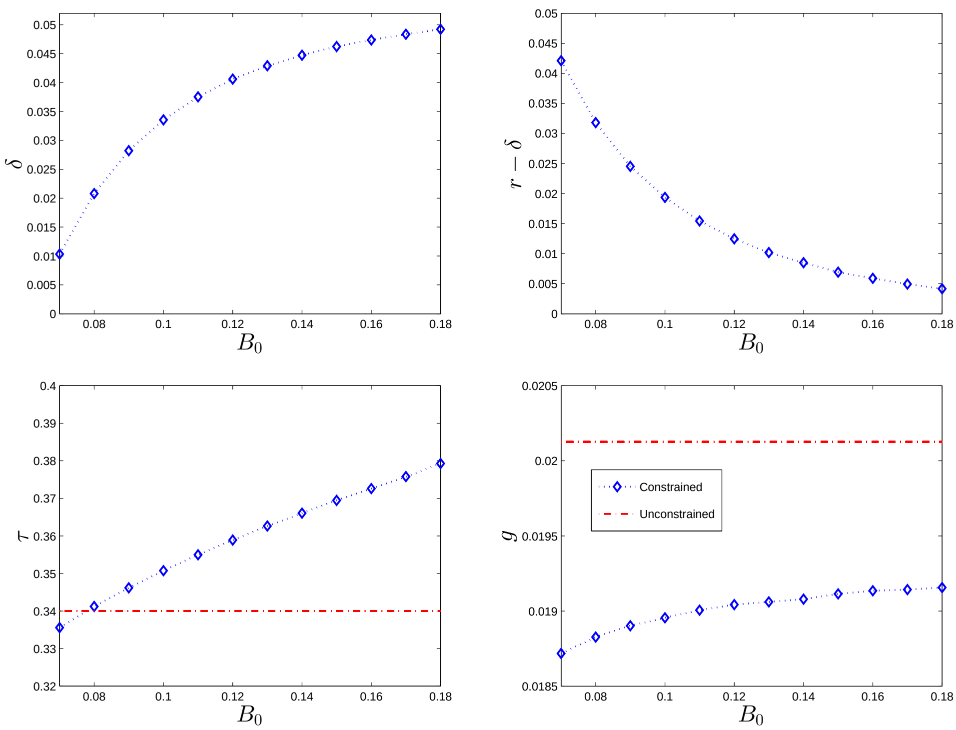

Figure 6: LIQUIDITY SUPPLY AND EQUILIBRIUM QUANTITIES.

The figure shows the equilibrium return on government debt δ, the liquidity premium $r - \delta$, the tax rate τ, and the economic growth rate g as a function of liquidity supply B_0. The red line represents the benchmark unconstrained economy, while the blue line represents the constrained economy.

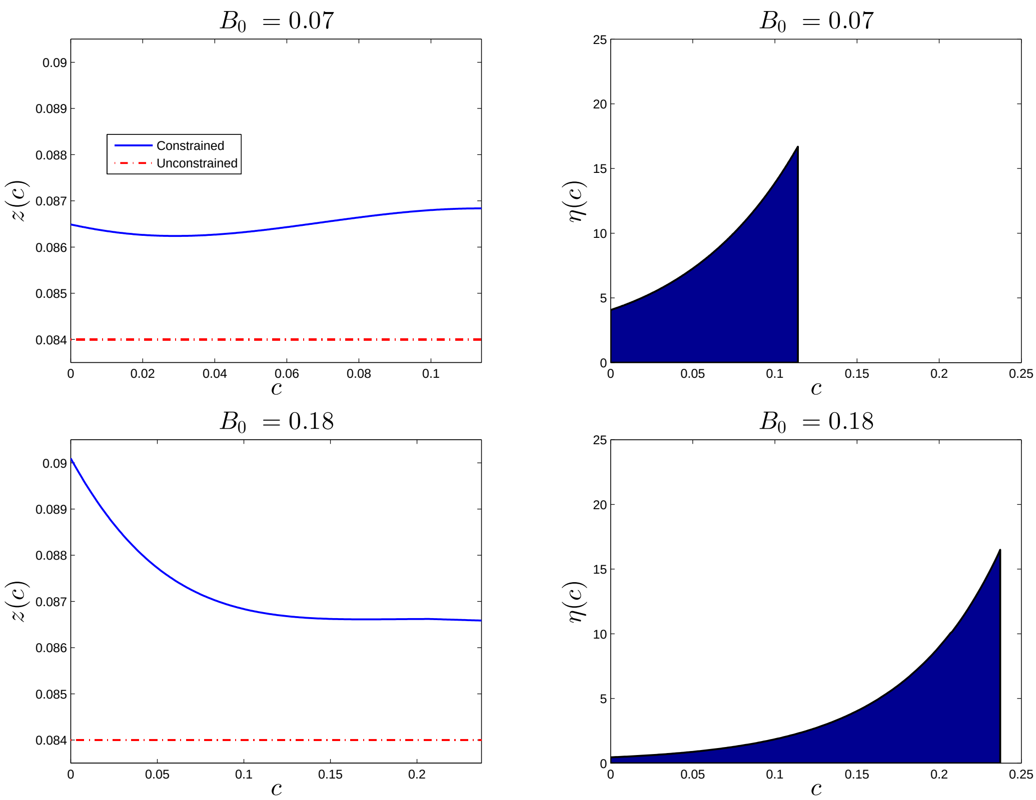

Figure 7: Liquidity supply and innovation.

The figure illustrates the innovation rate $z(c)$ (left panel) and the stationary cross-sectional distribution of liquidity $\eta(c)$ (right panel) as a function of $c \in [0, C^*]$ for low supply of liquidity ($B_0 = 0.07$, top panel) and high supply of liquidity ($B_0 = 0.18$, bottom panel). The red line represents the benchmark unconstrained economy, whereas the blue line represents the constrained economy.

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The Fellowship programme is named after Baron Alexandre Lamfalussy, the first President of the European Monetary Institute. Mr Lamfalussy is one of the leading central bankers of his time and one of the main supporters of a single capital market within the European Union.

Each year the programme sponsors five young scholars conducting a research project in the priority areas of the Network. The Lamfalussy Fellows and their projects are chosen by a selection committee composed of Eurosystem experts and academic scholars. Further information about the Network can be found at http://www.eufinancial-system.org and about the Fellowship programme under the menu point "fellowships".

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