EUROPEAN CENTRAL BANK

WORKING PAPER SERIES
NO. 542 / NOVEMBER 2005

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A FRAMEWORK OF ANALYSIS

by Livio Stracca

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ISSN 1561-0810 (print)
ISSN 1725-2806 (online)

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Abstract

This paper proposes a general equilibrium model with heterogeneous households and a financial market where each financial instrument provides liquidity services in addition to enabling a transfer of purchasing power over time. Importantly, liquidity services may be asymmetric according to whether the financial instrument is held as an asset or as a liability, and are also agent-specific. The main purpose of the study is to develop an analytical framework and a language for evaluating the effect of (broadly defined) liquidity factors on equilibrium rates of return and intertemporal allocation.

Keywords: Real interest rates, liquidity services, financial market, heterogeneity.
JEL codes: E40, E43

Non-technical summary

Traditional asset pricing models have emphasized the role of financial markets in allowing agents to transfer purchasing power over time and across states of the world. Asset pricing models based on the assumption that the only function of the financial markets is to transfer purchasing power over time and across states of the world certainly represent a useful benchmark, but overlook a number of additional benefits and costs which participation in financial markets entails and which are highly relevant in reality.

These additional elements may include "objective" factors such as the possibility of using financial assets to purchase goods and services or, in a lending relationship, the need of undertaking a monitoring activity on the creditworthiness of the recipient of the funds in a context of asymmetric information.

Moreover, "subjective" factors related to the extent to which the holding of certain assets and liabilities has an impact on agents' psychological wellbeing may also play an important role, as emphasized long ago by Keynes and, more recently, by behavioural economists.

The main purpose of the paper is to provide a systematic framework and a simple but rigorous language for evaluating the impact of these factors on equilibrium asset prices and rates of return. The paper, in particular, uses the concept of "liquidity" developed in the literature on monetary aggregation, and extends its meaning to *all* features of financial assets *and liabilities* which do *not* relate to the intertemporal transfer of purchasing power over time and across states of the world. It is notable that liquidity is seen as a concept not only applying to money (as in the literature on monetary aggregation), but also (and perhaps even more so) to *credit*.

One of the key applications of the proposed framework is the analysis of the pricing of "liquidity" in equilibrium. In the traditional, Keynesian theory of the "own rate of money interest" the monetary rate of interest can be defined (simplifying) as:

$$r = rr - l + s,$$

where *r* is the real interest rate on a financial instrument, *rr* is the real interest rate on a risk-free non-liquid asset, *l* is liquidity (inclusive of carrying costs) and *s* is a measure of risk. Therefore, in this framework an *improved* liquidity of financial instruments leads necessarily to a *fall* in equilibrium real interest rates, everything else remaining unchanged. This view is reflected in the idea that low liquidity, for example due to high cost of financial intermediation, must increase the return required by savers and hence real rates of return.

In the analytical framework proposed in this paper, instead, evaluating the impact of liquidity on equilibrium real interest rates is a more complex matter on account of the fact that money *and credit* can have liquidity services which have to be priced in equilibrium. In particular, the important element is the extent to which the liquidity services of any given financial instrument (seen both as an asset and as a liability) impact on the *excess* (rather than *gross*) demand for it. For example, a financial instrument which is very liquid for asset holders but which is even more liquid for liabilities holders will generally have a *higher* rate of return compared with the benchmark liquidity-free asset (namely, a *negative* user cost).

There are also other possible problems to which the proposed framework can be usefully applied. In the paper, the following are touched upon:

- Joint general equilibrium modelling of (endogenous) money and credit;
- Explain how a financial market comes to existence following financial innovation and liberalisation;
- Simultaneous holding of assets and liabilities by the same agent;
- Explain the contemporaneous increase in financial market liquidity and higher real returns.

A weak financial system – reflecting an underperforming banking system, poor investment protection and corporate governance, or fragile securities markets – yields a high cost of financial intermediation. For any given return on an investment project, savers' net return is lowered by the high cost of financial intermediation.

R.G. Hubbard, *The Wall Street Journal Europe*, 23 June 2005.

1 Introduction

Traditional asset pricing models have emphasized the role of financial markets in allowing agents to transfer purchasing power over time and across states of the world. Equilibrium asset prices and rates of return are typically derived in these models as those which ensure that a representative agent has achieved the optimal intertemporal allocation of consumption given the constraint represented by his lifetime income. As a result of this simplification, in these models a number of imperfections and frictions which characterize the actual functioning of financial markets are not explicitly considered.

Asset pricing models based on the assumption that the only function of the financial markets is to transfer purchasing power over time and across states of the world certainly represent a useful benchmark, but overlook a number of additional benefits and costs which participation in financial markets entails and which are highly relevant in reality. These additional elements may include "objective" factors such as the possibility of using financial assets to purchase goods and services or, in a lending relationship, the need of undertaking a monitoring activity on the creditworthiness of the recipient of the funds in a context of asymmetric information. Moreover, "subjective" factors related to the extent to which the holding of certain assets and liabilities has an impact on agents' psychological wellbeing may also play an important role, as emphasized long ago by Keynes (Keynes, 1936[1]) and, more recently, by behavioral economists (Thaler, 1990).

Providing a systematic framework for evaluating the impact of these factors on equilibrium asset prices and rates of return is the main purpose of this paper. The model aims at providing a general framework and a language useful for thinking about this type of issues in a systematic manner, and it is not the primary aim of the analysis to provide explicit analytical results in terms of closed form solutions.

The paper is organized as follows. Section 2 presents the set-up of the model and the optimization by individual households. Section 3 describes the equilibrium in the financial market. Section 4 illustrates an extension of the analysis to include government policy, notably the supply of outside money by the central bank. Section 5 deals with the pricing of liquidity services in equilibrium. Some additional problems to which the framework of analysis can be usefully applied are described in Section 6. Section 7 concludes.

2 The model

2.1 Basic features

In the same way as in standard intertemporal models, we derive equilibrium real rates of return in a model where agents solve an explicit dynamic optimization and goods and financial markets clear. However, unlike in standard models, we assume that the decision of holding financial assets and liabilities entails benefits and costs on its own, which contribute to agents' utility. This work is closely related to two thus far quite separate strands of literature. On the one hand, the model is a broad generalization of the asset pricing models with money in the utility function, for example as in Bakshi and Chen (1996).[2] On the other hand, in order to model the overall effect of the additional factors discussed above on asset returns, we make use of the concept of "liquidity services", drawing it from the theory of monetary aggregation pioneered by Barnett (1980). Consistent with the definition given by Barnett, in this paper liquidity services are assumed to encompass *all* benefits and costs implied by holding a financial instrument with the *only* exclusion of the possibility of transferring purchasing power over time.

Goods prices are completely flexible in this economy. In line with conventional assumptions, each household maximizes its lifetime utility subject to the usual budget constraint. Furthermore, for simplicity of exposition it is assumed that households are risk neutral. This allows to abstract from risk considerations in the description of the model, which are not central to the core issues being analyzed.

Another important feature of the model is that agents are assumed to be heterogeneous along a number of dimensions, including their appreciation of the liquidity services provided by different financial assets. So, the analysis does not hinge on the restrictive and implausible assumptions needed to consider exact linear aggregation across agents and a single homogeneous representative agent, as emphasized notably by Kirman (1992).[3] An important advantage of allowing for heterogeneity is that the model is able to determine both required returns and quantities of financial securities traded in the market. Moreover, we assume that the same financial instrument entails different liquidity services according to whether it is held as a financial asset or a financial liability. This is a highly realistic feature of the functioning of financial markets and also allows to justify agents' decision to hold assets and liabilities simultaneously (Greenwald and Stiglitz, 2003).

There is a long tradition especially in financial economics in linking the concept of liquidity to the degree of asymmetric information between borrowers and lenders in financial and credit markets. For example, it has been emphasized that the liquidity of the market is inversely related to the number of privately informed traders and adverse selection problems (Bagehot, 1971). Notably, adverse selection may increase in a financial crisis, leading to a disruption of liquidity. In the model we assume that all financial liabilities are repaid, and default is not possible.[4] However, we can also assume that establishing the creditworthiness of the recipient of credit (namely the holder of a financial liability) may be costly both from the lending side, for example due to screening and monitoring costs, and from the borrowing side, for example due to signalling costs (Greenwald and Stiglitz, 2003). Personal characteristics and situations also matter here. For instance, we may assume that an agent with low net worth (in terms of financial and human capital) might have comparatively more difficulties in signalling his creditworthiness, making financial liabilities relatively less liquid for him.

In addition to these objective factors, many benefits and costs associated with the holding of financial assets and liabilities may be related to their *subjective* impact on agents' psychological wellbeing. For example, having a fat bank account or experience stock market gains can give a sense of security and satisfaction to agents in itself, i.e. in addition to the effect on consumption. Running on debt, conversely, may cause anxiety independent of the probability that the debt may, or may not, be paid back (debt aversion). These factors have long been emphasized in the literature on saving (Browning and Lusardi, 1996) and have also received some attention by behavioral economists such as Thaler (1990). According to Thaler, agents may frame certain financial assets and liabilities into separate mental accounts, which implies that different types of financial wealth are more or less convertible into transaction balances. Moreover, certain features of financial assets might help households to solve self-control problems of the type described by Laibson (1997).[5]

It should be also emphasized that this study focuses on the macroeconomic equilibrium and the determination of the equilibrium interest rates in the economy. Thus, it departs from the focus traditionally maintained in the finance literature on the role of liquidity factors (such as transaction costs) in the pricing of individual asset prices (Brennan and Subrahmanyam, 1996; Pastor and Stambaugh, 2003). Moreover, the definition of liquidity is clearly broader than that traditionally assumed in the finance literature.

2.2 Set-up of the model

Following in particular Woodford (1996), we assume that the economy consists of a continuum of infinitely lived households indexed by j in $[0,1]$. Each household specializes in the production of a single differentiated good. The continuum of differentiated goods is denoted by z in $[0,1]$, where $z=j$ is the good supplied by household j. As noted, there is no government, and the economy is closed.[6] Moreover, the accumulation of capital and in particular its role in affecting the production function is not explicitly modelled. So, we can interpret consumption as including the consumption of fixed capital goods. Firms are not modelled explicitly, are owned by households and act on their behalf (Woodford, 1996).[7]

The economy includes a goods market and a financial market; prices are perfectly flexible and set competitively in both markets. In the following, $p(z)$ denotes the price of good z, with the general price level being normalized to one. Because nominal goods prices do not play any role in determining real quantities, we will always refer to real values in the continuation of this analysis. Financial assets are exchanged in the financial market, and households can theoretically have both assets and liabilities without constraints.[8] We denote by B_t^{ij} the *real* market value, expressed in net terms, of the instrument i, with $i=1,...,n$, held by household j, with $j=1,...,m$, where $B_t^{ij}>0$ indicates a net asset, and $B_t^{ij}<0$ a net liability. It should be noted that the index i refers to both the technical characteristics of the financial instrument and the identity of the borrower (in the case of financial assets) or the lender (in the case of financial liabilities). So, for example a financial asset i_1 might be a "bank loan to Mr. X", or a financial liability i_2 might be "credit received from firm Y".

From now on, we indicate with $A_t^{ij}\geq 0$ a financial instrument held as an asset by household j, and $L_t^{ij}\geq 0$ the same financial instrument held as a liability, with $B_t^{ij}=A_t^{ij}-L_t^{ij}$. The vectors $\underline{A}_t^j$, $\underline{L}_t^j$ and $\underline{B}_t^j$ will denote respectively the full set of financial assets, liabilities and net holdings in period t by household j, and the vector with the rates of return will be denoted by $\underline{R}_t$. $\sum_{i=1}^{n}B_t^{ij}$ is the current market value of the assets and liabilities portfolio, R_t^i is the ex post market value of asset i in time t compared with time $t-1$. Assuming for simplicity that all assets are zero-coupon, this is the gross rate of return on asset i. Agents are assumed to be risk neutral. For ease of exposition, the expectational term will be omitted in the continuation as it does not play any useful role in the analysis.

For each household j, the flow of funds in each period can be written down as follows:

$$c_t^j + \sum_{i=1}^{n} B_t^{ij} = y_t^j + \sum_{i=1}^{n} B_{t-1}^{ij} R_t^i, \qquad (1)$$

or, expressed in vector notation:

$$c_t^j + \underline{1}' \underline{B}_t^j = y_t^j + \underline{R}_t' \underline{B}_{t-1}^j, \qquad (2)$$

where $c_t^j = \int_0^1 p_t(z) c_t^j(z) dz$ is the consumption of household j, including capital goods, and $y_t^j = p_t(j) x_t^j$, where x_t^j is the output produced by household j. Obviously, expected returns are *given* for an individual household, but they are determined *endogenously* in the economy as a whole, as we shall see in Section 3.

2.3 Preferences and the optimal portfolio choice of the individual household

The main novel element in the model is represented by agents' preferences. Each household j has the following instantaneous utility function:

$$U_t^j = U_t^j(c_t^j, y_t^j, \underline{A}_t^j, \underline{L}_t^j), \qquad (3)$$

where $U_c^j > 0$, $U_{cc}^j < 0$, $U_y^j < 0$, $U_{yy}^j < 0$ (in line with standard assumptions). We do not impose any restriction on the signs of $U_{\underline{A}}^j$ and $U_{\underline{L}}^j$, as we want to allow for the possibility that the holding of financial assets and liabilities may imply both net benefits and net costs at the margin, depending on the type of asset and the agent concerned. Irrespective of whether they are positive or negative, benefits or costs are characterized by diminishing marginal returns, which appears to be a plausible assumption in most situations. Hence, we impose that the Hessian matrices $U_{\underline{AA}}^j$ and $U_{\underline{LL}}^j$ are negative definite. paper that we do not impose any form of separability in the utility function; so, the holdings of a certain financial asset or liability might in principle affect the liquidity of all other assets and liabilities, and in reality they often do. For example, the liquidity benefits of having a money market fund are greatly reduced if the agent already has a large bank account surplus.

It should be noted that the production function and the market for production factors are implicitly included in the U^j function, as they determine the quantity of output that is possible to produce for a given amount of leisure and, therefore, the optimal allocation of time between work and leisure. The utility function is indexed by j, which captures the idea that households may be heterogenous in tastes and production technology. Moreover, the utility function is indexed by t, i.e. it is time-varying. This reflects the fact, for example, that the trade-off between consumption and leisure changes over time due to technical progress (which allows more production for a given amount of effort).

Households maximize a lifetime utility function with instantaneous utility given by (3). The expected lifetime utility function is defined as follows:

$$\widetilde{U}_t^j = \sum_{h=t}^{\infty} \beta_j^{h-t} U_h^j, \tag{4}$$

where $0 < \beta_j < 1$ is the discount factor of household j. Rewriting (4) taking into account the budget constraint in (2):

$$U_t^j = U_t^j(y_t^j + \underline{B}_{t-1}^j \underline{R}_t - 1' \underline{B}_t^j, y_t^j, \underline{A}_t^j, \underline{L}_t^j) \tag{5}$$

Hence, the decision problem of the agent is:

$$Max_{y_t^j, \underline{A}_t^j, \underline{L}_t^j} \widetilde{U}_t^j = \sum_{h=t}^{\infty} \beta_j^{h-t} U_h^j(y_h^j + \underline{B}_{h-1}^j \underline{R}_t - 1' \underline{B}_h^j, y_h^j, \underline{A}_h^j, \underline{L}_h^j) \tag{6}$$

with $\underline{A}_h^j, \underline{L}_h^j \geq 0$. The first order condition for $\underline{A}_t^j$ is:

$$-1' U_{c,t}^j + \beta_j \underline{R}_{t+1} U_{c,t+1}^j + U_{\underline{A}}^j = 0, \tag{7}$$

and for $\underline{L}_t^j$:

$$1' U_{c,t}^j - \beta_j \underline{R}_{t+1} U_{c,t+1}^j + U_{\underline{L}}^j = 0 \tag{8}$$

Each agent j decides his optimal portfolio of financial assets and liabilities in order to satisfy the conditions set out in (7) and (8). The only difference with a standard intertemporal model is given the liquidity terms $U_{\underline{A}}^j$ and $U_{\underline{L}}^j$. This is evident when considering a theoretical financial asset s that provides no liquidity services either as an asset or as a liability, and only guarantees an automatic transfer of purchasing power over time.[9] This asset can be interpreted as the benchmark rate of return in the literature on monetary aggregation (Barnett and Serletis, 2000).

As in any intertemporal model, the marginal utility associated to this type of asset is given by:

$$-1' U_{c,t}^j + \beta_j R_{t+1}^s U_{c,t+1}^j = 0, \tag{9}$$

where R_{t+1}^s is the hypothetical rate of return on asset s. Therefore, considering (7), (8) and (9) jointly, we obtain that our household j chooses an asset and liability allocation $\{A_t^{ij}, L_t^{ij}\}$ such that:

$$\frac{U_{A_t^{ij}}^j}{\beta_j U_{c,t+1}^j} = -\frac{U_{L_t^{ij}}^j}{\beta_j U_{c,t+1}^j} = R_{t+1}^s - R_{t+1}^i, \tag{10}$$

and this is valid for any asset i and household j. The intuition behind this expression is that the term on the right hand side of the equation, a measure of the user cost of asset (or liability) i, is a premium for the liquidity services provided by i as a financial asset or liability, scaled by the marginal utility of future consumption.

In order to derive the demand functions in terms of order flows submitted to a hypothetical auctioneer in a Walrasian market, we can write equations (7) and (8) as Marshallian demand functions of the real rates of return:

$$A_t^{ij} = A_t^{ij}(R_{t+1}^i) \tag{11}$$

Generally, we cannot say anything certain on the sign of $\frac{\partial A_t^{ij}}{\partial R_{t+1}^i}$ (see Taylor, 1999), as an increase in the rate of return can have, in this model in the same way as in any model with heterogeneous agents, two opposing effects. On the one hand, the substitution effect implies that higher rates of return translate into a stronger demand for financial assets. On the other hand, for agents having a relatively large accumulated asset position a higher rate of return on a certain asset implies a positive wealth effect on expected lifetime income, which may have the opposite impact (namely lower the demand for the asset). Overall, the net effect at the household level (let alone the aggregate economy) is left indeterminate.

In any event, expression (11) cannot be directly the order flow since this has to be *non-negative*. Hence, the order flow for asset i by agent j, $A_t^{ij,d}$, is given by:

$$A_t^{ij,d} = \max\{0, A_t^{ij}(R_{t+1}^i)\} \tag{12}$$

And similarly for i as a financial liability:

$$L_t^{ij,d} = \max\{0, L_t^{ij}(R_{t+1}^i)\}, \tag{13}$$

where, again, the sign of the derivative $\frac{\partial L_t^{ij}}{\partial R_{t+1}^i}$ is in principle indeterminate.

The order flow of agent j will hinge on the traditional determinants of *net* borrowing demand emphasized in the traditional intertemporal models. Thus, depending on his discount factor and the degree to which his income is rising or declining through the lifetime, our agent will have a certain net borrowing demand. In addition, his order flow will also be significantly influenced by the liquidity services provided by the individual financial instruments, in this case in *gross* terms, namely distinguishing between assets and liabilities. This will in turn reflect personal characteristics as well as the technological and contractual features underlying financial instruments. The end-result of the combination of all these factors is the order flow equations which are shown in (12) and (13).

It should also be emphasized that, unlike in the standard approach which neglects the existence of the liquidity terms, the possibility exists that our household does not participate in the market for certain assets (or even in financial markets at all). This may happen if, for a certain agent j, $A_t^{ij}(R_{t+1}^i) < 0$ and, at the same time, $L_t^{ij}(R_{t+1}^i) < 0$, in which case $A_t^{ij,d} = L_t^{ij,d} = 0$.

This consideration suggests that the analytical framework developed here provides a simple way to model phenomena normally referred to as *limited participation* in the financial market. In particular, this setting should generalize the traditional limited participation assumption, i.e. that some agents are for some exogenous reason excluded from taking part in the financial market altogether, which has been often maintained in the literature (see for example Fuerst, 1992). On the same token, the model endogenously derives that agents may be willing to hold both assets and liabilities in their balance sheet even in the absence of risk considerations.[10]

Having described the problem of the optimal selection of financial assets for an individual household for *given* market returns, in the next section we set out to characterize the equilibrium in the economy as a whole.

3 The equilibrium in financial markets and real equilibrium interest rates

At the aggregate level, we impose the condition that each asset is in zero net supply, because in a closed economy with no government every financial asset for a certain household is also a financial liability for another household. We also assume the existence of a Walrasian auctioneer who matches the order flows of each individual household and is able to find a price (rate of return) for each asset so that the order flows match. Therefore, the relevant market equilibrium condition for each asset i is the following:

$$\int_0^1 B_t^{ij} dj = 0 \tag{14}$$

So, the price of asset i must ensure, for the equilibrium to be maintained, that the vector of returns, $\underline{R}_{t+1}$, is such that $\int_0^1 \underline{B}_t^j dj = 0$. This in turn implies:

$$\int_0^1 \underline{A}_t^{j,d} dj = \int_0^1 \underline{L}_t^{j,d} dj, \tag{15}$$

or:

$$\int_{j \in D_A^i} A_t^{ij}(R_{t+1}^i) dj = \int_{\in D_L^i} L_t^{ij}(R_{t+1}^i) dj, \tag{16}$$

where we refer to D_A^i as the subset of agents who participate in the market for asset i as a financial asset (i.e. for whom $A_t^{ij,d} > 0$), and to D_L^i in the market for asset i as a financial liability (i.e. for whom $L_t^{ij,d} > 0$). This expression identifies the level of real equilibrium interest rates. It should be noted that a unique equilibrium vector, say $\underline{R}_{t+1}^*$, ensuring that condition (16) holds exists only to the extent that, at the aggregate level, the excess demand curve, $\int_0^1 \underline{B}_t^j dj$, is strictly decreasing in the rates of return, namely $\partial(\int_0^1 \underline{B}_t^j dj)/\partial \underline{R}_{t+1} < 0$.

A unique equilibrium is of course not warranted for any conceivable financial instrument since even at the level of the individual household the relationship between the demand for assets and liabilities and real returns is not a straightforward one, due to the existence of substitution and wealth effects, as mentioned in the previous section. So, we assume that the financial market is able to clear for a subset of the theoretically possible financial assets, and we define the set of the available financial instruments, $i = 1, ..., n$, to be the one for which a market clearing is attainable. Generally speaking, as everything in the model is time-varying, the set of financial instruments for which market clearing is feasible (and therefore a market exists at all) will also be time-varying.

It is also worth stressing that in this model the *total* gross supply of, and demand for, financial assets are related primarily to the *heterogeneity* across households, while market prices (expected returns) ensure that demand and supply are equalized. This is a main advantage of considering heterogeneous agents in an asset pricing model, in that both prices and quantities may be derived.

4 Extension including government policy

The basic framework of analysis described so far can easily be extended to incorporate government intervention in the financial market as well as public expenditure and taxes. For illustration purposes, in this section we introduce a central bank in the model which is able to produce outside money at no cost. The central bank can transfer outside money (expressed in real terms), M_t, to the private sector either in outright money transfers or in exchange of other assets (open market operations).

Money and privately issued financial instruments now compete in the financial market.[11] We assume that money, which is held by all private agents as a financial asset, provides positive liquidity services with decreasing returns, so that $U_M^j > 0$, $U_{MM}^j < 0$.

Money is defined as an asset which delivers a zero *nominal* rate of return. Therefore, its *real* rate of return (determined endogenously in the financial market, taking into account the actions of the private sector), R_t^M, is linked one-to-one with the prevailing period-by-period inflation rate π_t:

$$\pi_t = -R_t^M \tag{17}$$

Therefore, in this model it is the demand for, and supply of, outside money which determines the inflation rate. While this is, in a way, a standard conclusion, the precise channel through which outside money influences the inflation rate (or, equivalently, its real rate of return) is somewhat more complex and richer than in other models.

At the level of the individual household, the only difference with the optimal asset and liability allocation expounded in Section 2 is given by the fact that desired money holdings enter in the flow budget constraint in (2), which now becomes:

$$c_t^j + \underline{1}'\underline{B}_t^j + M_t^j = y_t^j + \underline{R}_{t-1}'\underline{B}_{t-1}^j + (1 + R_{t-1}^M)M_{t-1}^j, \tag{18}$$

M_t^j being the money holdings of agent j.

Given the expanded choice set, the utility function of agent j can be rewritten as:

$$U_t^j = U_t^j(c_t^j, y_t^j, \underline{A}_t^j, \underline{L}_t^j, M_t^j) \tag{19}$$

The first order condition for money holdings is:

$$-1'U_{c,t}^j + \beta_j(1 + R_{t+1}^M)U_{c,t+1}^j + U_M^j = 0 \tag{20}$$

Following a similar reasoning as in Section 2.2, this can be expressed as a Marshallian demand function:

$$M_t^j = M_t^j(R_{t+1}^M) \tag{21}$$

with, again, uncertainty over the sign of $\frac{\partial M_t^j}{\partial R_{t+1}^M}$ due to the usual wealth and substitution effects. The order flow for money by agent j is given by:

$$M_t^{j,d} = \max\{0, M_t^j(R_t^M)\} \tag{22}$$

At the aggregate level, as in Section 3 the equilibrium is given by:

$$\int_0^1 M_t^{j,d} dj = M_t, \tag{23}$$

where M_t is the amount of outside money introduced into the system by the central bank. Condition (23) identifies the real rate of return on money and consequently the inflation rate prevailing in the economy.

5 Liquidity and real equilibrium interest rates

We have now set the stage for the analysis of the pricing of liquidity services in equilibrium. This is an important matter which has been extensively dealt with in the literature on monetary aggregation (Barnett and Serletis, 2000) as well as in an earlier Keynesian literature emphasizing the role of agents' preference for liquidity in the determination of the monetary rate of interest. In the Keynesian theory of the "own rate of money interest" (see Bibow, 1998, for a review of this concept), the monetary rate of interest can be defined (simplifying) as:

$$r = \overline{r} - l + \sigma, \tag{24}$$

where r is the real interest rate on a financial instrument, $\overline{r}$ is the real interest rate on a risk-free non-liquid asset, l is liquidity (inclusive of carrying costs) and σ is a measure of risk. Therefore, in this framework an *improved* liquidity of financial instruments leads necessarily to a *fall* in equilibrium real interest rates, everything else remaining unchanged. This view is reflected in the idea that low liquidity, for example due to high cost of financial intermediation, must increase the return required by savers and hence real rates of return (as evident in the quotation shown at the beginning of this note). In the present framework, however, evaluating the impact of liquidity on equilibrium real interest rates is a more complex matter on account of the heterogeneity across agents and the fact that both assets and liabilities can have liquidity services which have to be priced in equilibrium. In particular, the important element is the extent to which the liquidity services of the financial instrument i (seen both as an asset and as a liability) impact on the *excess* (rather than gross) demand for it, namely $\int_0^1 B_t^{ij} dj$. For example, a financial instrument which is very liquid for asset holders but which is even more liquid for liabilities holders will generally have a *higher* rate of return compared with the benchmark liquidity-free asset (namely, a negative user cost). In other words, the general equilibrium and heterogeneous agents nature of this model leads quite naturally to look at both sides of a financial contract, given that they both matter in the determination of the equilibrium rates of return.

5.1 The effect of improvements in liquidity due to technical and contractual progress

Financial and payment technological and contractual innovation may contribute over time to change the liquidity services provided by financial assets and the cost of producing them. In the Keynesian theory of the "own rate of money interest" described above, it might be argued that improved liquidity should contribute to *reducing* real equilibrium interest rates over time. This conclusion, as noted, is not necessarily warranted in a framework with heterogeneous agents and asymmetry in liquidity services between assets and liabilities as the one proposed in this paper. Notably, we should entertain the possibility that technological and contractual innovation also affects the liquidity services and costs for financial *liabilities*. For example, in most industrialized countries a debtor cannot be imprisoned anymore if he fails to pay back his debt. It can be argued that this type of legislative progress increases the liquidity services provided by all financial liabilities, and should *ceteris paribus* result in *higher* equilibrium real interest rates. Another interesting example is technological innovation in the banking sector. Suppose that agents are better able to monitor their bank accounts due to, say, the home banking technology. This factor, in itself, raises the demand for these assets and hence the real equilibrium rates of return on them. On the other hand, however, suppose that banks become better able to track movements in the current accounts of the customers, implying an improved liquidity of the liabilities side of their balance sheet. The net effect of this type of innovation will depend on the effect on the liquidity and therefore the desired holdings of assets and liabilities of each agent, which makes – in an heterogeneous agents model – the aggregate effect extremely uncertain.

5.2 Real equilibrium interest rates in a financial crisis

Let us now turn to the conceptually opposite case when unrest in financial markets leads to an exacerbation of agency costs and to lower liquidity. Trust between borrowers and lenders is eroded in these hard times. This is a situation which has been amply emphasized in the literature, where the financial system is thought to be unable to channel funds to those with the best investment opportunities (Mishkin, 1991) and information asymmetry problems deepen, leading to illiquidity (Glosten and Milgrom, 1985). Reflecting this view, the traditional reaction of central banks to financial unrest has been to provide highly liquid instruments (i.e., cash) to the market, notably through their lender of last resort function.

What happens to real equilibrium interest rates when markets become more illiquid? In the traditional Keynesian view, this translates into a lower l and this implies *higher* real interest rates in equilibrium, as evident from equation (24). However, it should now be clear from the previous discussion that the evaluation of the overall impact of an increase in agency costs onto real equilibrium interest rates is more complex than the Keynesian view suggests. Again, what matters is the impact of these developments on the excess, rather than gross, demand for each financial instrument, i.e. $\int_0^1 B_t^{ij} dj$ (and not $\int_0^1 A_t^{ij} dj$). For example, depending on the nature of financial contracts and the incentives faced by borrowers and lenders, an increase an agency costs may result in reduced liquidity for both asset holders (in terms of higher screening and monitoring costs) and for liability holders (in terms of signalling). The net effect of these factors onto equilibrium asset returns, also taking into account the heterogeneity across households and the aggregation biases that it implies, is once more far from straightforward.

In particular, it will be important to have a close look at the relative situation of debtors and creditors in the economy. This is especially relevant if, as is often the case in advanced economies, debtors and creditors in the economy typically belong to distinguished and well identified sectors (for example households and corporations). In this situation, a problem originating in a certain sector (say in the households sector) might have a considerably different impact on the level of the natural rate compared with the same problem experienced in another sector (say non-financial firms). Overall, the model presented in this paper provides a useful analytical framework to think in a rigorous way about this kind of issues, also from a policy perspective.

6 Other possible applications of the basic framework

We now turn to a more general assessment of the kind of possible problems to which the present framework of analysis can be usefully applied. In particular, we concentrate on the type of issues where the approaches proposed thus far in the literature do not appear to represent an equally satisfactory analytical tool, in terms of either simplicity or insight.

Joint general equilibrium modelling of (endogenous) money and credit. Thus far, the modelling of money and credit in general equilibrium has followed largely independent routes. The modelling of money in general equilibrium has often been based on the assumption that outside, government-issued money either enters as an argument in a representative household's utility function or is held due to a cash in advance constraint. The modelling of credit typically involves the existence of a representative financial intermediary who has to expend costly effort in order to monitor the creditworthyness of the recipient of the credit, to minimise the risk of default (Bernanke, Gertler and Gilchrist, 1999). Reflecting the different perspectives taken in dealing with money and credit frictions, the joint modelling of (especially inside) money and credit is seldom if ever attempted in general equilibrium. Yet, this analysis has demonstrated that the concept of liquidity (in the broad meaning of this paper) is a simple and elegant way to explain the existence of both (inside) money and credit in equilibrium, as well as the fact that there is a full range of assets having both a lower (money) and a higher (credit) rate of return compared with a theoretical benchmark rate ensuring a costless and frictionless transfer of purchasing power over time. In addition, the interpretation of liquidity given in this paper can explain the existence of assets with both a positive and a negative user cost, which at least for certain problems can represent an improvement on the literature on monetary aggregation, which typically assumes only positive user costs.

Explain how a financial market comes to existence following financial innovation and liberalisation. As we have seen, a model featuring agents heterogeneity in their perception of liquidity can explain why some agents do not participate in the financial market. Intuitively, this happens if both assets and liabilities are relatively illiquid, or if the agent would tend to demand for example liabilities in the absence of liquidity considerations (say, due to the expectation of a rising income during the lifetime) but such liabilities have negative liquidity for him. If this generalises to the whole population, it can imply that the financial market ceases to exist. Let us now assume that financial innovation increases the liquidity services of all assets and liabilities, so that both borrowing and lending becomes easier (financial development). In the standard Keynesian framework, this is translated into a increase in l, and in lower equilibrium rates of return, but the effect on the overall amount of financial assets and liabilities in circulation is indeterminate. In the present framework, financial innovation can be used to explain how financial assets and liabilities, and therefore a financial market, comes to existence.

Simultaneous holding of assets and liabilities by the same agent. This insight has been already touched upon earlier (see footnote 10). To the author's knowledge, no other analytical framework can explain in a straightforward way this phenomenon which is widespread in financially developed economies.[12]

Explain the contemporaneous increase in financial market

liquidity and higher real returns. As mentioned in the previous section, if financial innovation increases the liquidity of financial liabilities more than that of financial assets, it can be associated to higher, rather than lower real interest rates, while the standard Keynesian approach necessarily predicts lower real interest rates. The empirical relevance of this possibility is illustrated by the experience of financial liberalisation episodes in several countries, which intially led to a relaxation of borrowing constraints, and thereby to higher borrowing demand and real interest rates.[13]

7 Conclusions

This paper has proposed a simple general equilibrium intertemporal model with heterogeneous households and a financial market in which each financial instrument provides liquidity services in addition to the property of transferring purchasing power over time. The model proposes a view of the equilibrium real interest rates which reflects the traditional determinants (such as time preference and technology), but also the liquidity services provided by the financial assets and liabilities.

The analytical framework introduced in this paper appears to be particularly suited to study situations where "liquidity matters", and which the traditional intertemporal models are not able to address satisfactorily. In this note, we study the determination of equilibrium real interest rates when financial innovation improves the liquidity services provided by financial instruments, or alternatively when a financial crisis leads to a drying up of liquidity.

The framework proposed in this paper appears to provide a simple and useful setting to model, and a language to talk about, a series of imperfections and frictions in financial markets which are difficult to deal with in a simple manner in standard intertemporal models. Moreover, the model seems particularly appropriate to study issues related to heterogeneity which are normally set aside.[14] A notable feature of models allowing for heterogeneous agents is their ability to derive both quantities and prices (required returns) in the financial market, which is usually impossible for representative agent models.

Clearly, the analysis in this paper is only a first step and could be extended in several directions. Physical assets and open economy considerations could be explicitly included in the model, to have a more complete picture of the determination of equilibrium real interest rates under flexible prices in a realistic economy. Moreover, the role of risk and risk aversion might be integrated relatively straightforwardly in this framework. Finally, in this model we have assumed that liquidity services enter in the utility function with diminishing absolute marginal returns, but it is not difficult to envisage situations in which liquidity may be characterized by increasing absolute marginal returns. For example, an agent's debt aversion and therefore the marginal dis-utility associated with debt may actually increase, the larger the financial liability position in absolute terms. Studying the effects of absolute increasing returns may be intriguing since it is likely to endogenously give rise to non-convexities possibly leading to quantity rationing which have been emphasized in the information economics literature (Stiglitz, 1999).

Notes

[1]The reference is, in particular, to purely subjective motives for saving, such as the sense of independence and security that the holding of financial assets confers, or pure avarice, which are adequately emphasized in the *General Theory* (see also Browning and Lusardi, 1996).

[2]Tobin (1969) is an earlier classic reference on a general equilibrium approach to monetary theory.

[3]See also Barnett and Serletis (2000) for a review of issues related to the aggregation across heterogeneous agents.

[4]Note that adjusting the model to incorporate the possibility of default would not change its main features by much, as long as agency costs are taken into account in the no-default specification. So, instead of having a "lemons premium" in the model (Hubbard, 1998), we have a "cost of detecting lemons", which is in practice not that different.

[5]This would suggest the paradoxical conclusion that sometimes it is the apparent illiquidity of financial assets (namely the difficulty in converting them into transaction balances) which might *improve* their "liquidity services" as defined in this paper. This happens because illiquidity (in the traditional sense) helps agents solve their self-control problems and so results in a benefit for them (i.e. higher liquidity services). However, the use of the concept of lack of self-control should be used with some caution in this paper since we are assuming that agents are fully rational and apply standard exponential discounting.

[6]The basic framework can easily be extended to study government policy, as government intervention affects asset supplies and hence real rates of return.

[7]The same consideration is valid for the government, although with probably a somewhat smaller degree of realism.

[8]We assume that no-Ponzi conditions hold and that sustainability issues do not play any role in our economy.

[9]From the discussion in the foregoing it should be clear that this type of asset hardly exists in the real world; it is merely an artificial construct.

[10]The contemporaneous holding of financial assets and liabilities can of course be justified in different models, but all of them have some built-in form of heterogeneity. For example, Gertler (1999) derives simultaneous borrowing and lending in an overlapping generations model. It should be noted, however, that unlike in Gertler (1999) our model explains the possibility that the same *individual* holds both assets and liabilities, a point which is emphasised by Greenwald and Stiglitz (2003).

[11]Lagos and Rocheteau (2004) also consider a model where money and private capital compete to provide liquidity services, although liquidity has a quite different meaning compared with this study.

[12]As, for example, when a household contemporaneously holds a mortgage (liability), bank deposits, and financial securities (assets).

[13]See, for example, Jappelli and Pagano (1994) who argued that financial liberalisation can lead to increased household access to consumer credit or housing finance (i.e. financial liabilities), pushing up borrowing demand and real interest rates.

[14]Of course, this is not to say that limited participation in financial markets and heterogeneity have not been already dealt with altogether in the literature. For example, Brav, Constantinides and Geczy (2002) provide a very interesting analysis of the role of limited participation and incompleteness in financial markets in a heterogeneous agents economy.

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