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**NO 903 / MAY 2008**

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**A ROBUST CRITERION  
FOR DETERMINING  
THE NUMBER OF  
STATIC FACTORS IN  
APPROXIMATE FACTOR  
MODELS**

by Lucia Alessi, Matteo Barigozzi  
and Marco Capasso





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# A ROBUST CRITERION FOR DETERMINING THE NUMBER OF STATIC FACTORS IN APPROXIMATE FACTOR MODELS <sup>1</sup>

by Lucia Alessi <sup>2</sup>, Matteo Barigozzi <sup>3</sup>  
and Marco Capasso <sup>4</sup>



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## Abstract

We propose a refinement of the criterion by Bai and Ng [2002] for determining the number of static factors in factor models with large datasets. It consists in multiplying the penalty function by a constant which tunes the penalizing power of the function itself as in the Hallin and Liška [2007] criterion for the number of dynamic factors. By iteratively evaluating the criterion for different values of this constant, we achieve more robust results than in the case of fixed penalty function. This is shown by means of Monte Carlo simulations on seven data generating processes, including heteroskedastic processes, on samples of different size. Two empirical applications are carried out on a macroeconomic and a financial dataset.

**Keywords:** Approximate factor models, Information criterion, Number of factors.

**JEL-classification:** C52

## Nontechnical summary

The literature on factor models has been rapidly growing in the last years, and equally it has been growing the interest into criteria which can consistently estimate the number of common factors driving the data. Indeed, factor models are particularly useful for dimension reduction when datasets are large in both the time and the cross-section dimension. However, this is precisely the case in which the choice of the number of factors cannot be made by means of traditional information criteria which are not designed for diverging  $N$  and  $T$ . Given the vast use of factor models, determining the number of factors in large cross sections of time series is thus a hot topic. This paper provides a tool to address the issue, the theoretical properties of which are known and the empirical results are robust.

Relatively few authors have dealt with the model selection problem related to the number of common factors when both  $N$  and  $T$  diverge. Bai and Ng [2002] pioneered the literature by proposing a criterion, specified in six different forms, which basically modifies the AIC and BIC in order to take into account both dimensions of the dataset as arguments of the function penalizing overparametrization. The information criterion that we propose is a refinement of the criterion by Bai and Ng, drawing on the Hallin and Liška [2007] criterion for dynamic factors. The idea is fairly simple: we multiply the penalty function by a constant which tunes the penalizing power of the function itself. By evaluating the criterion for a whole range of values for this constant, we finally get an estimation of the number of static factors which is empirically more robust than it would be the constant being fixed. The consistency properties of our estimator are exactly the same of the original Bai and Ng estimator, the only difference being a multiplicative constant.

The motivation for our work is provided by the fact that there are cases in which the original criterion cannot give a precise answer, as we also show by means of a Monte Carlo Study whose experimental design is taken from Bai and Ng. By considering seven data generating processes, including heteroskedastic processes, we generate samples of different size and implement the original criteria by Bai and Ng and our modified criteria, in their six formulations (three *PC* and three *IC* criteria). The criteria require to set the maximum number of static factors allowed in the estimation prior to the estimation itself: in the first set of simulation experiments we keep this parameter fairly low, and show that the original *PC* criteria heavily depend upon the choice of this parameter, which is not the case for the modified *PC\** criteria. In the second set of simulations, we let the parameter vary according to the number of series in the panel, since in principle the number of static factors can be equal to  $N$ . We show that when the maximum number of static factors allowed is very - but still legitimately - large, also the original *IC* criteria lose robustness, although they do not depend explicitly on this parameter. The *IC\** criteria that we propose, on the contrary, remain reasonably accurate provided that the true number of factors is relatively low.

Finally, we carry out two empirical applications. The first application is on a macroeconomic dataset comprising 132 series of macroeconomic indicators of the US economy from January 1960 to December 2003: in this case our proposed criteria robustly suggest the existence of 6 factors. The second application is on a financial dataset including 89 daily asset returns from the London Stock Exchange: we show that in this case it is possible that the original *IC* criteria underestimate the number of factors.

# 1 Introduction

The literature on factor models has been rapidly growing in the last years, and equally there has been growing interest into criteria which can consistently estimate the number of common factors driving the data. Indeed, factor models are particularly useful for dimension reduction when datasets are large in both the time and the cross-section dimension. However, this is precisely the case in which the choice of the number of factors cannot be made by means of traditional information criteria which are not designed for diverging  $N$  and  $T$ . Given the vast use of factor models, determining the number of factors in large cross sections of time series is thus a hot topic. This paper provides a tool to address the issue, the theoretical properties of which are known and the empirical results are robust.

Relatively few authors have dealt with the model selection problem related to the number of common factors when both  $N$  and  $T$  diverge. Bai and Ng [2002] (henceforth BN) pioneered the literature by proposing a criterion, specified in six different forms, which basically modifies the AIC and BIC in order to take into account both dimensions of the dataset as arguments of the function penalizing overparametrization. Kapetanios [2005] takes a different approach, based on the limit of the empirical distribution of the eigenvalues of the sample covariance matrix: the idea is that the number of eigenvalues diverging as  $N$  diverges is equal to the number of static factors driving the dataset. Onatski [2007] adopts a third strategy and tests the null hypothesis of  $r_0$  static factors against the alternative of  $r_1$  static factors. The test is based on the few largest eigenvalues of the covariance matrix of a complex-valued sample derived from the original dataset, which asymptotically distribute as a Tracy-Widom.

Very recently, some criteria have been proposed to determine the number of dynamic common factors, which are the primitive shocks influencing each of the variables not only contemporaneously, but also via leads and lags. Amengual and Watson [2007] study the consistency properties of an estimator proposed in Stock and Watson [2005], which consists in projecting the data onto lagged values of principal components estimates of the static factors, and then applying the estimator proposed by BN to the residuals. Also the estimator by Bai and Ng [2007] builds on their criterion for the number of static factors, the main intuition being that the dynamic factors should explain the same percentage of variance as the  $r$  selected static factors. Breitung and Kretschmer [2005] apply canonical correlation analysis to the estimated static factors in order to tell which are the dynamic factors and which are just their lags. Finally, Hallin and Liška [2007] develop an information criterion based on the eigenvalues of the spectral density matrix of the observations. Indeed, the estimation of the Generalized Dynamic Factor Model by Forni et al. [2000] is carried out by means of dynamic principal components, and their number is equal to the number of diverging dynamic eigenvalues as  $N$  goes to infinity.

The information criterion that we propose is a refinement of the criterion by BN, drawing on the Hallin and Liška [2007] criterion for dynamic factors. The idea is fairly simple: we multiply the penalty function times a constant which tunes the penalizing power of the function itself. By evaluating the criterion for a whole range of values for this constant, we finally get an estimation of the number of static factors which is empirically more robust than it would be the constant being fixed. On the other hand, the consistency properties of our estimator are exactly the same of the original BN estimator, the only difference being a multiplicative constant.

The motivation for our work is provided by the fact that there are cases in which the original criterion, in its three *PC* and three *IC* specifications by BN, cannot give a precise answer.

For instance, Forni et al. [2007] implement the six BN specifications on an 89 series sample including macroeconomic and financial variables: if the maximum number of factors is set to 30, two *IC* specifications out of three do not converge; the *PC* specifications do not work either in this case, and give three different estimates in the cases in which they converge. As a second example, in Alessi et al. [2006] we apply the BN criterion on a panel of 89 stock return series and find that when the maximum number of factors is high, that same value is returned as an estimate; when it is low, the *IC* criteria indicate the existence of two static factors while the *PC* criteria point to numbers between seven and fourteen.

The paper is structured as follows. In the next section we outline the factor model and briefly recall the assumptions for consistency of the estimator. In section 3 we present our criterion and a practical guide to the algorithm. In section 4 we validate our method and compare it to the original criterion on the basis of a Monte Carlo study on seven data generating processes, including heteroskedastic processes. In section 5 we present two empirical applications of the criterion and in section 6 we conclude.

## 2 The factor model

A general approximate dynamic factor model in its static representation can be written as

$$\mathbf{x}_t = \mathbf{\Lambda}\mathbf{F}_t + \boldsymbol{\xi}_t, \quad (1)$$

where  $\mathbf{x}_t$  is a large panel composed of  $N$  time series,  $\mathbf{\Lambda}$  is the  $N \times r$  matrix of factor loadings,  $r$  being the number of static factors, and  $\mathbf{F}_t$  is the  $r \times 1$  vector of static factors.  $\mathbf{x}_t$  is thus represented as the sum of two mutually orthogonal components, i.e.  $\mathbf{\Lambda}\mathbf{F}_t$  which is called the common component, and  $\boldsymbol{\xi}_t$  which is the idiosyncratic component. For the formal statement of the assumptions of the model we refer to BN and limit ourselves to a brief overview of the main points.

1. Each factor is assumed to have an impact on each of the variables of the panel. Assuming that  $\mathbf{\Lambda}'\mathbf{\Lambda}/N$  converges to a positive definite limiting matrix rules out the possibility that some factors are loaded with zero coefficient by some of the variables in the panel.
2. The model is approximate since it allows for a small amount of cross-sectional correlation across the idiosyncratic terms. To formally state this assumption, let us denote by  $\lambda_\xi$  the largest eigenvalue of the contemporaneous covariance matrix of the idiosyncratic component,  $\mathbf{\Gamma}_\xi$ . While an exact dynamic factor model requires  $\mathbf{\Gamma}_\xi$  to be diagonal, an approximate factor model only requires non-pervasiveness of the idiosyncratic component, i.e. that all eigenvalues of  $\mathbf{\Gamma}_\xi$  are bounded for  $N$  going to infinity. This is the same as to assume that there exist a real  $M$  such that  $\lambda_\xi \leq M$  for any  $n \in \mathbb{N}$ .
3. We say that the common component of  $\mathbf{x}_t$  has reduced static rank  $r$ : this means that  $\mathbf{\Lambda}'\mathbf{\Lambda}/N$  has full rank  $r$ .<sup>1</sup>
4. The  $r$  static factors  $\mathbf{F}_t$  are identified up to a rotation, or in other words, the finite dimensional vector space spanned by the static factors is identified. The common component  $\mathbf{\Lambda}\mathbf{F}_t$  belongs to this space.

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<sup>1</sup>In the dynamic specification of the factor model, the  $r$  static factors are actually just  $q < r$  dynamic factors together with their lags. In this case we say that the common component of  $\mathbf{x}_t$  has reduced dynamic rank  $q$ .



5. The idiosyncratic parts are allowed to be autocorrelated.
6. Heteroskedasticity is allowed in both the time and the cross-section dimension.
7. Stationarity is not required.

In large cross-sections, the  $r$  static factors can be consistently estimated by means of principal components, the cross-sectional correlation across the idiosyncratic components being not enough to survive aggregation. The principal component estimation of the factors is static, fully carried out in the time domain, as in Stock and Watson [1998]. The principal component estimator of the loading matrix  $\mathbf{\Lambda}$  is the matrix  $\mathbf{A}$  which minimizes the residual sum of squares:

$$\sum_{t=1}^T (\mathbf{x}_t - \mathbf{A}\mathbf{f}_t)'(\mathbf{x}_t - \mathbf{A}\mathbf{f}_t) \quad \text{s.t.} \quad \mathbf{A}'\mathbf{A} = \mathbf{I}_r \quad (2)$$

The columns of  $\mathbf{A}$  turn out to be the  $r$  largest eigenvectors of the sample covariance matrix  $\mathbf{\Gamma}_x = T^{-1} \sum_{t=1}^T (\mathbf{x}_t - \bar{\mathbf{x}}_t)'(\mathbf{x}_t - \bar{\mathbf{x}}_t)$ .<sup>2</sup> However, BN show that their criterion works for a more general class of estimators, and our criterion inherits this property. We refer to their paper for the discussion of this latter result, as well as for the proof of the theorem establishing the asymptotic properties of the estimated factors.

### 3 Determining the number of factors

BN propose an information criterion to determine the number of static factors. They assume the static factor model (1) with  $r$  factors for an  $N$ -dimensional vector process of finite time length  $T$ . Common factors  $\mathbf{F}_t^{(k)}$  and their loadings  $\mathbf{\Lambda}^{(k)}$  are estimated using static principal components. We use the superscript  $k$  when we choose  $k$  static factors. The information criterion is aimed at minimizing the residual variance of the idiosyncratic components which is computed as a function of  $k$ . Namely the static factors and their loadings must minimize

$$V(k) = \frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T (x_{it} - \lambda_i^{(k)'} \mathbf{F}_t^{(k)})^2, \quad (3)$$

computed for all the possible numbers of static factors  $k \in [0, r_{\max}]$  up to  $r_{\max} = \min\{N, T\}$ . The minimization is subject to the normalization  $\mathbf{\Lambda}^{(k)'} \mathbf{\Lambda}^{(k)} / N = \mathbf{I}_k$  or  $\mathbf{F}^{(k)'} \mathbf{F}^{(k)} / T = \mathbf{I}_k$ . Actually it is enough to minimize (3) only for  $\mathbf{\Lambda}^{(k)}$ , given a previous estimate of the static factors  $\hat{\mathbf{F}}_t^{(k)}$ . What we get is a function  $V(k, \hat{\mathbf{F}}_t^{(k)})$  that does not depend on the estimator used for the factors as long as it satisfies Theorem 1 in BN. Indeed all estimators satisfying such theorem span the same space  $V(k, \hat{\mathbf{F}}_t^{(k)})$  is a quantity that cannot increase as  $k$  approaches  $r_{\max}$ . Overparametrizing is avoided by introducing a penalty function  $p(N, T)$  which counterbalances the fit improvement due to the inclusion of additional common factors. BN propose two classes of criteria

$$PC_N^T(k) = V(k, \hat{\mathbf{F}}_t^{(k)}) + k\sigma^2 p(N, T),$$

$$IC_N^T(k) = \log \left[ V(k, \hat{\mathbf{F}}_t^{(k)}) \right] + kp(N, T).$$

<sup>2</sup>Alternatively, the static factors can be estimated via a two-step estimator which first exploits dynamic principal components for the estimation of the covariance matrix of the common component in the frequency domain and then turns to the static representation, as in Forni et al. [2005]. In this case stationarity of  $\mathbf{x}_t$  is required.

According to Theorem 2 in BN, the number of static factors is consistently estimated if the penalty function satisfies the two conditions

$$\lim_{\substack{N \rightarrow \infty \\ T \rightarrow \infty}} p(N, T) = 0 \quad \text{and} \quad \lim_{\substack{N \rightarrow \infty \\ T \rightarrow \infty}} p(N, T) [\min(\sqrt{N}, \sqrt{T})]^2 = \infty .$$

Depending on the chosen criterion, the estimated number of factors is

$$\begin{aligned} \hat{r}_N^T &= \operatorname{argmin}_{0 \leq k \leq r_{\max}} PC_N^T(k) \\ \text{or} \\ \hat{r}_N^T &= \operatorname{argmin}_{0 \leq k \leq r_{\max}} IC_N^T(k). \end{aligned}$$

Notice that in the PC criteria BN introduce a scaling factor  $\sigma^2$ , which is the variance of the residuals associated with principal component estimates. As an estimate of  $\sigma^2$ , they suggest  $V(r_{\max}, \hat{\mathbf{F}}_t^{(r_{\max})})$ . This introduces a direct dependence of the PC criteria on the maximum number of static factors. Moreover, in all empirical applications both the PC and IC criteria proposed by BN turn out to heavily depend on the choice of  $r_{\max}$  (e.g. see Forni et al. [2007]). The PC specifications depend explicitly on it while the IC specifications depend on it only when implementing them in practice.

By analogy with the criterion by Hallin and Liška [2007] for dynamic factors, we propose to multiply the penalty function by a positive constant  $c$ . Indeed if  $p(N, T)$  is an appropriate penalty function, then  $cp(N, T)$  is appropriate as well for any positive constant  $c$ . We consider six different criteria analogous to those studied by BN:

$$\begin{aligned} PC_1^{*c}(k) &= V(k, \hat{\mathbf{F}}_t^{(k)}) + ck \left( \frac{N+T}{NT} \right) \log \left( \frac{NT}{N+T} \right); \\ PC_2^{*c}(k) &= V(k, \hat{\mathbf{F}}_t^{(k)}) + ck \left( \frac{N+T}{NT} \right) \log(\min\{\sqrt{N}, \sqrt{T}\})^2; \\ PC_3^{*c}(k) &= V(k, \hat{\mathbf{F}}_t^{(k)}) + ck \frac{\log(\min\{\sqrt{N}, \sqrt{T}\})^2}{\min\{\sqrt{N}, \sqrt{T}\}^2}; \\ IC_1^{*c}(k) &= \log \left[ V(k, \hat{\mathbf{F}}_t^{(k)}) \right] + ck \left( \frac{N+T}{NT} \right) \log \left( \frac{NT}{N+T} \right); \\ IC_2^{*c}(k) &= \log \left[ V(k, \hat{\mathbf{F}}_t^{(k)}) \right] + ck \left( \frac{N+T}{NT} \right) \log(\min\{\sqrt{N}, \sqrt{T}\})^2; \\ IC_3^{*c}(k) &= \log \left[ V(k, \hat{\mathbf{F}}_t^{(k)}) \right] + ck \frac{\log(\min\{\sqrt{N}, \sqrt{T}\})^2}{(\min\{\sqrt{N}, \sqrt{T}\})^2}. \end{aligned}$$

As before, depending on the chosen criterion, the estimated number of factors is

$$\begin{aligned} \hat{r}_{c,N}^T &= \operatorname{argmin}_{0 \leq k \leq r_{\max}} PC_{a,N}^{T*}(k) \\ \text{or} \\ \hat{r}_{c,N}^T &= \operatorname{argmin}_{0 \leq k \leq r_{\max}} IC_{a,N}^{T*}(k) \quad \text{with } a = 1, 2, 3. \end{aligned}$$

The degree of freedom represented by  $c$  can be exploited when implementing the criterion in practice. The only information we have about the asymptotic behavior of  $\hat{r}_{c,N}^T$  comes from

considering subsamples of sizes  $n_j \leq N$  and  $T_j \leq T$  with  $j = 0, \dots, J$ . For each value of  $c$  we compute the number of factors  $r_{c,n_j}^{T_j}$  for all possible subsamples. This number has a variability

$$S_c = \frac{1}{J} \sum_{j=1}^J \left[ r_{c,n_j}^{T_j} - \frac{1}{J} \sum_{j=1}^J r_{c,n_j}^{T_j} \right]^2 .$$

The procedure for selecting the number of static factors basically explores the behavior of the variance  $S_c$  of the estimated number of factors for  $N$  and  $T$  going to infinity, for a whole interval of values for the constant  $c$ . We refer to the paper by Hallin and Liška [2007] for an extensive explanation of the role of the constant  $c$  and the other parameters used, and we just outline here the necessary steps for implementing the algorithm.

1. Set the maximum number of static common factors  $r_{\max}$ . We will show in the next section that our criterion does not heavily depend on this choice. Thus, in practice, to be sure to find the right number of static factors, we can choose a very high value for  $r_{\max}$  (in principle also  $r_{\max}$  approaching  $N$  is a feasible choice).
2. Set an upper bound for the constant  $c$ , i.e.  $c \in [0, c_{\max}]$ .
3. For each considered value of  $c$ , perform the following:
  - (a) choose randomly different subsamples of increasing dimension  $n_j = N - j$ , where  $j = 0, \dots, J$ , with  $j$  integer such that  $N - j$  is not too small (subsamples of different increasing time dimension  $t_j$  could also be considered);
  - (b) minimize the  $PC^*$ s or  $IC^*$ s with respect to the number of static factors  $k$ ;
  - (c) compute the variance  $S_c$  of the estimated number of factors as  $n_j \rightarrow N$  and in case also as  $t_j \rightarrow T$ .

When  $c = 0$  we always get  $\hat{r}_{c,N}^T = r_{\max}$  and  $S_c = 0$ ; when  $c$  increases we find stability intervals, but also values of  $c$  with high variability. As  $c$  increases we increase the penalization. In order to tune the penalty function, we look for intervals of  $c$  for which no dependence on the sample size is present, i.e.  $S_c = 0$ . Moreover, we ask for a constant number of factors for  $n_j = N$  and  $t_j = T$  for all values of  $c$  in the considered intervals. This number is the estimated number of static factors  $\hat{r}_{c,N}^T$ . The value  $r_{\max}$  is never considered as it is a boundary solution and we want to avoid the case in which  $c$  does not penalize at all the criterion, thus giving as result the maximum possible number of factors. Figure 1 shows how the  $IC_2^*$  criterion works. As the constant  $c$  increases, the solid line provides the suggested number of factors. A plateau of the solid line means a region where the suggested number of factors  $r_{c,N}^T$  is stable across different values of  $c$ . On the other side, the dashed line provides a measure of the instability of  $r_{c,N}^T$  when different subsamples of the dataset are considered. When the dashed line goes to zero, the value provided by the solid line is stable across different subsamples, i.e. is not biased by the whole sample size. Therefore, we have to choose the smallest value of  $c$  for which both a plateau of the solid line (not including the extreme left one) and a zero of the dashed line occur. The traditional  $IC$  criteria by BN implicitly consider only the case  $c = 1$ . The example of figure 1 when  $c = 1$  suggests a number of factors  $r_{1,N}^T = 4$  which is smaller than the true one  $r = 5$ , moreover when  $c = 1$  we have  $S_c \neq 0$ . Our refinement of the  $IC$  criterion considers also different values of  $c$ , thus finding a value  $c = 0.6$  for which the number of factors suggested by the solid line is the correct one ( $r_{c,N}^T = 5$ ). The estimated number in

this case seems to be stable across adjacent values of  $c$  (plateau of the solid line) and across different subsample sizes (zero of the dashed line). This way, we avoid an overpenalization of the number of factors and success in finding the true number of static factors.

## 4 Monte Carlo study

In this section we conduct a set of simulation experiments to evaluate in finite samples the performance of our refined criterion, relative to that of the original BN criterion. The experimental design is the same as in BN where seven data-generating processes (DGPs) are considered.

The baseline model is:

$$x_{it} = \sum_{j=1}^r \lambda_{ij} F_{tj} + \sqrt{\theta} e_{it} \quad i = 1, \dots, N \quad t = 1, \dots, T,$$

with factors and factor loadings normally distributed with zero mean and unit variance. We summarize the different specifications.

DGP1) Homoskedastic idiosyncratic component, with the same variance for the common component and the idiosyncratic component:

$$e_{it} \sim N(0, 1) \quad \text{and} \quad r = \theta.$$

DGP2) Heteroskedastic idiosyncratic component, with the same variance for the common component and the idiosyncratic component:

$$e_{it} = \begin{cases} e_{it}^1 & \text{if } t \text{ odd} \\ e_{it}^1 + e_{it}^2 & \text{if } t \text{ even} \end{cases}, \quad e_{it}^1, e_{it}^2 \sim N(0, 1) \quad \text{and} \quad r = \theta.$$

DGP3) Homoskedastic idiosyncratic component, with the variance of the common component larger than the variance of the idiosyncratic component:

$$e_{it} \sim N(0, 1) \quad \text{and} \quad r = 2\theta.$$

DGP4) Homoskedastic idiosyncratic component, with the variance of the common component smaller than the variance of the idiosyncratic component:

$$e_{it} \sim N(0, 1) \quad \text{and} \quad r = \frac{\theta}{2}.$$

DGP5) Allow for small cross-section correlation across idiosyncratic parts, with the same variance for the common component and the idiosyncratic component:

$$e_{it} = v_{it} + \sum_{j \neq 0}^J \beta v_{i-jt}, \quad v_{it} \sim N(0, 1) \quad \text{and} \quad r = \theta.$$

DGP6) Allow for serial correlation across idiosyncratic parts, with the variance of the common component smaller than the variance of the idiosyncratic component:

$$e_{it} = \rho e_{it-1} + v_{it}, \quad e_{it} \sim N(0, 1), \quad v_{it} \sim N(0, 1), \quad r = \theta \quad \text{and} \quad r < \frac{\theta}{1 - \rho^2}.$$



DGP7) Allow for serial and small cross-section correlation across idiosyncratic parts, the variance of the common component is larger than the variance of the idiosyncratic component:

$$e_{it} = \rho e_{it-1} + v_{it} + \sum_{h \neq 0, h=-H}^H \beta v_{i-h,t}, \quad e_{it} \sim N(0, 1), \quad v_{it} \sim N(0, 1), \quad r = \theta \quad \text{and} \quad r < \frac{\theta}{1 - \rho^2}.$$

For each model we set  $r = 1, 3, 5, 8, 10, 15$  (compatibly with  $r < \min\{N, T\}$ ). The values of the parameters are chosen as in BN:  $\rho = 0.5$ ,  $\beta = 0.2$ , and  $H = \max\{N/20, 10\}$ . We generate samples with  $N, T = 40, 50, 60, 100, 200, 500$ . For each model and each (standardized) sample we implement the six criteria considered by BN and our six criteria when setting  $r_{\max} = 8, 15, 20$ ,  $n_J = \frac{3}{4}N$  and  $c_{\max} = 13$  with step size of 0.01. For each of the 12 criteria, we compute the average number of factors returned as a result over 500 Monte Carlo replications together with its standard deviation.

Table 1 shows the average result over the three  $PC$  criteria and the average result over the three  $PC^*$  criteria, as well as the average result over the three  $IC$  criteria and the average result over the three  $IC^*$  criteria, over the 86 generated samples. The same information is summarized in figure 2, where the simulated samples on the horizontal axis are ordered by increasing  $r$ .

The plots show that both the original and the modified criteria become in general less and less reliable once the true number of static factors exceeds 5. However, when the true number of factors is small, in five DGPs out of seven the  $PC^*$  criteria perform on average better than the  $PC$  criteria. Indeed, for a given  $r$ , the latter ones always give a very variable result which depend on the size of the sample, while this problem affects  $PC^*$  criteria only when small cross-section correlation across idiosyncratic parts is allowed for (DGP 5 and DGP 7). Another possible explanation for the lack of robustness of the  $PC$  criteria is the following: although both  $PC$  and  $IC$  criteria need a maximum number of factors  $r_{\max}$  as an input, only  $PC$  criteria explicitly take into account its resulting minimum squared distance. This makes the  $PC$  criteria less robust to the choice of the  $r_{\max}$ . In order to investigate whether  $PC^*$  criteria are also heavily influenced by the choice of the  $r_{\max}$ , in figure 3 we break down simulation results on the basis of the  $r_{\max}$ , taking DGP 1 as an example. An important dependence of the  $PC^*$  criteria on the  $r_{\max}$  seems not to be the case, at least when the true number of static factors is not large.

Gains over the  $IC$  criteria are not as striking as in the  $PC$  case. However, there is at least one case, DGP 6, in which the  $IC^*$  criteria on average dominate the  $IC$  criteria when the true number of static factors is up to 5. Moreover, as shown in table 2, for five DGPs the average RMSE difference between the  $IC$  and the  $IC^*$  criteria is positive, i.e. the  $IC$  RMSE is on average higher than the  $IC^*$  RMSE. These are precisely the more realistic DGPs, where either the variance of the common component is not larger than the variance of the idiosyncratic component or small cross-section correlation and/or serial correlation across idiosyncratic parts are allowed for.<sup>3</sup> Finally, since in principle the maximum number of static factors  $r_{\max}$  is equal to the smaller between  $N$  and  $T$ , we also run a set of simulations in order to check the robustness of the original and the modified criteria when this parameter is large. We have already shown by means of the first set of simulations that the original  $PC$  criteria are not robust in this sense, even when  $r_{\max}$  is relatively small (up to 15). In this second set of simulations, we generate the same samples with the same DGPs and parameters as above, but

<sup>3</sup> Matlab codes and disaggregated results are available at [http://www.barigozzi.eu/ABC\\_crit.zip](http://www.barigozzi.eu/ABC_crit.zip) and [http://www.barigozzi.eu/Tables\\_Refined\\_BaiNg\\_r5.pdf](http://www.barigozzi.eu/Tables_Refined_BaiNg_r5.pdf), respectively.

let  $r_{\max}$  vary according to the number of series in the panel. In particular, we set again the size of the smallest random subsample at  $n_J = \frac{3}{4}N$  and set  $r_{\max} = \min\{\frac{3}{4}N, T - 1\}$ . Thus, for example, for samples of size  $N = 500$  and  $T = 100$ ,  $r_{\max} = 99$ , while for samples of size  $N = T = 200$ ,  $r_{\max} = 112$ . Figure 4 reads in the same way as figure 2 and summarizes the results over 70 samples. It shows that when  $r_{\max}$  is allowed to take legitimately large values, also the original  $IC$  criteria show a serious dependence on this parameter, which is not the case for  $IC^*$  criteria. Basically, when  $r_{\max}$  is large, the  $\hat{r}_{c,N}^T$  estimated by the  $IC$  criteria is equal to  $r_{\max}$ , i.e. the  $IC$  criteria do not penalize at all the inclusion of additional factors. This happens when the number of series in the panel is very large ( $N = 200, 500$ ) or when the cross section dimension is not large enough with respect to the time dimension ( $N = 100, T = 40$ ). In these problematic samples, the  $IC^*$  are not able to deliver a consistent estimate either, since they often return  $\hat{r}_{c,N}^T = 1$ . In general, however, when  $r_{\max}$  takes large values the  $IC^*$  criteria seem to be more accurate if the true number of factors  $r$  is low.

## 5 Empirical applications

We test the performance of our criterion by means of two empirical applications on macroeconomic and financial datasets. In the first case we take a dataset which has been studied by Stock and Watson [2005] and used in many applications of factor models, including Hallin and Liška [2007]. This dataset (downloadable at <http://www.princeton.edu/~mwatson>) comprises 132 series of macroeconomic indicators of the US economy from January 1960 to December 2003 for a total of 528 time observations. In a second exercise we consider 89 daily asset returns from the London Stock Exchange, from 1<sup>st</sup> October 2001 to 31<sup>st</sup> July 2003 for a total of 456 time observations.

In table 3 we report results obtained for the macroeconomic application when using the  $PC$  and  $IC$  criteria by BN and when using the modified criteria  $PC^*$  and  $IC^*$  for  $r_{\max} = 10, 20, 30, 40, 50, 60, 100$ . As expected the  $PC$  criteria are not reliable while the  $IC$ s and our criteria are more robust with respect to  $r_{\max}$ . Indeed, our six criteria indicate the presence of 6 static factors in 40 out of 42 cases, while in two cases they point to 5 factors. As for the original criteria, only  $IC_1$  and  $IC_2$  are able to provide a robust result, i.e. 7 factors. However, for  $r_{\max} = 100$  the  $IC$  criteria lose their power and yield the maximum possible number of factors, while our criteria are still robust suggesting always 6 factors. In figure 5(a) we show the result for the  $IC_1^*$  criterion and  $r_{\max} = 30$ .

In table 4 we report the results for the financial dataset, with  $r_{\max}$  up to 80. Our six criteria indicate the presence of 6 factors in all 42 cases but 6, where they point to smaller numbers. Also in this application the  $PC$  criteria are dependent on  $r_{\max}$ , while the  $IC$ s always indicate 2 factors. However this result could underestimate the true number of factors, as shown in figure 5(b). This plot refers to the  $IC_1^*$  criterion with  $r_{\max} = 30$ : it is clear that the original  $IC_1$  criterion is identifying the largest plateau, while our criterion finds another smaller plateau corresponding to 6 factors and  $c = 0.28$ . Note that in a dynamic specification of the factor model we would expect the  $r$  static factors to explain the same amount of variance of the  $q$  static factors. Indeed, in Alessi et al. [2006] on this same financial dataset we apply the criterion by Hallin and Liška [2007] for determining  $q$  and find that between 5 and 6 static factors have to be included in order to explain the same percentage of variance that is explained by the two selected dynamic factors.

## 6 Conclusions

This paper proposes an information criterion for the determination of the number of static factors in approximate factor models. It refines the Bai and Ng [2002] (BN) criterion, which is one of the most popular criteria available for addressing this issue. The appeal of our criterion stands in the fact that it builds on a well known criterion, the theoretical properties of which have been proved, and inherits them all. In addition, our criterion improves the finite sample performance of the original criterion, being capable of giving an answer even when the BN criterion does not converge and yielding generally more robust results. Indeed, both criteria in their six formulations have been compared on the basis of a large number of simulations, whose results are encouraging.

The major gains from using the criteria we propose versus the criteria by BN concern samples driven by a relatively small number of static factors (less than 5), which is also the case in which the BN criteria performance is better if compared to their performance on samples driven by a large number of factors. In the case of *PC* criteria, the accuracy improvement is striking in five models out of seven, while in the case of *IC* criteria the advantages from our proposed criteria are more modest. However, with a second set of simulations, we show that when the maximum number of factors allowed in the estimation becomes very - but still legitimately - large, also the original *IC* criteria lose robustness, while the modified *IC* criteria we propose still perform quite well if the true number of factors is low. Finally, we carry out two empirical applications on macroeconomic and financial data comparing the results of the original and our refined criteria.

The potential applications of our criteria go beyond the estimation of the number of static factors. Indeed, in principle those estimators for the number of dynamic factors which implement the BN criterion for the number of static factors would work as well with the criterion we propose. For example, Amengual and Watson [2007] show that the criterion by BN is still consistent when applied to variables measured with error, provided that the (estimation) error is sufficiently small. Straightforwardly, this result holds for our criterion too.

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$r_{max}$	AVERAGE $PC$			AVERAGE $PC^*$			AVERAGE $IC$			AVERAGE $IC^*$		
	8	15	20	8	15	20	8	15	20	8	15	20
$r$	<b>DGP 1</b>											
1	1.41	3.58	6.59	1.04	1.04	1.04	1.00	1.00	1.00	1.00	1.00	1.00
3	3.20	4.77	7.29	3.03	3.03	3.03	3.00	3.00	3.00	3.01	3.00	3.00
5	5.06	6.21	8.35	5.02	5.02	5.02	4.96	4.96	4.96	5.01	5.00	4.99
8	7.63	8.65	10.28	5.21	5.23	5.23	7.10	7.10	7.10	3.43	3.42	3.42
10	NA	10.23	11.72	NA	9.08	9.07	NA	7.62	7.62	NA	8.53	8.44
15	NA	12.54	14.47	NA	5.84	5.85	NA	7.67	7.73	NA	4.50	4.52
$r$	<b>DGP 2</b>											
1	1.77	4.12	7.10	1.34	1.35	1.34	1.28	1.28	1.28	1.27	1.27	1.27
3	3.67	5.49	8.10	3.45	3.46	3.46	3.39	3.39	3.39	3.41	3.41	3.40
5	5.50	7.06	9.37	5.10	5.11	5.11	4.90	4.90	4.90	4.75	4.73	4.67
8	6.88	8.92	10.91	4.79	5.27	5.28	5.38	5.46	5.47	3.13	3.78	3.69
10	NA	9.30	11.47	NA	7.51	7.57	NA	4.60	4.63	NA	6.58	6.56
15	NA	9.95	12.91	NA	3.52	3.50	NA	3.92	4.03	NA	2.24	2.19
$r$	<b>DGP 3</b>											
1	1.69	4.55	8.11	1.04	1.04	1.04	1.05	1.10	1.27	1.01	1.00	1.00
3	3.36	5.44	8.32	3.03	3.03	3.03	3.04	3.05	3.05	3.02	3.02	3.01
5	5.15	6.57	8.97	5.02	5.02	5.02	5.03	5.03	5.03	5.04	5.03	5.02
8	7.98	8.81	10.60	5.33	5.38	5.38	7.97	7.99	8.00	2.46	2.59	2.56
10	NA	10.49	11.93	NA	9.80	9.80	NA	9.85	9.87	NA	9.73	9.61
15	NA	14.54	15.53	NA	7.39	7.43	NA	13.26	13.49	NA	5.25	5.28
$r$	<b>DGP 4</b>											
1	1.27	3.06	5.73	1.05	1.05	1.04	1.00	1.00	1.00	1.00	1.00	1.00
3	3.12	4.46	6.75	3.04	3.03	3.03	2.95	2.95	2.95	3.00	3.00	2.99
5	4.85	6.04	8.01	4.98	4.98	4.97	4.13	4.13	4.13	4.87	4.82	4.74
8	5.89	8.05	10.02	4.63	4.66	4.69	3.76	3.77	3.77	3.26	3.22	3.21
10	NA	8.54	10.93	NA	7.13	7.16	NA	3.00	3.00	NA	6.23	6.06
15	NA	8.59	11.31	NA	3.46	3.40	NA	2.74	2.74	NA	2.39	2.33
$r$	<b>DGP 5</b>											
1	7.55	12.34	15.76	5.64	8.61	9.03	7.42	10.45	11.43	4.40	8.99	9.90
3	7.93	12.98	16.45	5.56	9.75	10.75	7.91	11.84	13.18	3.40	9.71	11.87
5	8.00	13.59	17.22	4.91	10.37	12.22	8.00	12.94	14.79	1.76	8.31	12.98
8	8.00	14.54	18.20	4.51	7.88	9.35	7.98	14.16	16.54	1.80	5.12	8.51
10	NA	14.83	18.73	NA	6.29	8.40	NA	14.15	16.93	NA	3.01	7.01
15	NA	14.76	19.57	NA	6.37	6.58	NA	12.57	16.06	NA	3.20	3.40
$r$	<b>DGP 6</b>											
1	3.40	7.91	12.01	1.58	1.63	1.65	1.85	2.54	3.20	1.08	1.09	1.09
3	4.48	8.57	12.35	3.38	3.44	3.45	3.67	4.37	4.96	3.09	3.09	3.08
5	5.73	9.34	12.90	5.17	5.27	5.27	5.33	6.12	6.77	4.88	4.89	4.85
8	7.46	10.65	13.92	4.90	5.23	5.22	6.47	7.46	8.18	2.96	3.11	3.08
10	NA	11.49	14.64	NA	7.54	7.58	NA	7.26	8.03	NA	5.97	5.90
15	NA	12.56	16.04	NA	4.30	4.31	NA	6.27	7.02	NA	2.03	2.06
$r$	<b>DGP 7</b>											
1	7.71	13.39	17.54	5.80	8.52	8.82	7.54	11.29	12.86	4.58	8.57	9.32
3	7.97	13.75	17.97	5.54	9.67	10.49	7.94	12.40	14.46	3.12	9.03	10.95
5	8.00	14.13	18.38	4.93	9.81	11.69	8.00	13.34	15.85	1.96	7.46	11.71
8	8.00	14.72	18.90	4.81	7.36	8.07	7.98	14.32	17.28	2.33	4.38	6.25
10	NA	14.86	19.19	NA	6.51	7.32	NA	14.21	17.49	NA	3.50	5.55
15	NA	14.79	19.67	NA	6.95	7.06	NA	13.03	16.70	NA	4.40	4.51

Table 1: Average estimated number of factors for  $PC$  and  $PC^*$  criteria and for  $IC$  and  $IC^*$  criteria.

DGP	RMSE $PC_1$	RMSE $PC_2$	RMSE $PC_3$	RMSE $IC_1$	RMSE $IC_2$	RMSE $IC_3$	Average RMSE for $PC$	Average RMSE for $IC$
1	-0.34	<b>0.00</b>	<b>1.86</b>	-0.86	-0.66	-0.67	<b>0.27</b>	-0.67
2	-0.10	<b>0.24</b>	<b>2.04</b>	-0.35	<b>0.13</b>	<b>0.09</b>	<b>0.43</b>	<b>0.09</b>
3	-0.23	<b>0.48</b>	<b>2.76</b>	-1.34	-1.85	-1.81	<b>0.80</b>	-1.85
4	-0.08	-0.09	<b>1.24</b>	<b>0.21</b>	<b>0.77</b>	<b>0.71</b>	<b>0.09</b>	<b>0.75</b>
5	<b>0.53</b>	<b>1.43</b>	<b>4.58</b>	-0.01	<b>0.08</b>	<b>1.73</b>	<b>2.01</b>	<b>0.29</b>
6	<b>0.58</b>	<b>2.00</b>	<b>5.11</b>	-0.02	<b>0.52</b>	<b>1.39</b>	<b>2.23</b>	<b>0.56</b>
7	<b>0.63</b>	<b>2.15</b>	<b>5.93</b>	<b>0.17</b>	<b>0.94</b>	<b>3.01</b>	<b>2.78</b>	<b>1.09</b>

Table 2: RMSE differences for the number of static factors, computed as the value by BN minus the value obtained with our refined criterion.

$r_{\max}$	$PC_1$	$PC_1^*$	$PC_2$	$PC_2^*$	$PC_3$	$PC_3^*$	$IC_1$	$IC_1^*$	$IC_2$	$IC_2^*$	$IC_3$	$IC_3^*$
<b>10</b>	9	6	8	6	10	6	7	6	7	6	10	6
<b>20</b>	16	6	15	6	20	6	7	6	7	6	20	6
<b>30</b>	25	6	23	6	30	6	7	6	7	6	30	6
<b>40</b>	34	5	33	6	40	6	7	5	7	6	40	6
<b>50</b>	48	6	47	6	50	6	7	6	7	6	50	6
<b>60</b>	60	6	59	6	60	6	7	6	7	6	60	6
<b>100</b>	100	6	100	6	100	6	100	6	100	6	100	6

Table 3: Estimated number of factors for the macroeconomic application.

$r_{\max}$	$PC_1$	$PC_1^*$	$PC_2$	$PC_2^*$	$PC_3$	$PC_3^*$	$IC_1$	$IC_1^*$	$IC_2$	$IC_2^*$	$IC_3$	$IC_3^*$
<b>10</b>	2	4	2	3	2	6	2	4	2	3	2	6
<b>20</b>	2	6	2	6	4	6	2	6	2	6	2	6
<b>30</b>	7	6	6	6	11	6	2	6	2	6	2	6
<b>40</b>	19	6	16	6	24	6	2	6	2	6	2	6
<b>50</b>	35	4	33	6	42	6	2	4	2	6	2	6
<b>60</b>	56	6	53	6	60	6	2	6	2	6	2	6
<b>80</b>	80	6	80	6	80	6	2	6	2	6	2	6

Table 4: Estimated number of factors for the financial application.

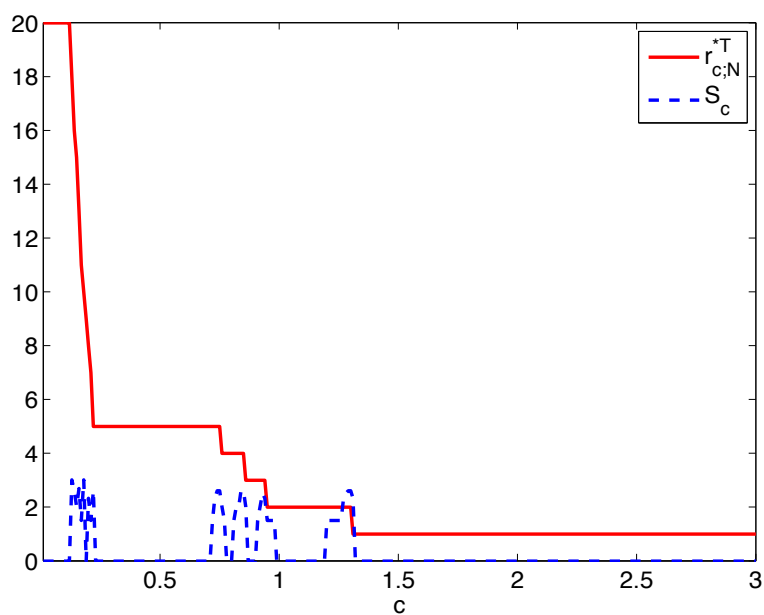
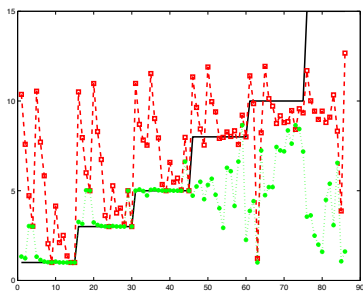
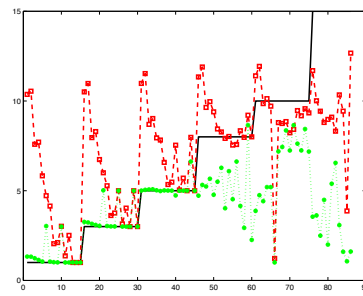


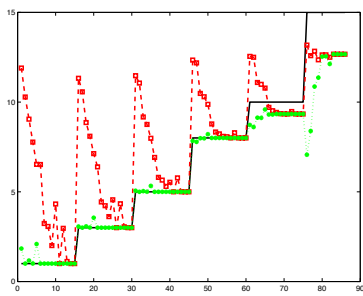
Figure 1: An example of the application of  $IC_2^*$  criterion (true number of factors:  $r = 5$ ).



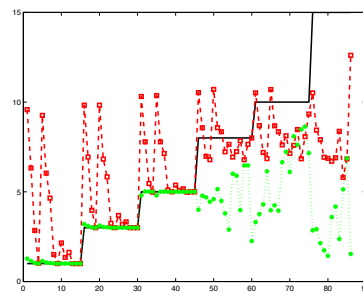
(a) DGP1



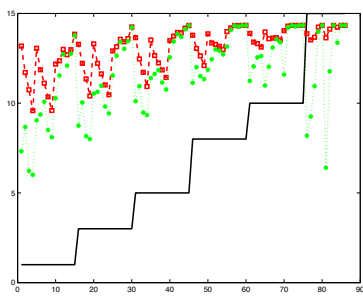
(b) DGP2



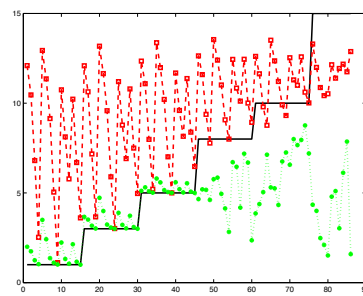
(c) DGP3



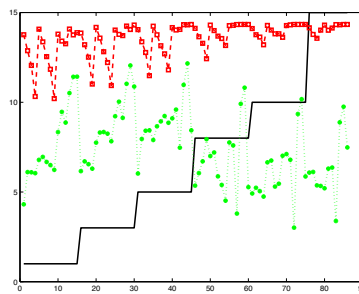
(d) DGP4



(e) DGP5

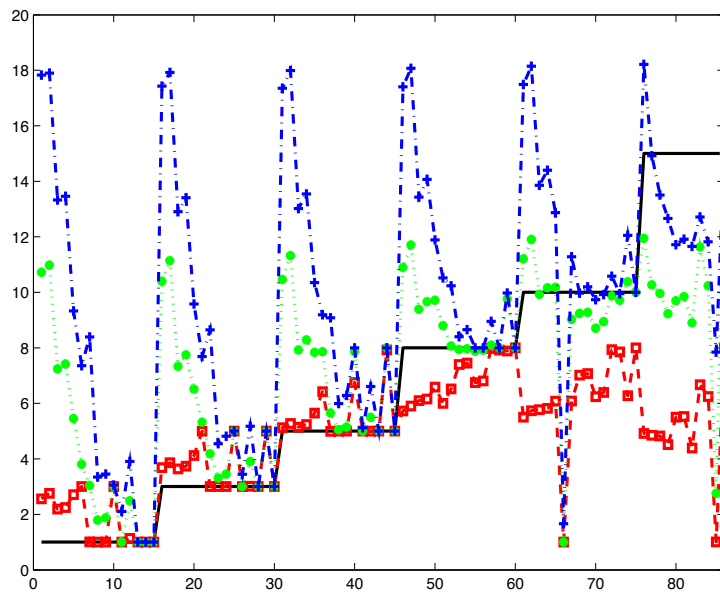


(f) DGP6

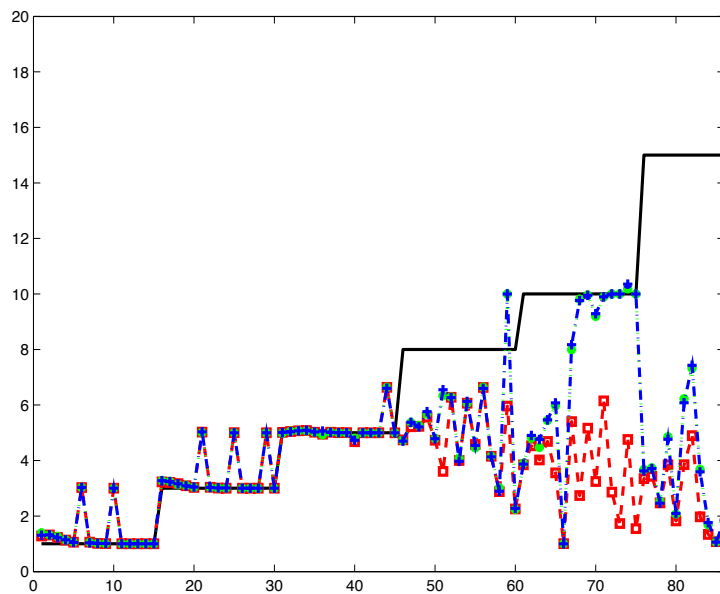


(g) DGP7

Figure 2:  $PC$  and  $PC^*$  criteria: average estimated number of factors when  $r_{\max} = 8, 15, 20$ . Horizontal axis: 86 generated samples ordered by increasing  $r$ . Solid line:  $r$ , dashed line:  $\hat{r}_{c,N}^T$  estimated by  $PC$ , dotted line:  $\hat{r}_{c,N}^T$  estimated by  $PC^*$ .

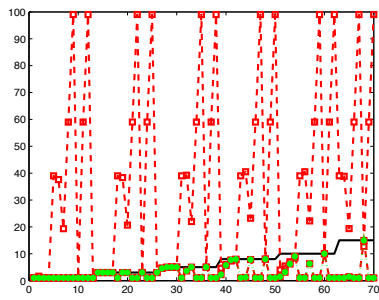


(a) DGP1  $PC$  criteria

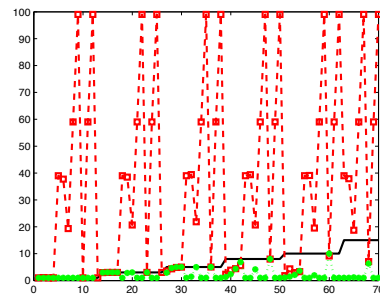


(b) DGP1  $PC^*$  criteria

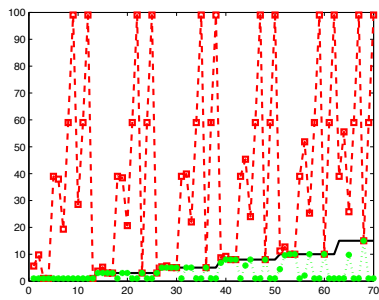
Figure 3: Average estimated number of factors. Horizontal axis: 86 generated samples ordered by increasing  $r$ . Solid line:  $r$ , dashed line:  $\hat{r}_{c,N}^T$  when  $r_{\max} = 8$ , dotted line:  $\hat{r}_{c,N}^T$  when  $r_{\max} = 15$ , dashed-dotted line:  $\hat{r}_{c,N}^T$  when  $r_{\max} = 20$ .



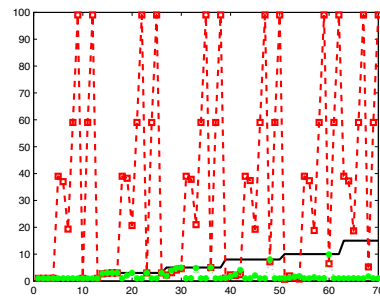
(a) DGP1



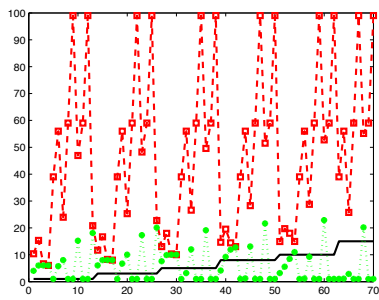
(b) DGP2



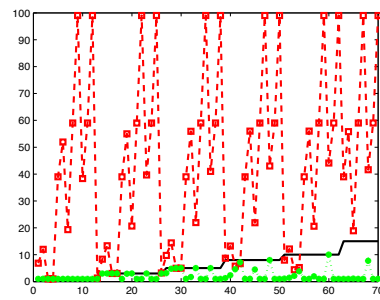
(c) DGP3



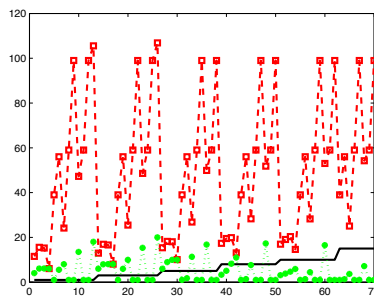
(d) DGP4



(e) DGP5

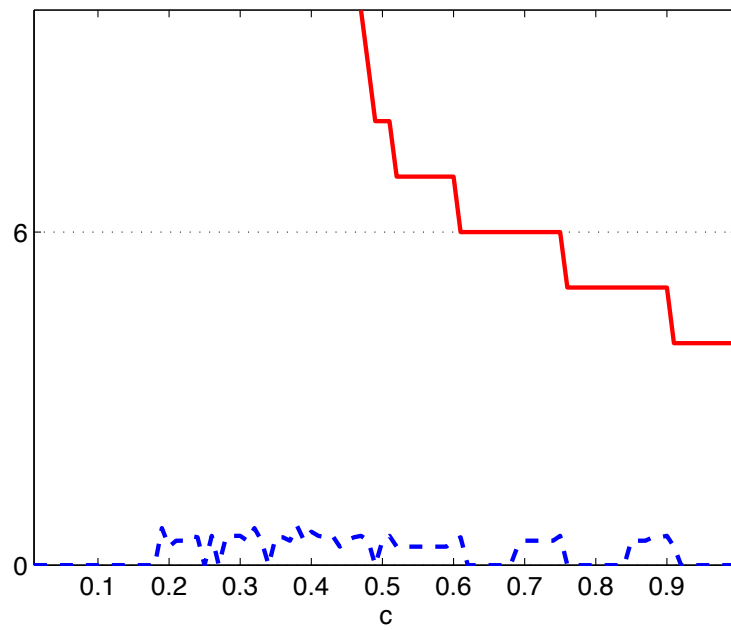


(f) DGP6

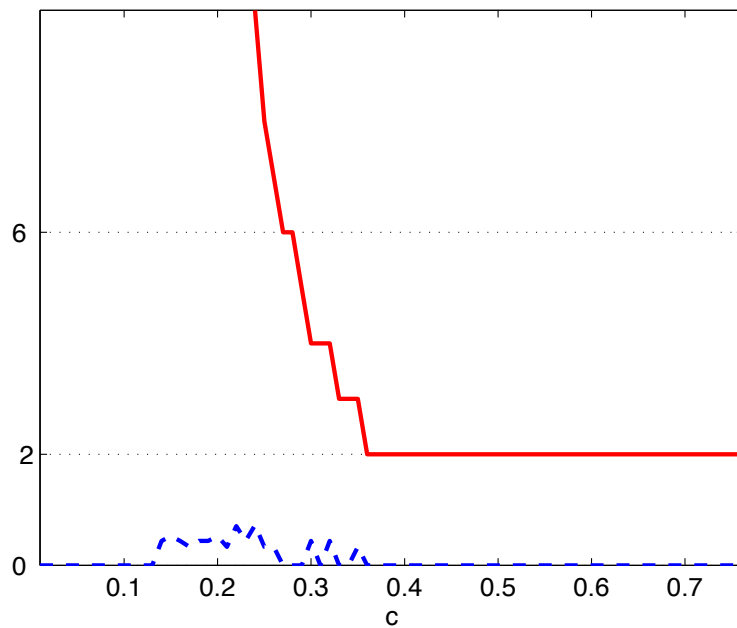


(g) DGP7

Figure 4:  $IC$  and  $IC^*$  criteria: average estimated number of factors when  $r_{\max}$  is as large as possible. Horizontal axis: 70 generated samples ordered by increasing  $r$ . Solid line:  $r$ , dashed line:  $\hat{r}_{c,N}^T$  estimated by  $IC$ , dotted line:  $\hat{r}_{c,N}^T$  estimated by  $IC^*$ .



(a) Macroeconomic case



(b) Financial case

Figure 5:  $IC_1^*$  criteria in the two empirical cases.

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